Diquark Feynman Propagator in Mixed Nuclear-Quark Matter
Nucleon-Quark Diquark-exchange Interaction

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In this note the Feynman-propagator for axial-vector diquark (D) exchange is derived. The result is a (-)-sign w.r.t. quark-antiquark exchange. This difference is due to a (-)-sign for a closed fermion loop, which is present for *e.g* vector and axial-vector exchange in the quark-model, but which is absent in the case of D-exchange. The calculations in these notes follow closely that for vacuum polarization in the literature. Taking into account that the diquark D is a color $\{\bar{3}\}_c$-state gives a factor +2. The result is an effective diquark propagator in momentum space

$$i(\widetilde{\Delta})^{ab}_{\mu\nu}(k) \;=\; +2i\delta_{ab}\frac{\left(\eta_{\mu\nu}-k_\mu k_\nu/m_\chi^2\right)}{k^2-m_\chi^2+i\epsilon}.$$

Application to $QN \to NQ$ gives a repulsive potential for the axial-vector $\gamma_5\gamma_\mu$ NDQ-coupling, which can be used in mixed nuclear-quark matter calculations.

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I. INTRODUCTION

In these notes we derive the diquark (D) propagator in the context of the standard quantum field theory (QFT). The axial-vector diquark field originates from presentation of the proton current $\eta^{(1)}(x)$ in terms of three quarks [1, 2]. It is pointed out that there is an important difference between quark-quark objects and mesons, which are quark-antiquark states in the quark-model. This shows up in the Wick-theorem giving a sign difference. Application to the process $N+Q \to Q+N$ for diquark-exchange leads to a (universal) axial-vector repulsive interaction.
The content of these notes is as follows. In section II the diquark-field and the diquark propagator are defined. The latter using the standard formalism in field theory [3–6]. In section III the diquark propagator is given using its composite quark-quark structure, exploiting the Wick-theorem. Here, we note the important difference with for example ρ and A_1 exchange within the quark-model. In section IV the calculation in the momentum space is carried out, using the Pauli-Villars regularization method [8]. We follow the treatment in [3] for the vacuum polarization and renormalize the diquark coupling. This results in a dispersion presentation of the diquark propagator. In section V for $NQ \to QN$ the diquark exchange is worked out. In section VI we give a short discussion and conclusion.
In Appendix A the Wick-expansion in relation with the Feynman propagator is reviewed in some detail. In Appendix B details for meson-exchange are given, showing the important sign difference with di-quark exchange. In Appendix C the potential for a scalar-diquark and pseudoscalar-diquark exchange are derived. They arise from a second proton three-quark current $\eta^{(2)}(x)$ discussed in [2].

II. QFT SECOND-QUANTIZATION FORMALISM

The triquark representation of the nucleon current in the context of the QCD sum rules has been introduced in [1]

$$\eta_N^{(1)}(x) = \left[\widetilde{q}^a(x)C\gamma^\mu q^b(x)\right]\gamma_5\gamma_\mu q^c(x)\epsilon^{abc}, \tag{2.1}$$

where C is the charge-conjugation matrix in Dirac-spinor space, and a,b,c denote the color indices. The composite diquark (D) field $\chi^a_\mu(x)$ is introduced in [7] using the current by writing

$$\eta_N^{(1)}(x) \;=\; (\hbar c)^2\gamma_5\gamma^\mu q^a(x)\cdot\chi^a_\mu(x), \quad \chi^a_\mu(x) \equiv \epsilon^{abc}\widetilde{q}^b(x)C\gamma_\mu q^c(x)/(\hbar c)^2. \tag{2.2}$$

The diquark D-field Feynman propagator is

$$i(\Delta_F)^{ab}_{\mu\nu}(x'-x) \;=\; (0|T\left[\chi^a_\mu(x')\chi^{b\dagger}_\nu(x)\right]|0) \tag{2.3}$$

where the diquark fields are [1]

$$\chi^a_\mu(x) \;=\; \widetilde{q}^c(x) C\gamma_\mu q^d(x)\ \varepsilon^{acd}\ ,\ \chi^{b\dagger}_\mu(x) = -\bar{q}^e(x)\gamma_\mu C\ \widetilde{\bar{q}}^f(x)\ \varepsilon^{bef} \tag{2.4}$$

The Dirac indices are contracted, and C is the charge conjugation Dirac matrix which satisfies $C^{-1}\gamma_\mu C = -\gamma_\mu^T$ [3]. The Dirac equation reads $\gamma^\mu\partial_\mu q(x) = -imq(x)$ and for the transposed $\partial^\mu \widetilde{q}(x)\widetilde{\gamma}_\mu = -im\widetilde{q}(x)$.

$$\partial^\mu \chi^a_\mu(x) \;=\; \left[\left(\partial^\mu \widetilde{q}^c(x)\right) C\gamma_\mu q^d(x) + \widetilde{q}^c C\gamma_\mu \partial^\mu q^d(x)\right]\ \varepsilon^{acd} = 0, \tag{2.5}$$

as follows from $\partial^\mu \widetilde{q}(x) C\gamma_\mu = +im\ \widetilde{q}(x)C$, due to the properties of the charge conjugation matrix C [3]. It follows that

$$\partial^\mu (\Delta_F)^{ab}_{\mu\nu}(x-x') \;=\; (0|\left[\chi^a_0(x), \chi^{b\dagger}_\nu(x')\right]\big|_{x_0=x'_0}|0) = 0 \tag{2.6}$$

since this is $\propto \Delta(x-x')\big|_{x_0=x'_0} = 0$.
Suppressing the color indices, the plane wave expansion of the quark field, see *e.g.* [3], reads

$$q(x) \;=\; \sum_s \int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_p}}\left[b(p,s)\ u(p,s)\ e^{-ip.x} + d^\dagger(p,s)\ v(p,s)\ e^{ip.x}\right], \tag{2.7}$$

$$\bar{q}(x) \;=\; \sum_s \int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_p}}\left[b^\dagger(p,s)\ \bar{u}(p,s)\ e^{ip.x} + d(p,s)\ \bar{v}(p,s)\ e^{-ip.x}\right], \tag{2.8}$$

$$\tag{2.9}$$

It is convenient to introduce the positive and negative frequency parts

$$q^{(+)}(x) \;=\; \sum_s \int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_p}}\left[b(p,s)\ u(p,s)\ e^{-ip.x}\right], \tag{2.10}$$

$$q^{(-)}(x) \;=\; \sum_s \int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_p}}\left[d^\dagger(p,s)\ v(p,s)\ e^{ip.x}\right], \tag{2.11}$$

which are the annihilation and creation operators for a quark and a anti-quark respectively. Similarly

$$\bar{q}^{(-)}(x) \;=\; \sum_s \int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_p}}\left[b^\dagger(p,s)\ \bar{u}(p,s)\ e^{ip.x}\right], \tag{2.12}$$

$$\bar{q}^{(+)}(x) \;=\; \sum_s \int \frac{d^3p}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_p}}\left[d(p,s)\ \bar{v}(p,s)\ e^{-ip.x}\right], \tag{2.13}$$

which are the creation and annihilation operators for a quark and a anti-quark respectively. The vacuum is defined as

$$q^{(+)}(x)|0) = \bar{q}^{(+)}(x)|0) = 0. \tag{2.14}$$

The annihilation and creation operators $b(p,s)$ and $d^\dagger(p,s)$ satisfy the anti-commutation relations unequal to zero are

$$\left\{b(p,s), b^\dagger(p',s')\right\} \;=\; \delta_{ss'}\delta^3(\mathbf{p}-\mathbf{p}'), \tag{2.15a}$$

$$\left\{d(p,s), d^\dagger(p',s')\right\} \;=\; \delta_{ss'}\delta^3(\mathbf{p}-\mathbf{p}'), \tag{2.15b}$$

The anti-commutator is [3]

$$\left\{q^a_\alpha(x), \bar{q}^b_\beta(x')\right\} = -i\delta_{a,b}S(x-x')_{\alpha\beta}, \tag{2.16a}$$

$$S_{\alpha\beta}(z) = \left(i\gamma^\mu\partial_\mu + m_Q\right)\ \Delta(z, m_Q^2). \tag{2.16b}$$

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[1] Note that because $\gamma_0 C\gamma_0 = -C$ the diquark field is an axial-vector field. Here and in most of the following we use units $\hbar = 1, c = 1$.

and the vacuum expectation of T-product is related to the quark Feynman-propagator, see [3] section 13.6,

$$(0|T\left[q^a_\alpha(x)\bar{q}^b_\beta(x')\right]|0) = -i\delta_{ab}S_F(x-x')_{\alpha\beta}, \tag{2.17a}$$
$$S_{F\alpha\beta}(z) = (i\gamma^\mu\partial_\mu + m_Q)\ \Delta_F(z,m_Q^2). \tag{2.17b}$$

For the invariant function $S_F(x-x')$ see [3, 5]. The hermitian conjugate of $\chi^a_\mu(x) = \widetilde{q}^c(x)C\gamma_\mu q^d(x)\ \varepsilon^{acd}$ is

$$\chi^{a\dagger}_\mu(x) \ = \ \widetilde{q}^{d\dagger}(x)\,(C\gamma_\mu)^\dagger\, q^{c\dagger}(x)\ \varepsilon^{acd} = \widetilde{\bar{q}}^d(x)\gamma_\mu C\ \bar{q}^c\ \varepsilon^{acd}, \tag{2.18a}$$

which will be used extensively in the rest of these notes.

