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Determination of the Residue at the Deuteron Pole in an np Phase-Shift Analysis*

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Abstract

In an energy-dependent phase-shift analysis of all low-energy *np* scattering data below $T_{\text{lab}} = 30$ MeV we reach $\chi^2/N_{\text{DF}} = 1.1$, where N_{DF} is the number of degrees of freedom. In our fit we determine the S-matrix elements in the coupled ${}^3S_1 + {}^3D_1$ channels, which allows us to compute the residue at the deuteron pole. Expressed in terms of the deuteron parameters, we find for the asymptotic normalization of the ${}^3S_1$ state $A_S = 0.8838(4)$ fm$^{-1/2}$ and for the asymptotic D/S ratio $\eta = 0.027\,12(22)$. Compared with other determinations, there seems to be some indication for the presence of closed isobar channels and/or energy-dependent potentials in the deuteron system.

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For several years now there has been a renewed interest in the precise determination of the deuteron parameters. These serve as an important constraint on the description of the np interaction. Special attention [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] has been given to the asymptotic D-to-S-state ratio η and, more recently, to the asymptotic normalization A_S of the S state [13, 14]. Most of the determinations of these two quantities come from analyses of either pd elastic scattering or (d,p) stripping reactions and (p,d) pickup reactions on various nuclei. Some discrepancies between the various determinations have shown up, especially for the value of η.

In this Letter we present a very accurate determination of η and A_S with the help of an energy-dependent phase-shift analysis of all np scattering data below $T_{\rm lab} = 30$ MeV. In this way the values for A_S and η are obtained purely from the two-body np scattering data, thereby circumventing the typical many-body problems arising in many of the other analyses.

In our phase-shift analysis the scattering matrix S and the K matrix are determined, where $S = (1+iK)(1-\ iK)^{-1}$. To study the deuteron, special attention is given to the coupled ${}^3S_1 + {}^3D_1$ channels. Time-reversal invariance allows us to choose the relative phases between the ${}^3S_1$ and ${}^3D_1$ channel such that the S and K matrices are symmetric above as well as below the threshold $E = 0$, with

$$E = (k^2 + M_p^2)^{1/2} + (k^2 + M_n^2)^{1/2} - (M_p + M_n)$$

the c.m. energy, k the relative c.m. momentum, and M_p and M_n the proton and neutron mass, respectively.

Unitarity requires that above the threshold ($E > 0$, $k > 0$) the S matrix is unitary, and therefore the K matrix Hermitean. Below the threshold ($E < 0$), the S matrix is real and the K matrix purely imaginary. The S and K matrices can be diagonalized simultaneously by a real, orthogonal matrix

$$U = \begin{pmatrix} \cos\epsilon & -\sin\epsilon \\ \sin\epsilon & \cos\epsilon \end{pmatrix} ,$$

where ϵ is the Blatt and Biedenharn [15] mixing parameter. The eigenvalues of S and K are $S_\lambda = \exp(2i\delta_\lambda)$ and $K_\lambda = \tan\delta_\lambda$ with $\lambda = 0$ or 2, and δ_λ the Blatt and Biedenharn eigenphase shifts. Next we introduce the projection operators $P_\lambda = P_\lambda^2$ on the scattering eigenstates, where $P_0 + P_2 = 1\!\!1$ and

$$P_0 = \begin{pmatrix} \cos^2\epsilon & \cos\epsilon\sin\epsilon \\ \cos\epsilon\sin\epsilon & \sin^2\epsilon \end{pmatrix} . \tag{1}$$

This allows us to write $S = \sum_\lambda S_\lambda P_\lambda$ and $K = \sum_\lambda K_\lambda P_\lambda$.

The existence of the deuteron at [16] $E = -B = -2.224\,575(9)$ MeV means that in the complex momentum plane the eigenvalue S_0 has a pole at $k = i\alpha$, and so at this position one has $K_0 = -i$. Here α is given by the relativistic deuteron radius (or decay length) $R = 1/\alpha = 4.318\,963$ fm. In the neighborhood of this deuteron pole the S matrix can be written as a pole part plus a regular function:

$$S = N_p^2 P_0/(\alpha + ik) + \text{reg. fn.} , \tag{2}$$

with

$$N_p^2 = -2/(\partial K_0/\partial k)_{k=i\alpha} \ . \tag{3}$$

When one has a parametrization of the S and K matrices valid in the neighborhood of this deuteron pole, then one can easily obtain the following deuteron parameters: the binding energy B as the energy where $K_0 = -i$, the mixing parameter ϵ by diagonalization of the K matrix at this energy, and N_p^2 by use of Eq. (3). The asymptotic normalizations $A_{S,p}$ of the S state and $A_{D,p}$ of the D state, and the asymptotic D/S ratio η_p, are then given by

$$\begin{aligned} &\eta_p = -\tan\epsilon \ , \ \ A_{S,p} = N_p/(1+\eta_p^2)^{1/2} \ , \\ &A_{D,p} = \eta_p A_{S,p} \ , \end{aligned} \tag{4}$$

where the subscript p indicates that the quantities are determined from the residue at the deuteron pole.