III. DIQUARK PROPAGATOR

From the detailed derivation of the Wick-expansion in Appendix A for the axial-vector diquark propagator we have

$$\left(0|T\left[\chi^a_\mu(x')\chi^{b\dagger}_\nu(x)\right]|0\right) \Rightarrow \quad X_{\mu\nu} = -\varepsilon^{acd}\varepsilon^{bef}\Bigg[(0|T\left[q^d_\beta(x')\bar{q}^e_\kappa(x)\right]|0)(0|T\left[q^c_\alpha(x')\bar{q}^f_\lambda(x)\right]|0)$$
$$-(0|T\left[q^c_\alpha(x')\bar{q}^e_\kappa(x)\right]|0)(0|T\left[q^d_\beta(x')\bar{q}^f_\lambda(x)\right]|0)\Bigg]O^{\alpha\beta}_\mu O'^{\kappa\lambda}_\nu \tag{3.1}$$

with

$$O^{\alpha\beta}_\mu = (C\gamma_\mu)^{\alpha\beta}\ ,\ O'^{\kappa\lambda}_\nu = (\gamma_\nu C)^{\kappa\lambda}$$

The color factors in (3.1) are

$$-\varepsilon^{acd}\varepsilon^{bef}\delta_{de}\delta_{cf} = +2\delta_{ab}, \quad \text{and} \quad -\varepsilon^{acd}\varepsilon^{bef}\delta_{ce}\delta_{df} = -2\delta_{ab}. \tag{3.2}$$

which give

$$X_{\mu\nu} = +2\delta_{ab}\Bigg[(0|T\left[q_\beta(x')\bar{q}_\kappa(x)\right]|0)(0|T\left[q_\alpha(x')\bar{q}_\lambda(x)\right]|0)$$
$$+(0|T\left[q_\alpha(x')\bar{q}_\kappa(x)\right]|0)(0|T\left[q_\beta(x')\bar{q}_\lambda(x)\right]|0)\Bigg]O^{\alpha\beta}_\mu O'^{\kappa\lambda}_\nu \tag{3.3}$$

where $(0|....|0)$ is diagonal in color, and a single component has to be used. Using the Feynman propagators [3] we obtain

$$(0|T\left[q^d_\beta(x')\bar{q}^e_\kappa(x)\right]|0)(0|T\left[q^c_\alpha(x')\bar{q}^f_\lambda(x)\right]|0) = -\delta_{de}\delta_{cf}S_{F\beta\kappa}(x'-x)\ S_{F\alpha\lambda}(x'-x), \tag{3.4a}$$
$$(0|T\left[q^c_\alpha(x')\bar{q}^e_\kappa(x)\right]|0)(0|T\left[q^d_\beta(x')\bar{q}^f_\lambda(x)\right]|0) = -\delta_{ce}\delta_{df}S_{F\alpha\kappa}(x'-x)\ S_{F\beta\lambda}(x'-x) \tag{3.4b}$$

The result for $X_{\mu\nu}$ is [2)]

$$X_{\mu\nu} \ = \ -2\delta_{ab}\Big[S_{F\beta\kappa}(x'-x)\ S_{F\alpha\lambda}(x'-x) + S_{F\alpha\kappa}(x'-x)\ S_{F\beta\lambda}(x'-x)\Big]\ (C\gamma_\mu)_{\alpha\beta}(\gamma_\nu C)_{\kappa\lambda}$$
$$= \ -2\delta_{ab}\ Tr\left[\gamma_\mu S_F\gamma_\nu(C\widetilde{S}_F C) + S_F\gamma_\nu(C\widetilde{S}_F C)\ \gamma_\mu\right] = -4\delta_{ab}Tr\left[\gamma_\mu S_F\gamma_\nu(C\widetilde{S}_F C)\right] \tag{3.5}$$

where in last lines we used the shorthand notation $S_{F\alpha\beta} \equiv S_{F\alpha\beta}(x'-x)$. Now,

$$C\widetilde{S}_F C = C\,(i\widetilde{\gamma}\cdot\partial + m_Q)\ C = -\,(i\gamma\cdot\partial - m_Q)\,,$$

giving

$$X_{\mu\nu} \ = \ 4\delta_{ab}Tr\left[\gamma_\mu\,(i\gamma\cdot\partial^y + m_Q)\,\gamma_\nu\,(i\gamma\cdot\partial^z - m_Q)\right]\ \Delta_F(y)\cdot\Delta_F(z)$$
$$= \ 4\delta_{ab}\,\mathrm{Tr}\left[-\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta\partial^y_\alpha\partial^z_\beta - \gamma_\mu\gamma_\nu m_Q^2\right]\ \Delta_F(y)\cdot\Delta_F(z) \tag{3.6}$$

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[2] **Note: the factor 2 due to color!**

Using
$$\mathrm{Tr}\left[\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta\right] = 4\left(\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}\right)$$
we obtain
$$X_{\mu\nu} = -16\delta_{ab}\left[\left(\partial^y_\mu\partial^z_\nu + \partial^y_\nu\partial^z_\mu\right) - \partial^y_\alpha\partial^\alpha_z\eta_{\mu\nu} + \eta_{\mu\nu}m_Q^2\right]\ \Delta_F(y)\cdot\Delta_F(z). \tag{3.7}$$
We introduced temporarily the differentiation variables y, z, which in the end will be put to $y = z = x' - x$.

Remark: For $q\bar{q}$-exchange the Wick-theorem gives a (-)-sign similar to that for a closed fermion-loop.

The diquark field Feynman propagator is [3]
$$\begin{aligned} i(\Delta_F)^{ab}_{\mu\nu}(x'-x) &= (0|T\left[\chi^a_\mu(x')\chi^{b\dagger}_\nu(x)\right]|0) = -16\delta_{ab}\Big\{\Big[\left(\partial_\mu\Delta_F(y)\cdot\partial_\nu\Delta_F(z) + \partial_\nu\Delta_F(y)\cdot\partial_\mu\Delta_F(z)\right) \\ &\quad -\partial_\alpha\Delta_F(y)\cdot\partial^\alpha\Delta_F(z)\eta_{\mu\nu} + \eta_{\mu\nu}m_Q^2\Delta_F(y)\cdot\Delta_F(z)\Big]\Big\}_{y=z=x'-x} \end{aligned} \tag{3.8}$$

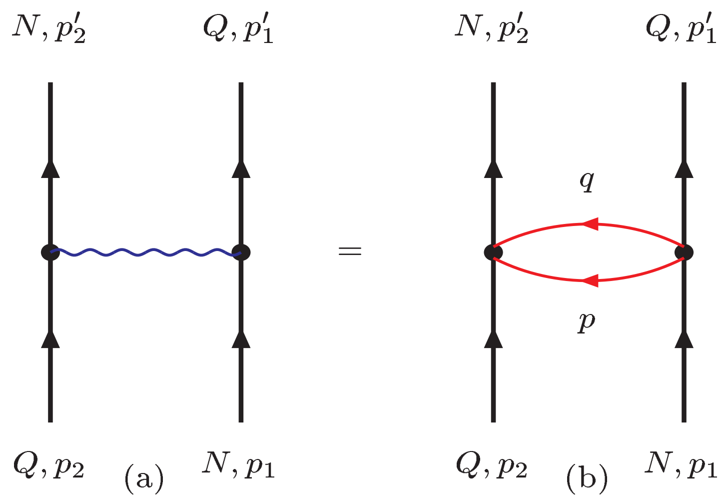

FIG. 1: Diquark-exchange NQ → QN. Panel (a): χ^a_μ-exchange. Panel (b): D-exchange.

IV. PROPAGATOR DISPERSION RELATION

The spectral representation of the Diquark Feynman propagator is
$$i(\Delta_F)^{ab}_{\mu\nu}(x'-x) = (0|T\left[\chi^a_\mu(x')\chi^{b\dagger}_\nu(x)\right]|0) = i\int_{s_0}^\infty ds\ (\Delta_F)^{ab}_{\mu\nu}(x'-x;s)\ \rho(s). \tag{4.1}$$
In Fig. 1 the momenta of the nucleons and quarks are shown in panel (b). Then, the diquark-exchange propagator in momentum space involves the integral
$$\begin{aligned} \widetilde{I}_{\mu\nu}(k;m) &= \int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\ (2\pi)^4\delta^4(p+q-k)\left[-\left(p_\mu q_\nu + p_\nu q_\mu\right)\right. \\ &\quad \left.+\left(p\cdot q + m^2\right)\eta_{\mu\nu}\right]\ \times\left[p^2 - m^2 + i\epsilon\right]^{-1}\left[q^2 - m^2 + i\epsilon\right]^{-1}. \end{aligned} \tag{4.2}$$

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[3] We follow the conventions of [3], Appendix B and C in part II, in definition functions $\Delta_F(x)$ etc and Feynman rules.

This integral has been evaluated in [3] exployting the Pauli-Villars regularization [8], which means

$$\widetilde{I}_{\mu\nu}(k;m) \;\to\; \widehat{I}(k) = \widetilde{I}_{\mu\nu}(k;m) + \sum_i C_i(M_i^2)\; \widetilde{I}_{\mu\nu}(k;M_i^2) \equiv \sum_i c_i\; \widetilde{I}_{\mu\nu}(k;m_i^2), \tag{4.3}$$

where the M_i are large masses and the C_i are chosen such that the integrals converge. Using the Schwinger parameterization

$$\left[p^2 - m^2 + i\epsilon\right]^{-1} \;=\; -i \int_0^\infty dz\; \exp\left[iz\left(p^2 - m^2 + i\epsilon\right)\right] \tag{4.4}$$

the integral (4.2) takes the form

$$\begin{aligned} \widetilde{I}_{\mu\nu}(k) \;=\; &+\int_0^\infty dz_1 \int_0^\infty dz_2 \int \frac{d^4p}{(2\pi)^4} \cdot \\ &\times \left[p_\mu(k-p)_\nu + p_\nu(k-p)_\mu - \left(p\cdot(k-p) + m^2\right) - i\epsilon)\;\eta_{\mu\nu}\right] \cdot \\ &\times \exp\left[iz_1\left(p^2 - m^2 + i\epsilon\right) + iz_2\left((k-p)^2 - m^2 + i\epsilon\right)\right] \end{aligned} \tag{4.5}$$

Making the standard shift

$$p_\mu \to l_\mu = p_\mu - \frac{z_2}{z_1+z_2} k_\mu = (p-k)_\mu + \frac{z_1}{z_1+z_2}\; k_\mu, \tag{4.6}$$

the denominator has l^2 and no linear l_μ term, which enables the integrals

$$\int \frac{d^4p}{(2\pi)^4}\left[1, l_\mu, l_\mu l_\nu\right]\; \exp\left[il^2(z_1+z_2)\right] = \frac{1}{16\pi^2 i}\frac{1}{(z_1+z_2)^2}\left[1, 0, \frac{i\eta_{\mu\nu}}{2(z_1+z_2)}\right] \tag{4.7}$$

leading to

$$\begin{aligned} \widetilde{I}_{\mu\nu}(k) = &-i\sum_i \frac{c_i}{4\pi^2}\int_0^\infty dz_1 \int_0^\infty \frac{dz_2}{(z_1+z_2)^2} \cdot \\ &\times \left(\exp\left\{i\left[k^2\frac{z_1z_2}{z_1+z_2} - (m^2 - i\epsilon)(z_1+z_2)\right]\right\}\right) \cdot \\ &\times \left\{2\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right)\frac{z_1z_2}{(z_1+z_2)^2} + \eta_{\mu\nu}\left[\frac{-i}{(z_1+z_2)} - \frac{k^2 z_1 z_2}{(z_1+z_2)^2} + m_i^2\right]\right\}. \end{aligned} \tag{4.8}$$

It appears that the $\eta_{\mu\nu}[\cdots]$-term vanishes, see [3], and so

$$\begin{aligned} \widetilde{I}_{\mu\nu}(k) = &-2i\sum_i \frac{c_i}{4\pi^2}\int_0^\infty \int_0^\infty \frac{dz_1 dz_2}{(z_1+z_2)^2}\frac{z_1z_2}{(z_1+z_2)^2} \quad \cdot \\ &\times \left(\exp\left\{i\left[k^2\frac{z_1z_2}{z_1+z_2} - (m_i^2 - i\epsilon)(z_1+z_2)\right]\right\}\right)\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right) \end{aligned} \tag{4.9}$$

Using the identity

$$1 \;=\; \int_0^\infty \frac{d\lambda}{\lambda}\delta\left(1 - \frac{z_1+z_2}{\lambda}\right) \tag{4.10}$$

the remaining contribution to $\widehat{I}_{\mu\nu}(k)$ becomes

$$\begin{aligned} \widehat{I}_{\mu\nu}(k) \;=\; &-\frac{2i}{4\pi^2}\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right)\int_0^\infty \int_0^\infty dz_1 dz_2 z_1 z_2 \delta(1 - z_1 - z_2)\int_0^\infty \frac{d\lambda}{\lambda} \cdot \\ &\times \sum_i c_i \exp\left[i\lambda\left(k^2 z_1 z_2 - m_i^2 + i\epsilon\right)\right] \end{aligned} \tag{4.11}$$

The λ-integral diverges logarithmically and is evaluated by applying the cut-off procedure by choosing $C_1 = -1, C_i = 0$ $(i > 1)$. This gives

$$\begin{aligned} \widehat{I}_{\mu\nu}(k) &= \widetilde{I}_{\mu\nu}(k;m^2) - \widetilde{I}_{\mu\nu}(k;M^2) \\ &\approx -\frac{2i}{4\pi^2}\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right)\int_0^1 dz\ z(1-z)\ \ln\left[\frac{M^2}{m^2 - z(1-z)\ k^2}\right] \\ &= -\frac{i}{12\pi^2}\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right) \times \left[\ln\left(\frac{M^2}{m^2}\right) - 6\int_0^1 dz\ z(1-z)\ \ln\left(1 - z(1-z)\ \frac{k^2}{m^2}\right)\right]. \end{aligned} \tag{4.12}$$

We write $\widehat{I}_{\mu\nu}(k) \equiv -i\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right)\ \Pi_2(k^2)$. The (unrenormalized) diquark propagator becomes

$$i\widetilde{\Delta}^{(0)}_{\mu\nu}(k) = -\frac{i}{12\pi^2}\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right) \times \left[\ln\left(\frac{M^2}{m^2}\right) - 6\int_0^1 dz\ z(1-z)\ \ln\left(1 - z(1-z)\ \frac{k^2}{m^2}\right)\right]/(\hbar c)^4 \tag{4.13a}$$

$$\equiv -i\left(\eta_{\mu\nu}k^2 - k_\mu k_\nu\right)\ \Pi_2(k^2)/(\hbar c)^4. \tag{4.13b}$$

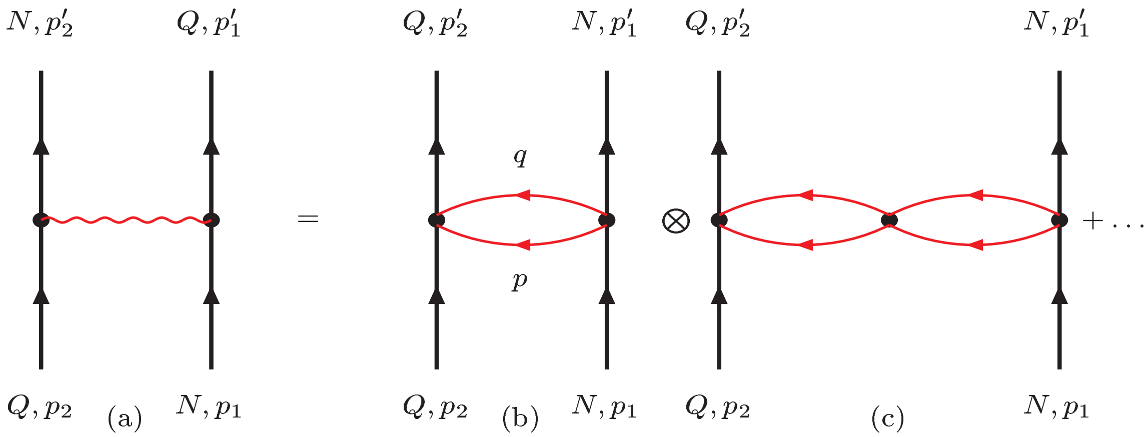

FIG. 2: Diquark-exchange NQ → QN. Panel (a): axial-vector χ^a_μ-exchange. Panel (b) and (c): QQ-pair exchange and iterations.

Higher-order contributions to the propagator come from diagrams as depicted in Fig. 2, *i.e.* the iteration of the second order diquark contribution gives $\Pi_2(k^2) \to \Pi(k^2) = \Pi_2(k^2)/\left[1 - \Pi_2(k^2)\right]$. This is similar to that for the vacuum polarization in QED, see [11]. The divergency's in the diquark-propagator can be removed by a coupling constant renormalization, analogously to QED for the vacuum polarization. Schematically, the matrix element for $NQ \to QN$ reads

$$\begin{aligned} M(k) &\sim g_0^2 \Gamma_5^\mu \Delta_{\mu\nu}(k)\ \Gamma_5^\nu \sim \Pi_{\mu\nu}(k^2) = \left(\eta_{\mu\nu} - k_\mu k_\nu/k^2\right)\ \Pi(k^2)\ \ \text{with} \\ \Pi(k^2) &= \Pi_2(k^2)\left[1 - \Pi_2(k^2)\right]^{-1} \end{aligned}$$

and considering this at $k^2 = 0$ the amplitude gets a factor $M(k^2=0) \sim g_0^2/(1-\Pi_2(0)) \sim g_0^2/(1-\Pi_0(0)) \sim g_R^2$ where g_R^2 is the renormalized coupling. Then, $g_0^2 = g_R^2(1-\Pi_2(0))$ Then, for the amplitude we get

$$M(k) \sim g_R^2\left[\Gamma_5^\mu\left(\eta_{\mu\nu} - k_\mu k_\nu/k^2\right)\ \Gamma_5^\nu\right]\ \left\{1 - \left[\Pi_2(k^2) - \Pi_2(0)\right]\right\}^{-1}$$

where is used $(1-\Pi_2(0))/(1-\Pi_2(k^2)) \approx \left\{1 - \left[\Pi_2(k^2) - \Pi_2(0)\right]\right\}^{-1}$ [11].
The χ^a_μ-field used thus far is not normalized to dimension [MeV]. We now normalize by redefining $\chi^a_\mu(x) \to \chi^a_\mu(x)/m_\chi^2$. Furthermore, since in perturbation theory the in-fields are used, i.e. $(\Box + m_\chi^2)\chi^a_\mu(x) = 0$, we take in the projection

operator $k^2 = m_\chi^2$. Then, the propagator becomes

$$i\widetilde{\Delta}_{\mu\nu}(k) = +\frac{2i}{4\pi^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_\chi^2}\right) \times \left[\int_0^1 dz\; z(1-z)\; \ln\left(1 - z(1-z)\;\frac{k^2}{m^2}\right)\right]/m_\chi^2 \tag{4.14a}$$

$$= -\frac{i}{12\pi^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_\chi^2}\right)\widetilde{\Pi}(k^2)/m_\chi^2, \tag{4.14b}$$

where $\widetilde{\Pi}(k^2) = \Pi_2(k^2) - \Pi_2(0)$.

The z-integral is elementary and can be found in the literature, see *e.g.* [15, 16], and with $A = m^2/k^2$ reads

$$\int_0^1 dz\; z(1-z)\;\ln\left(1 - z(1-z)\;\frac{k^2}{m^2}\right) = \frac{1}{6}\left\{(1+2A)\sqrt{1-4A}\;\ln\left[\frac{1+\sqrt{1-4A}}{|1-\sqrt{1-4A}|}\right] - \left(4A + \frac{5}{3}\right)\right\}. \tag{4.15}$$

It can be verified that for $k^2 \to 0$ the -4A-term in canceled, and there is no pole. The discontinuity in the complex k-plane is

$$Disc\;\widetilde{I}(k) = \frac{2\pi i}{6}\;(1 + \frac{2m^2}{k^2})\sqrt{1 - \frac{4m^2}{k^2}}\;\theta\left(1 - \frac{4m^2}{k^2}\right) \tag{4.16}$$

Then, in momentum space, using a cut-off s_{max},

$$i(\widetilde{\Delta}_F)_{\mu\nu}(k) = +\frac{i}{12\pi^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_\chi^2}\right) \times \frac{1}{m_\chi^2}\int_{4m^2}^{s_{max}} ds\;\left(1 + \frac{2m^2}{s}\right)\sqrt{1 - \frac{4m^2}{s}}\left[k^2 - s + i\epsilon\right]^{-1}. \tag{4.17}$$

Separation the finite and divergent part is achieved by using [15]

$$\frac{1}{k^2 - s + i\epsilon} = \frac{k^2}{s(k^2 - s + i\epsilon)} - \frac{1}{s - i\epsilon}$$

which gives for the finite part, *i.e.* the "renormalized propagator" [4],

$$i(\widetilde{\Delta}_F)_{\mu\nu}(k) = +\frac{i}{12\pi^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_\chi^2}\right) \times \frac{k^2}{m_\chi^2}\int_{4m^2}^{\infty} ds\;\left(1 + \frac{2m^2}{s}\right)\sqrt{1 - \frac{4m^2}{s}}\left[s\left(k^2 - s + i\epsilon\right)\right]^{-1}. \tag{4.18}$$

In the process $N \to Q + D$ we approximate $k^2 \approx k_0^2 \approx (m_N - m_Q)^2$ valid for low-momentum transfer.
The spectral representation of the Diquark Feynman propagator is

$$i(\Delta_F)^{ab}_{\mu\nu}(x'-x) = (0|T\left[\chi^a_\mu(x')\chi^{b\dagger}_\nu(x)\right]|0) = i\int_{4m^2}^{s_{max}} ds\;(\Delta_F)^{ab}_{\mu\nu}(x'-x;s)\;\rho(s)$$

$$= -16i\;\delta_{ab}\;D(x'-x;m_\chi,\Lambda)\;\left[\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_\chi^2}\right]. \tag{4.19}$$

The Fourier transforms, with $m_\chi \sim 2m_Q$, are

$$\widetilde{D}(k^2, m_\chi^2) = -\int_{4m_Q^2}^{s_{max}} ds\;\rho(s)\;\widetilde{\Delta}_F(k^2,\Lambda^2;s), \tag{4.20a}$$

$$\widetilde{\Delta}_F(k^2;s) = \left[k^2 - s + i\epsilon\right]^{-1},\quad \rho(s) = \frac{1}{12\pi^2}\left(1 + \frac{2m^2}{s}\right)\sqrt{1 - \frac{4m^2}{s}}/s. \tag{4.20b}$$

With $\sqrt{s} = E(\mathbf{q}) = \sqrt{\mathbf{q}^2 + m_\chi^2}$ we have for space-like $k^2 = -\mathbf{k}^2$

$$\widetilde{D}(\mathbf{k}^2, m_\chi^2) = -4\pi\int_0^\infty q^2 dq\;\rho(s)\;\Delta_F(\mathbf{k}^2,\Lambda^2;s), \tag{4.21a}$$

$$\widetilde{\Delta}_F(\mathbf{k}^2,\Lambda^2;s) = -\exp\left[-\mathbf{k}^2/\Lambda^2\right]\left[\mathbf{k}^2 + \mathbf{q}^2 + m_\chi^2\right]^{-1}, \tag{4.21b}$$

[4] This dispersive method of "renormalization" is from [15], and is an alternative to the Pauli-Villars method.

where we added a gaussian form-factor as usual in the Nijmegen potentials. In configuration space

$$D(\mathbf{x}^2, m_Q^2) = \int_{4m_Q^2}^{s_{max}} ds\ \rho(s) \left[\frac{m(s)}{4\pi}\ \phi_C^0\left(m(s), \Lambda; |\mathbf{x}|\right)\right] \quad (4.22)$$

where $m(s) = \sqrt{s - m_\chi^2}$.

This result implies a repulsive QN-potential!

Gaussian Approximation:

The volume integral D_V of the D(x)-function is

$$D_V = \left(1 + 2\frac{m_Q}{\Delta m_{NQ}}\right) \int_{m_Q^2}^{\infty} ds\ \rho(s)/m^2(s). \quad (4.23)$$

where we included a factor which accounts for the mass-difference at the NQ-vertex giving $k_0 \to m_n - m_Q$. For a gaussian approximation

$$D_G(|\mathbf{x}|) = g \exp\left[-|\mathbf{x}|^2/\Lambda^2\right], \quad (4.24)$$

having the same volume integral one has

$$2\pi\sqrt{\pi}\ g\Lambda^3 = \left(1 + 2\frac{m_Q}{\Delta m_{NQ}}\right) \int_{m_Q^2}^{\infty} ds\ \rho(s)/m^2(s). \quad (4.25)$$

V. DI-QUARK EXCHANGE NUCLEON-QUARK INTERACTION

As the proton and neutron presentation [1] shows the diquark field has isospin one. So, $\chi_\mu^a(x)$ is an isovector vector-field. Therefore, we introduce the fields

$$\mathbf{D}_\mu^a(x) \equiv \boldsymbol{\chi}_\mu^a(x) = \varepsilon^{abc}\tilde{Q}^b(x)C\gamma_\mu\boldsymbol{\tau}Q^c(x)/(\hbar c)^2, \quad (5.1)$$

where $Q = (u, d)$ is the isospin-spinor $SU_I(2)$ doublet and a $SU_C(3)$ triplet. Similarly for the di-quark $\mathbf{D}_\mu^a$. The di-quark NQ-vertex is given by the interaction Lagrangian

$$\mathcal{L}_{int}^{(1)} = -\lambda_3\left\{(\bar{\psi}(x)\gamma_5\gamma^\mu\boldsymbol{\tau}q^a)\cdot\mathbf{D}_\mu^a(x) + h.c.\right\}, \quad (5.2)$$

The field theoretical study of the diquark-propagator in this paper shows that the Feynman-rule for the diquark χ^a-propagator for the $\eta_{\mu\nu}$-term is found in (4.9)

$$i(\widetilde{\Delta_F})_{\mu\nu}^{ab}(\boldsymbol{\Delta}) = -i\delta_{ab}\widetilde{D}(\boldsymbol{\Delta}^2)\left(\eta_{\mu\nu} - \frac{\boldsymbol{\Delta}_\mu\boldsymbol{\Delta}_\nu}{m_\chi^2}\right),\quad \widetilde{D}(\boldsymbol{\Delta}^2) > 0, \quad (5.3)$$

where $\Delta = p_1' - p_1 = p_2' - p_2$. In the gaussian approximation

$$\widetilde{D}(\boldsymbol{\Delta}^2) \approx exp\left[-\boldsymbol{\Delta}^2/\Lambda^2\right]/\mathcal{M}^2. \quad (5.4)$$

The second-order amplitude for Fig. 3 using the interaction (5.2) can be described (effectively) by

$$M^{(2)}(p_1', s_1', p_2', s_2'; p_1, s_1, p_2, s_2) = -\frac{\lambda_3^2}{2!}\ [\bar{u}_Q(p_1', s_1')\gamma_5\gamma^\mu\boldsymbol{\tau}u_N(p_1, s_1)]\cdot[\bar{u}_N(p_2', s_2')\gamma_5\gamma_\mu\boldsymbol{\tau}u_Q(p_2, s_2)]\ \widetilde{D}(\boldsymbol{\Delta}^2), \quad (5.5)$$

where $\Delta = p_1' - p_1 = p_2' - p_2$. In the low momentum transfer region the approximation $\widetilde{D} \approx \exp\left[-\boldsymbol{\Delta}^2/\Lambda^2\right]/\mathcal{M}^2$ leads to a gaussian contact interaction.

$$V(p_1', s_1', p_2', s_2'; p_1, s_1, p_2, s_2) = -(\lambda_3^2/2)\,[\bar{u}_Q(p_1', s_1')\gamma_5\gamma^\mu\boldsymbol{\tau}u_N(p_1, s_1)]\cdot[\bar{u}_N(p_2', s_2')\gamma_5\gamma_\mu\boldsymbol{\tau}u_Q(p_2, s_2)]\ \widetilde{D}(\boldsymbol{\Delta}^2). \quad (5.6)$$

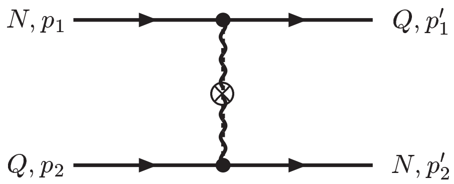

FIG. 3: Diquark-exchange for $NQ \to QN$ reaction.

Using Pauli-spinor matrix elements

$$\bar{u}(\mathbf{p}'\gamma_5\gamma_0 u(\mathbf{p}) = -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{\mathcal{E}'}+\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}}\right], \tag{5.7}$$

$$\bar{u}(\mathbf{p}')\gamma_5\boldsymbol{\gamma} u(\mathbf{p}) = -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\boldsymbol{\sigma}+\frac{(\boldsymbol{\sigma}\cdot\mathbf{p}')\ \boldsymbol{\sigma}\ (\boldsymbol{\sigma}\cdot\mathbf{p})}{\mathcal{E}'\mathcal{E}}\right] \approx -\boldsymbol{\sigma}, \tag{5.8}$$

where M',M are the quark or the nucleon mass, and $\mathcal{E} = E_p + M$. Note that the leading term from the vertex factors [....][....] in (5.6) has $-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$. In momentum space we write $\widetilde{V}_{QN} \equiv \widetilde{V}^{(a)}_{QN} + \widetilde{V}^{(b)}_{QN}$, where (a) is similar to the axial-exchange in NN-potentials, and (b) represents terms emphasizing the nucleon and quark mass, and obtain

$$\begin{aligned}\widetilde{V}^{(a)}_{QN} &= +2\lambda_3^2\Bigg[\left(1-\frac{2\mathbf{k}^2}{3M_QM_N}+\frac{3(\mathbf{q}^2+\mathbf{k}^2/4)}{2M_QM_N}\right)\ \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{1}{4M_QM_N}\left((\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})-\frac{1}{3}\mathbf{k}^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\\ &\quad +\frac{i}{4M_QM_N}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{k}\Bigg]\cdot\widetilde{g}(\mathbf{k}^2),\end{aligned} \tag{5.9a}$$

$$\begin{aligned}\widetilde{V}^{(b)}_{QN} &= -2\lambda_3^2\Bigg[\frac{(M_N-M_Q)^2}{4M_N^2M_Q^2}\left\{\left(\mathbf{q}^2+\mathbf{k}^2/4\right)-\mathbf{k}^2/2\right\}\ \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\\ &\quad -\frac{i}{4}\left(\frac{M_N^2-M_Q^2}{8M_N^2M_Q^2}\right)\ (1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}\Bigg]\cdot\widetilde{g}(\mathbf{k}^2),\end{aligned} \tag{5.9b}$$

where $\widetilde{g}(\mathbf{k}^2) = \exp\left(-\mathbf{k}^2/\Lambda^2\right)/\mathcal{M}^2$. Here, we added the gaussian cut-off and a scale parameter $\mathcal{M}$.
Note: $V^{(a)}_{QN}$ is similar to axial-vector exchange in NN and YN. $V^{(b)}_{QN}$ is the "extra term" proportional to the $M_N - M_Q$ mass difference, which is not small in the QN-potential.

In configuration space, taking into account the exchange character of the potential we have a factor $P_f P_\sigma$. Since the physical states satisfy $P_f P_\sigma P_x = -1$, this leads to a factor $-P_x$ and a sign-change in the antisymmetric spin-orbit. Then, we obtain for the central, spin-spin, tensor, and spin-orbit-potentials, see *e.g.* Ref. [12],

$$V_{QN}^{(a)}(r) = -2\lambda_3^2\,\frac{\Lambda}{8\pi}\left[\left(\phi_C^0(r) - \frac{\Lambda^2}{6M_N M_Q}\phi_C^1(r)\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) - \frac{3}{4M_Q M_N}\left(\boldsymbol{\nabla}^2\phi_C^0(r) + \phi_C^0(r)\boldsymbol{\nabla}^2\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right.$$
$$\left. - \frac{\Lambda^2}{16M_N M_Q}\,\phi_T^0(r)\,S_{12} + \frac{\Lambda^2}{8M_N M_Q}\phi_{SO}^0(r)\,\mathbf{L}\cdot\mathbf{S}\right](\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\,P_x, \tag{5.10a}$$

$$V_{QN}^{(b)}(r) = -2\lambda_3^2\,\frac{\Lambda}{8\pi}\left[\frac{(M_N-M_Q)^2}{4M_N M_Q}\left\{+\frac{\Lambda^2}{8M_N M_Q}\phi_C^1(r) + \frac{1}{2M_N M_Q}\left(\boldsymbol{\nabla}^2\phi_C^0(r) + \phi_C^0(r)\boldsymbol{\nabla}^2\right)\right\}\cdot\right.$$
$$\left.\times(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) - \frac{\Lambda^2}{4M_N M_Q}\frac{(M_N^2-M_Q^2)}{4M_N M_Q}\phi_{SO}^0(r)\cdot\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L}\right](\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\,P_x, \tag{5.10b}$$

where

$$\phi_C^0(r) = \frac{1}{\sqrt{\pi}}\frac{\Lambda^2}{\mathcal{M}^2}\ \exp\left[-\frac{1}{4}\Lambda^2 r^2\right], \tag{5.11a}$$
$$\phi_C^1(r) = \frac{2}{\sqrt{\pi}}\frac{\Lambda^2}{\mathcal{M}^2}\ \left(3-\Lambda^2 r^2/2\right)\exp\left[-\frac{1}{4}\Lambda^2 r^2\right], \tag{5.11b}$$
$$\phi_T^0(r) = \frac{1}{6\sqrt{\pi}}\frac{\Lambda^2}{\mathcal{M}^2}\ (\Lambda r)^2\exp\left[-\frac{1}{4}\Lambda^2 r^2\right], \tag{5.11c}$$
$$\phi_{SO}^0(r) = \frac{2}{\sqrt{\pi}}\frac{\Lambda^2}{\mathcal{M}^2}\ \exp\left[-\frac{1}{4}\Lambda^2 r^2\right]. \tag{5.11d}$$

We introduced a gaussian cut-off with the parameter Λ. This parameter is a free parameter and can be used to tune the di-quark exchange potential which is also the case with λ_3. The non-local potential is

$$V^{(n.l.)}(r) = -\left[\boldsymbol{\nabla}^2\frac{\phi(r)}{2M_{red}} + \frac{\phi(r)}{2M_{red}}\boldsymbol{\nabla}^2\right]P_x,\ \text{with}\ \phi(\mathrm{r}) = -\frac{\lambda_3^2}{4\pi}\frac{3\Lambda}{4\mathrm{M_Q M_N}}\phi_{\mathrm{C}}^0(\mathrm{r})\ (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\ (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2). \tag{5.12}$$

For the statistical average S-wave potential we obtain from Eq. (5.10)

$$\bar{V}(CQM) = \frac{1}{4}V(^1S_0) + \frac{3}{4}V(^3S_1) = +\frac{3\lambda_3^2}{4\pi}\,\Lambda\left(\phi_C^0(r) - \frac{\Lambda^2}{6M_N M_Q}\left\{1 - \frac{3(M_N-M_Q)^2}{16M_N M_Q}\right\}\phi_C^1(r)\right) \tag{5.13}$$

which result comes from $(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) = -3$ for both $^1S_0$ and $^3S_1$.

The confinement-deconfinement transition can be parametrized as $\lambda_3 \to \gamma_D\lambda_3$ with *e.g.*

$$\gamma_D(\rho_N,\rho_D) = \left[\exp\left\{+\gamma_3\left(\rho_N/\rho_D - 1\right)\right\} - 1\right]\ \theta(\rho_N-\rho_D), \tag{5.14}$$

where ρ_D is the deconfinement threshold. In [7, 13, 14] a similar form is used for the density dependence of the constituent quark mass.

Notes: *1. The S-wave quark-nucleon repulsion (5.13) is repulsive and becomes strong for high densities. 2. The* $^1P_1$*-wave has* $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 P_x = -9$ *giving strong repulsion. For* $^3P_J(J=0,1,2)$ *the spin-isospin and the exchange operator give a factor -1, giving again a (weaker) repulsion. 3. The di-quark exchange potential gives a repulsive wall for the nucleons between the nucleon- and quark-phase.*

VI. DISCUSSION AND CONCLUSION

The difference between diquark-exchange and quark-antiquark-exchange, in the case of for example ρ, A_1-exchange, is the absence of the (-)-sign due to the closed fermion-loop for diquark-exchange. This is clear from the analysis of the Wick-expansion of the T-product $(0|T\left[\chi_\mu^{(a)}(x)\chi_\nu^{b\dagger}(x')\right]|0)$. Therefore, in the Feynman-rule for diquark-exchange there is the factor $+iP_{\mu\nu}$ instead of $-iP_{\mu\nu}$. This, and the exchange-character of the $QN \to NQ$ interaction, is the source of the repulsion of the diquark-exchange with the $\gamma_5\gamma_\mu$-coupling. The diquark is in the $\{\bar{3}\}$-irrep of SU$_c$(3), which gives a factor $C_F = 2$.
The vector and axial-vector mesons ρ and A_1 show up as resonances $\pi\pi$ and $\pi\rho$ channels. In the quark-model they are antiquark-quark systems, and a calculation similar to that for the qq-system in this paper applies to them as well. In this view the ρ and A_1 propagators are effective ones just as for the diquark. Hence, likewise we can represent diquark exchange by an effective propagator

$$i(\widetilde{\Delta})^{ab}_{\mu\nu}(k) \;=\; +2i\delta_{ab}\frac{\left(\eta_{\mu\nu}-k_\mu k_\nu/m_\chi^2\right)}{k^2-m_\chi^2+i\epsilon},$$

where m_χ is an effective mass $\approx 2m_Q$. We note the (-)-sign difference with a ρ and A_1 propagator which is explained in these notes. Therefore, the derivation in these notes confirms the repulsive diquark potential used in [7] succesfully for mixed nuclear-quark matter, without any ambiguity.
A second proton current $\eta^{(2)}(x)$ is treated in Appendix C 1 and contains a scalar $\chi_S^a(x)$ and a pseudoscalar $\chi_5^a(x)$ diquark field, which introduces extra NDQ-vertices. They lead to a scalar and pseudoscalar NQ-potential. The scalar field allows the appearance of a condensate $\langle 0|\chi_S|0\rangle \neq 0$ [17], a possibility studied extensively in the literature. Whereas for the axial-vector diquark-exchange we found a repulsion in all partial waves, for the scalar-pseudoscalar diquarks the QN-potentials are a mixture of attraction and repulsion.

APPENDIX A: DIQUARK FEYNMAN-PROPAGATOR AND WICK-EXPANSION

Diquark Feynman-propagator:

The diquark field Feynman propagator is

$$i(\Delta_F)^{ab}_{\mu\nu}(x'-x) \;=\; (0|T\left[\chi^a_\mu(x')\chi^{b\dagger}_\nu(x)\right]|0) \tag{A1}$$

where the diquark fields are

$$\chi^a_\mu(x) \;=\; \widetilde{q}^b(x)C\gamma_\mu q^c(x)\,\varepsilon^{abc}\;,\;\chi^{a\dagger}_\mu(x) = -\bar{q}^b(x)\gamma_\mu C\,\widetilde{\bar{q}}^c(x)\,\varepsilon^{abc} \tag{A2}$$

The Wick-expansion of the T-product into normal-ordered N-products for the operators A,B,C, and D occurring in (A1) reads, in the notation of [4],

$$\begin{aligned} T(ABCD) &= N(ABCD) + N(\widehat{AB}\,CD) + N(AB\,\widehat{CD}) + N(\widehat{AB}\,\widehat{CD}) - N(\widehat{AC}\,\widehat{BD}) + N(\widehat{AD}\,\widehat{BC}) \\ &\Rightarrow -N(\widehat{AC}\,\widehat{BD}) + N(\widehat{AD}\,\widehat{BC}), \end{aligned}$$

where the terms in the last line survives after taking the vacuum expectation value. In the notation of [3] this reads

$$\begin{aligned} T(ABCD) &= :ABCD: + \langle 0|T(AB)|0\rangle :CD: + :AB: \langle 0|T(CD)|0\rangle + \langle 0|T\rangle(AB)|0\rangle\langle|T(CD)|0\rangle \\ &\quad -\langle 0|T(AC)|0\rangle\langle 0|T(BD)|0\rangle + \langle 0|T(AD)|0\rangle\langle 0|T(BC)|0\rangle \\ &\Rightarrow -\langle 0|T(AC)|0\rangle\langle 0|T(BD)|0\rangle + \langle 0|T(AD)|0\rangle\langle 0|T(BC)|0\rangle \end{aligned}$$

So,

$$\begin{aligned} i(\Delta_F)^{ab}_{\mu\nu}(x'-x) &= (0|T\left[\chi^a_\mu(x')\chi^{b\dagger}_\nu(x)\right]|0) \\ &\Leftarrow -\langle 0|T(AC)|0\rangle\langle 0|T(BD)|0\rangle + \langle 0|T(AD)|0\rangle\langle 0|T(BC)|0\rangle \end{aligned}$$

since in our case $\langle 0|T(AB)|0\rangle = \langle|T(CD)|0\rangle = 0$. for the evaluation of these see Eqn. (3.4).

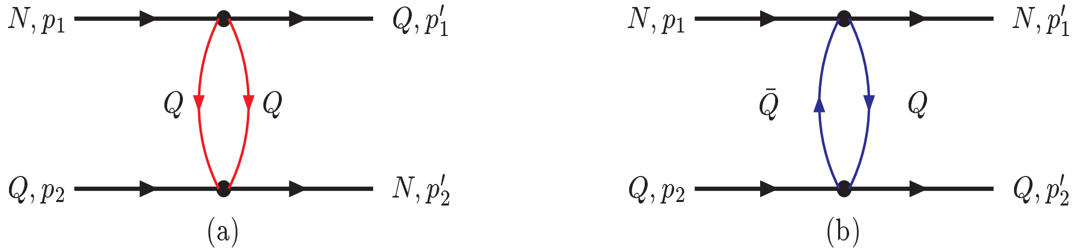

FIG. 4: Scalar Diquark-exchange and Meson-exchange. Panel (a): diquark exchange. Panel (b): antiquark-quark exchange.

APPENDIX B: COMPARISON SCALAR DIQUARK-EXCHANGE AND MESON-EXCHANGE

For meson-exchange we have an antiquark-quark state

$$\psi^a(x) \;=\; \bar{q}^c(x)q^d(x)\epsilon^{acd}\;,\; \bar{\psi}^a(x) = \bar{q}^d(x)q^c(x)\;\epsilon^{acd} = -\bar{q}^e(x)q^f(x)\epsilon^{aef}.$$

Then for the propagator

$$(0|T\left[\psi^a(x')\bar{\psi}^b(x)\right]|0) = -\epsilon^{acd}\epsilon^{bef}\;(0|T\left[\bar{q}^c(x')q^d(x')\cdot\bar{q}^e(x)\;q^f(x)\right]|0) \to$$
$$+\epsilon^{acd}\epsilon^{bef}\;(0|T\left[q^d(x')\bar{q}^e(x)\right]|0)(0|T\left[q^f(x)\bar{q}^c(x')\right]|0)$$

The color factor becomes $+\epsilon^{acd}\epsilon^{bef}\delta_{de}\delta_{cf} = -2\delta_{ab}$, So, we get

$$(0|T\left[\psi^a(x')\bar{\psi}^b(x)\right]|0) = -2\delta_{ab}(0|T\left[q(x')\bar{q}(x)\right]|0)(0|T\left[q(x)\bar{q}(x')\right]|0)$$

which differs indeed a (-)-sign with the corresponding (3.3). Since the rest of the calculations are identical this results in an overall (-)-sign.
The difference between a meson and the diquark is the assumed absence of a bound state. In general, see *e.g.* [6],

$$\rho(s) \;=\; Z\delta(s-m_R^2)+\sigma(s)\;\theta(s-4m_R^2)$$

with $Z \geq 0$ and $\sigma(s) \geq 0$ and

$$1-Z \;=\; \int_{4m_R^2}^{\infty} ds\;\sigma(s)\;,\;0 \leq Z \leq 1.$$

Because of the positive signs of both terms the presence or absence of a bound state for a vector, axial-vector, etc mesons makes no difference.
Conclusion: For the antiquark-quark exchange we get an extra (-)-sign compared to the diquark-exchange.

APPENDIX C: SCALAR AND PSEUDOSCALAR DIQUARKS

In [17] a new form of ordering at high density, with color and flavor degrees of freedom correlated, where a condensate of scalar diquarks occurs. Therefore, we work out the exchange of such diquarks and introduce the isoscalar scalar field [5] In [2] three representations of the proton as a three-quark system are discussed. For the representation with

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[5] Since $\gamma_0 C\gamma_0 = -C$ the $\chi^a(x)$ is a scalar field.

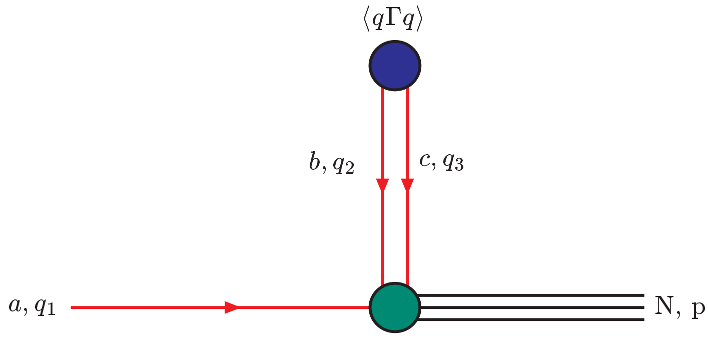

FIG. 5: Di-quark-Quark-Nucleon vertex, Di-quark condensate $\langle \widetilde{q}^b \Gamma q^c \varepsilon^{abc} \rangle$ with $\Gamma = C\gamma_5$ from Cooper-pairing.

the axial-vector $\chi^a_\mu(x)$, we used the $\eta^{(1)}$-current. Here we analyze the second current

$$\begin{aligned} \eta^{(2)} &= (\widetilde{u}^a C\sigma_{\mu\nu} u^b)\sigma^{\mu\nu}\gamma_5 d^c \varepsilon^{abc} = -4\left[(\widetilde{u}^a_R C d^b_R)u^c_R - (\widetilde{u}^a_L C d^b_L)u^c_L\right]\ \varepsilon^{abc} \\ &= -2\left[\left(\widetilde{u}^a C\gamma_5 d^b\right) u^c + \left(\widetilde{u}^a C d^b\right)\gamma_5 u^c\right]\varepsilon^{abc} \equiv -2\left[\chi^a_S u^a + \chi^a_5 u^a\right], \end{aligned} \tag{C1}$$

Note that in the first step $u \leftrightarrow d$ interchange has been performed using a Fierz-transformation in Dirac and isospin space. This current contains the composite diquark combinations

$$\chi^a_S = (\widetilde{u}^b C\gamma_5 d^c)\varepsilon^{abc},\ \chi^a_5 = (\widetilde{u}^b C d^c)\varepsilon^{abc}, \tag{C2}$$

where χ^a_S has spin-parity $J^{PC} = 0^{++}$ and χ^a_5 has $J^{PC} = 0^{-+}$, Furthermore, $ud = [|1,1/2) + |0,1,2)]/\sqrt{2}$ *i.e.* a combination of I=0,1. In quark matter a condensate with $\langle 0|\chi_S|0\rangle \neq 0$ is possible, see [17] and the illustration in Fig. C3.
The interaction Lagrangian for the NDQ-transition is

$$\begin{aligned} \mathcal{L}^{(2)}_I &= -\lambda_2\{\bar{\psi}\eta^{(2)} + \bar{\eta}^{(2)}\psi\} \\ &= 2\lambda_2\left\{\bar{\psi}\left[\chi^a_S(x)u^a(x) + \chi^a_5(x)u^a(x)\right] + h.c.\right\}. \end{aligned} \tag{C3}$$

$$\chi^a_S(x) = \widetilde{q}^c(x) C\gamma_5 q^d(x)\ \varepsilon^{acd}\ ,\ \chi^{b\dagger}_S(x) = -\bar{q}^e(x)\gamma_5 C\widetilde{\bar{q}}^f(x)\ \varepsilon^{bef}. \tag{C4}$$

Following the same steps as for the $\chi^a_\mu(x)$ propagator, analogous to (3.6) we have

$$\begin{aligned} X_S &= 4\delta_{ab} Tr\left[\gamma_5\left(i\gamma\cdot\partial^y + m_Q\right)\gamma_5\left(i\gamma\cdot\partial^z - m_Q\right)\right]\ \Delta_F(y)\cdot\Delta_F(z) \\ &= 4\delta_{ab}\ \mathrm{Tr}\left[\gamma_\alpha\gamma_\beta\partial^y_\alpha\partial^z_\beta - m^2_Q\right]\ \Delta_F(y)\cdot\Delta_F(z) \\ &= -16\delta_{ab}\left[-\partial^y_\alpha\partial^\alpha_z + m^2_Q\right]\ \Delta_F(y)\cdot\Delta_F(z), \end{aligned} \tag{C5}$$

and

$$\begin{aligned} X_5 &= 4\delta_{ab} Tr\left[\left(i\gamma\cdot\partial^y + m_Q\right)\left(i\gamma\cdot\partial^z - m_Q\right)\right]\ \Delta_F(y)\cdot\Delta_F(z) \\ &= -4\delta_{ab}\ \mathrm{Tr}\left[\gamma_\alpha\gamma_\beta\partial^y_\alpha\partial^z_\beta + m^2_Q\right]\ \Delta_F(y)\cdot\Delta_F(z) \\ &= -16\delta_{ab}\left[\partial^y_\alpha\partial^\alpha_z + m^2_Q\right]\ \Delta_F(y)\cdot\Delta_F(z), \end{aligned} \tag{C6}$$

where the differentiation variables, which in the end will be put to $y = z = x' - x$.

1. Scalar Diquarks

The spectral representation of the Diquark Feynman propagator is

$$i(\Delta_F)_{S,ab}(x'-x) \;=\; (0|T\left[\chi^a(x')\chi^{b\dagger}(x)\right]|0) = i\int_{s_0}^{\infty} ds\;(\Delta_F)_{ab}(x'-x);s)\;\rho(s). \tag{C7}$$

In momentum space the propagator leads to the integral

$$\widetilde{I}_S(k;m) \;=\; \int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\,(2\pi)^4\delta^4(p+q-k)\left[p\cdot q+m^2\right]\;\times\left[p^2-m^2+i\epsilon\right]^{-1}\left[q^2-m^2+i\epsilon\right]^{-1}. \tag{C8}$$

Following same steps below Eqn. (4.5) leads now to

$$\begin{aligned}\widetilde{I}_S(k) \;&=\; -i\sum_i\frac{c_i}{16\pi^2}\int_0^\infty\int_0^\infty\frac{dz_1dz_2}{(z_1+z_2)^2}\left[-\frac{2i}{(z_1+z_2)}+\frac{z_1z_2k^2}{(z_1+z_2)^2}+m_i^2\right]\cdot\\ &\quad\times\exp\left\{i\left[k^2\frac{z_1z_2}{z_1+z_2}-(m_i^2-i\epsilon)(z_1+z_2)\right]\right\}.\end{aligned} \tag{C9}$$

Now, see [3], Eqn. (8.17),

$$\begin{aligned}\widetilde{J}(k) \;&=\; \int_0^\infty\int_0^\infty\frac{dz_1dz_2}{(z_1+z_2)^2}\left[m^2-\frac{i}{(z_1+z_2)}-\frac{k^2z_1z_2}{(z_1+z_2)^2}\right]\cdot\\ &\quad\times\exp\left\{i\left[k^2\frac{z_1z_2}{z_1+z_2}-(m_i^2-i\epsilon)(z_1+z_2)\right]\right\}=0.\end{aligned} \tag{C10}$$

So, under the integral the linear combination

$$0\equiv m^2-\frac{i}{(z_1+z_2)}-\frac{k^2z_1z_2}{(z_1+z_2)^2} \tag{C11}$$

and we have three equivalent expressions for $\widetilde{I}_5(k)$:

$$\begin{aligned}(i)\;\widetilde{I}_S(k) \;&=\; -i\sum_i\frac{c_i}{16\pi^2}\int_0^\infty\int_0^\infty\frac{dz_1dz_2}{(z_1+z_2)^2}\left[2m_i^2-\frac{3i}{(z_1+z_2)}\right]\cdot\\ &\quad\times\exp\left\{i\left[k^2\frac{z_1z_2}{z_1+z_2}-(m_i^2-i\epsilon)(z_1+z_2)\right]\right\}\end{aligned} \tag{C12a}$$

$$\begin{aligned}(ii)\;\widetilde{I}_S(k) \;&=\; -i\sum_i\frac{c_i}{16\pi^2}\int_0^\infty\int_0^\infty\frac{dz_1dz_2}{(z_1+z_2)^2}\left[-\frac{i}{(z_1+z_2)}+2k^2\frac{z_1z_2}{(z_1+z_2)^2}\right]\cdot\\ &\quad\times\exp\left\{i\left[k^2\frac{z_1z_2}{z_1+z_2}-(m_i^2-i\epsilon)(z_1+z_2)\right]\right\}\end{aligned} \tag{C12b}$$

$$\begin{aligned}(iii)\;\widetilde{I}_S(k) \;&=\; -i\sum_i\frac{c_i}{16\pi^2}\int_0^\infty\int_0^\infty\frac{dz_1dz_2}{(z_1+z_2)^2}\left[3k^2\frac{z_1z_2}{(z_1+z_2)^2}-m_i^2\right]\cdot\\ &\quad\times\exp\left\{i\left[k^2\frac{z_1z_2}{z_1+z_2}-(m_i^2-i\epsilon)(z_1+z_2)\right]\right\}.\end{aligned} \tag{C12c}$$

Using the identity (4.10) in $\widehat{I}_{\mu\nu}(k)$ and subsequently scaling $z_i\to\lambda z_i$, one gets for (iii) the expression

$$\begin{aligned}\widetilde{I}_S(k) \;&=\; \frac{-i}{16\pi^2}\sum_i c_i\int_0^\infty\int_0^\infty dz_1dz_2\;\delta(1-z_1-z_2)\int_0^\infty\frac{d\lambda}{\lambda}\left[3z_1z_2k^2-m_i^2\right]\cdot\\ &\quad\times\exp\left[i\lambda\left(z_1z_2k^2-m_i^2+i\epsilon\right)\right].\end{aligned} \tag{C13}$$

The λ-integral diverges logarithmically and is evaluated by applying the cut-off procedure by choosing $C_1 = -1, C_i = 0$ $(i > 1)$. This gives

$$\begin{aligned}\widehat{I}_S(k) &= \widetilde{I}_S(k;m^2) - \widetilde{I}_S(k;M^2) \\ &\approx -i(16\pi^2)^{-1} \sum_i c_i \int_0^\infty \int_0^\infty dz_1 dz_2\ \delta(1-z_1-z_2)\ \left[3z_1 z_2 k^2 - m_i^2\right] \cdot \\ &\quad \times \int_0^\infty \frac{d\lambda}{\lambda} \exp\left[i\lambda\left(z_1 z_2 k^2 - m_i^2 + i\epsilon\right)\right] \qquad \text{(C14)} \\ &= +(16\pi^2)^{-1} \sum_{i=0}^{1} c_i \int_0^1 dz\ \left[3z(1-z)k^2 - m_i^2\right] \ln\left[1 - z(z-1)\ k^2/m_i^2\right] \qquad \text{(C15)}\end{aligned}$$

where in the last step we scaled $\lambda \to m_i^2 \lambda$. Working this further out we get

$$\begin{aligned}\widehat{I}_S(k) &= +(16\pi^2)^{-1} \int_0^1 dz \left\{ \left[3z(1-z)k^2\right]\ \ln\left[\frac{1-z(1-z)k^2/m^2}{1-z(1-z)k^2/M^2}\right] \right. \\ &\quad \left. - \left[m^2 \ln\left\{1-z(1-z)k^2/m^2\right\} - M^2 \ln\left\{1-z(1-z)k^2/M^2\right\}\right] \right\} \qquad \text{(C16)}\end{aligned}$$

Using the approximation $\ln(1 - a/m_i^2) \approx -a/m_i^2$ for $m_i = m, M$ we obtain

$$\begin{aligned}\widehat{I}_S(k) &\approx +(3/16\pi^2) \int_0^1 dz\ \left[z(1-z)k^2\right] \left(\ln\frac{M^2}{m^2} + \ln\left[\frac{m^2 - z(1-z)k^2}{M^2 - z(1-z)k^2}\right]\right) \\ &\approx +(3/16\pi^2) \int_0^1 dz\ \left[z(1-z)k^2\right] \left(\ln\frac{M^2}{m^2} - \ln\left[\frac{M^2}{m^2 - z(1-z)k^2}\right]\right) \\ &= +(3/16\pi^2)k^2 \left\{\frac{1}{6}\ln\frac{M^2}{m^2} - \int_0^1 dz\ z(1-z)\ \ln\left[\frac{M^2}{m^2 - z(1-z)k^2}\right]\right\} \qquad \text{(C17a)}\end{aligned}$$

Since for the nucleon-quark vertex $k^\mu \approx (m_N - m_Q, \mathbf{k})$ we take $k^2 \cong (m_N - m_Q)^2$ in the front factor. The z-integral is

$$I_2(k) = \int_0^1 dz\ z(1-z) \ln\left[1 - z(1-z)k^2/m^2\right] = \frac{1}{6}\left\{(1+2A)\sqrt{1-4A}\ \ln\left[\frac{1+\sqrt{1-4A}}{\left|1-\sqrt{1-4A}\right|}\right] - \left(4A + \frac{5}{3}\right)\right\}.$$

where the I_2-integral has been taken from (4.15) and $A = m^2/k^2$. So, we finally get

$$\widehat{I}_S(k) \approx +(3/16\pi^2)k^2\ I_2(k) \qquad \text{(C18)}$$

The (unrenormalized) diquark propagator becomes

$$i\left(\widetilde{\Delta}_F^{(0)}\right)_{S,ab}(k) = -\frac{3i}{\pi^2}\ k^2\ \times \left[\int_0^1 dz\ z(1-z)\ \ln\left(1 - z(1-z)\ \frac{k^2}{m^2}\right)\right]\ \delta_{ab}. \qquad \text{(C19)}$$

The χ^a-field used thus far is not normalized to dimension [MeV]. We now normalize by redefining $\chi^a(x) \to \chi^a(x)/m_\chi^2$. Then, the "unrenormalized" propagator becomes

$$i(\widetilde{\Delta}_F)_{S,ab}(k) = -\frac{3i}{\pi^2}\ (k^2/m_\chi^2)\ \times \left[\int_0^1 dz\ z(1-z)\ \ln\left(1 - z(1-z)\ \frac{k^2}{m^2}\right)\right]\ \delta_{ab} \qquad \text{(C20)}$$

which gives for the "renormalized propagator", *i.e.* the finite part [6],

$$i(\widetilde{\Delta}_F)_{S,ab}(k) = -\frac{i}{2\pi^2}\frac{k^2}{m_\chi^2}\int_{4m^2}^\infty ds\ \left(1 + \frac{2m^2}{s}\right)\sqrt{1 - \frac{4m^2}{s}}\left[s\left(k^2 - s + i\epsilon\right)\right]^{-1}\delta_{ab}. \qquad \text{(C21)}$$

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[6] This dispersive method of "renormalization" is from [15], and is an alternative to the Pauli-Villars method.

In the process $N \to Q + D$ we approximate $k^2 \approx k_0^2 \approx (m_N - m_Q)^2$ valid for low-momentum transfer.
The spectral representation of the Diquark Feynman propagator is

$$i(\Delta_F)_{S,ab}(x'-x) = (0|T\left[\chi^a(x')\chi^{b\dagger}(x)\right]|0) = i\int_{4m^2}^{\infty} ds\ (\Delta_F)_{S,ab}(x'-x;s)\ \rho(s)$$
$$= -i\delta_{ab}\left(\frac{m_N - m_Q}{m\chi}\right)^2 D(x'-x; m_\chi, \Lambda). \tag{C22}$$

The Fourier transforms, with $m_\chi \sim 2m_Q$, are

$$\widetilde{D}(k^2, m_\chi^2) = -\int_{4m_Q^2}^{\infty} ds\ \rho(s)\ \widetilde{\Delta}_F(k^2, \Lambda^2; s), \tag{C23a}$$

$$\widetilde{\Delta}_F(k^2; s) = \left[k^2 - s + i\epsilon\right]^{-1},\quad \rho(s) = \frac{1}{12\pi^2}\left(1 + \frac{2m^2}{s}\right)\sqrt{1 - \frac{4m^2}{s}}/s. \tag{C23b}$$

With $\sqrt{s} = E(\mathbf{q}) = \sqrt{\mathbf{q}^2 + m_\chi^2}$ we have for space-like $k^2 = -\mathbf{k}^2$

$$\widetilde{D}(\mathbf{k}^2, m_\chi^2) = 4\pi\int_0^{\infty} q^2 dq\ \rho(s)\ \widetilde{\Delta}_F(\mathbf{k}^2, \Lambda^2; s), \tag{C24a}$$

$$\widetilde{\Delta}_F(\mathbf{k}^2, \Lambda^2; s) = \exp\left[-\mathbf{k}^2/\Lambda^2\right]\left[\mathbf{k}^2 + \mathbf{q}^2 + m_\chi^2\right]^{-1}, \tag{C24b}$$

where we added a gaussian form-factor as usual in the Nijmegen potentials. In configuration space

$$D(\mathbf{x}^2, m_Q^2) = \int_{4m_Q^2}^{s_{max}} ds\ \rho(s)\left[\frac{m(s)}{4\pi}\ \phi_C^0\left(m(s), \Lambda; |\mathbf{x}|\right)\right] \tag{C25}$$

where $m(s) = \sqrt{s - m_\chi^2}$.
Approximating the propagator with an effective one for $k^2 < 0$,

$$(\widetilde{\Delta}(k^2))_{S,ab} \cong -\left(\frac{\Delta_{NQ}}{m_\chi}\right)^2 \frac{F(k^2)}{k^2 - \bar{m}_\chi^2 + i\epsilon} \approx -\left(\frac{\Delta_{NQ}}{m_\chi}\right)^2 \frac{\exp(-\mathbf{k}^2/\Lambda^2)}{\mathbf{k}^2 + \bar{m}_\chi^2}, \tag{C26}$$

where $\Delta_{NQ}^2 = m_N^2 - m_Q^2$, and $\bar{m}_\chi^2 = m_\chi^2 - \Delta_{NQ}^2 > 0$, the scalar diquark-exchange, see Fig. 3, gives the QN-potential, see *e.g.* [12],

$$V_{QN}(r) = +\lambda_S^2\frac{\bar{m}_\chi}{4\pi}\left\{\left[\phi_C^0 - \frac{\bar{m}_\chi^2}{4m_N m_Q}\phi_C^1\right] + \frac{\bar{m}_\chi^2}{2m_N m_Q}\phi_{SO}^0\mathbf{L}\cdot\mathbf{S} + \frac{\bar{m}_\chi^4}{16m_N^2 m_Q^2}\cdot\right.$$
$$\times\frac{3}{(\bar{m}_\chi r)^2}\phi_T^0 Q_{12} + \frac{m_Q^2}{m_N m_Q}\left[\frac{(m_N^2 - m_Q^2)}{4m_N m_Q}\right]\ \phi_{SO}^0\cdot\frac{1}{2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot\mathbf{L}$$
$$\left.+\frac{1}{4m_N m_Q}\left(\boldsymbol{\nabla}^2\phi_C^0 + \phi_C^0\boldsymbol{\nabla}^2\right)\right\}\ P_x. \tag{C27}$$

and with a factor $\langle 1 + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\rangle/2 = 2I - 1$. For $NQ \to QN$ this gives for S-waves, averaging over the spin with factors 1/4 and 3/4 for the singlet and triplet respectively, the spin-isospin factor is -1/2, which gives attraction.

2. Pseudoscalar Diquarks

The spectral representation of the Diquark Feynman propagator is

$$i(\Delta_F)_{5,ab}(x'-x) = (0|T\left[\chi_5^a(x')\chi_5^{b\dagger}(x)\right]|0) = i\int_{s_0}^{\infty} ds\ (\Delta_F)_{ab}(x'-x);s)\ \rho(s). \tag{C28}$$

In momentum space the propagator leads to the integral

$$\widetilde{I}_5(k;m) = \int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\ (2\pi)^4\delta^4(p+q-k)\left[-p\cdot q + m^2\right]\ \times\left[p^2 - m^2 + i\epsilon\right]^{-1}\left[q^2 - m^2 + i\epsilon\right]^{-1}. \tag{C29}$$

Following same steps below Eqn. (4.5) leads now to

$$\widetilde{I}_5(k) = -i\sum_i \frac{c_i}{16\pi^2}\int_0^\infty\int_0^\infty \frac{dz_1dz_2}{(z_1+z_2)^2}\left[+\frac{2i}{(z_1+z_2)}-\frac{z_1z_2k^2}{(z_1+z_2)^2}+m_i^2\right]\cdot \times\exp\left\{i\left[k^2\frac{z_1z_2}{z_1+z_2}-(m_i^2-i\epsilon)(z_1+z_2)\right]\right\}. \tag{C30}$$

Similar to (C13) we now get

$$\widetilde{I}_5(k) = \frac{+3i}{16\pi^2}\sum_i c_i\int_0^\infty\int_0^\infty dz_1dz_2\ \delta(1-z_1-z_2)\int_0^\infty\frac{d\lambda}{\lambda}\left[z_1z_2k^2-m_i^2\right]\cdot \times\exp\left[i\lambda\left(z_1z_2k^2-m_i^2+i\epsilon\right)\right]. \tag{C31}$$

Analogous to the scalar case treated above, the finite part

$$i(\widetilde{\Delta}_F)_{5,ab}(k) = +\frac{i}{2\pi^2}\frac{k^2}{m_\chi^2}\int_{4m^2}^\infty ds\ \left(1+\frac{2m^2}{s}\right)\sqrt{1-\frac{4m^2}{s}}\left[s\left(k^2-s+i\epsilon\right)\right]^{-1}\delta_{ab}. \tag{C32}$$

and we can follow the same steps (C22) $\rightarrow$ (C25) giving for the "effective" propagator

$$(\widetilde{\Delta}(k^2))_{5,ab} \cong +\left(\frac{\Delta_{NQ}}{m_\chi}\right)^2\frac{F(k^2)}{k^2-\bar{m}_\chi^2+i\epsilon}\approx\left(\frac{\Delta_{NQ}}{m_\chi}\right)^2\frac{\exp(-\mathbf{k}^2/\Lambda^2)}{\mathbf{k}^2+\bar{m}_\chi^2}, \tag{C33}$$

which is analogous to (C26).
Then, the pseudoscalar diquark-exchange, see Fig. 3, gives the QN-potential, see *e.g.* [12],

$$V_{QN}(r) = -\lambda_5^2\frac{m_\chi^2}{4\pi}\left[\frac{m_\chi^2}{4m_Qm_N}\left(\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\phi_C^1(r)+S_{12}\phi_T^0\right)\right]\ P_x. \tag{C34}$$

and with a factor $\langle 1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\rangle/2 = 2I-1$. This gives, with $m_\chi = 2m_Q$, for $NQ\rightarrow QN$ in ${}^1S_0$ a coupling $+\lambda_5^2/3$ and in ${}^3S_1$ a coupling $+\lambda_5^2/3$. So, repulsion for S-waves. *We notice that the volume integral of this potential is zero, which is different from χ_μ^a-exchange. Therefore, it is expected to be weaker than the axial-vector and scalar diquark-exchange potential.* Note that from (C3) the scalar and pseudoscalar couplings are equal: $\lambda_S = \lambda_5 = 2\lambda_2$.

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