Let us next consider the deuteron wave function. We assume the simplest model with only NN channels and no admixture of other channels like $\Delta\Delta, NN^*$ (Roper), six-quark states, etc. The radial wave functions $u(r)$ in the $^3S_1$ channel and $w(r)$ in the $^3D_1$ channel are normalized such that $\int_0^\infty dr(u^2+w^2) = 1$. For $r \rightarrow \infty$ one has the asymptotic behavior

$$u \sim A_{S,d}e^{-r/R} \ , \ \ w \sim A_{D,d}e^{-r/R}(1+3R/r+3R^2/r^2) \ .$$

Now the deuteron normalization N_d and the asymptotic D/S ratio η_d are given by

$$\eta_d = A_{D,d}/A_{S,d} \ , \ \ N_d^2 = A_{S,d}^2 + A_{D,d}^2 \ , \tag{5}$$

where the subscript d indicates that the quantities are determined from the wave functions. When we assume moreover that the interaction is described by a local potential, then the deuteron parameters (with subscript p) as defined via the residue are equal to the analogous parameters (with subscript d) as defined via the wave functions. This is also the case if the potential contains a momentum dependence $\Delta\phi(r) + \phi(r)\Delta$, where Δ is the Laplacian. In the presence of explicitly energy-dependent potentials $V(r, E)$, however, $N_p \neq N_d$, because then

$$N_d^2 = N_p^2 \left[1 - \frac{2M_{\rm red}}{\hbar^2}\int_0^\infty dr \tilde{\psi}_d \left(\frac{\partial V}{\partial k^2}\right)_{k^2=-\alpha^2} \psi_d\right] \ , \tag{6}$$

where $\tilde{\psi}_d = (u, w)$ is the transpose of the radial deuteron wave function and $M_{\rm red}$ is the reduced mass. Similarly, if the deuteron system contains closed isobar channels, then again $N_p \neq N_d$. We would like to stress therefore that for theoretical models one should not only compute the deuteron parameters via the deuteron wave function, but one should also compute the residue at the deuteron pole.

From an energy-dependent phase-shift analysis of all np scattering data below $T_{\rm lab} = 30$ MeV one obtains a parametrization of the S and K matrices valid in the c.m. energy interval $0 \leq E \leq 14.95$ MeV. To get the K matrix in the neighborhood of the deuteron pole one must extrapolate from the scattering region $E > 0$ to $E = -B = -2.224\,575$ MeV. One way to do such an extrapolation is to take the effective-range expansion of the effective-range function

$$F(k^2) = k^{2l+1}\cot\delta = -1/a + \tfrac{1}{2}rk^2 + \ldots$$

The effective-range function F is a real, analytic function of k^2 which is regular in the neighborhood of $E = 0$ and has left-hand cuts due to one-pion exchange (OPE) starting at $E_0 = -4.9$ MeV and $E_+ = -5.2$ MeV. The presence of these nearby OPE singularities raises questions about the accuracy of the approximation over the whole energy range and also about the validity of the extrapolation to the deuteron pole. It is exactly for these reasons that we do not use effective-range expansions in our analysis, but prefer to use the P matrix in which these cuts can be removed explicitly.

In our analysis the coupled $^3S_1 + {}^3D_1$ channels are parametrized by the P matrix at $r = b = 1.4$ fm. This P matrix is the logarithmic derivative of the wave-function matrix at $r = b$: $P(E) = b[(\partial\psi/\partial r)\psi^{-1}]_{r=b}$. For $r > b$ we take the OPE potential into account exactly. This implies that $P(E)$ is a real analytic function in the complex energy plane, regular in the neighborhood of the scattering region. The nearest left-hand singularity in $P(E)$ is now due to two-pion exchange and is a cut starting at $E_L = -19.5$ MeV. $P(E)$ has also right-hand cuts (due to pion production) starting at $E_R = 132.7$ MeV.

In the c.m. energy region $0 < E < 15$ MeV where we analyzed the experimental data, this P matrix is sufficiently well parametrized by three parameters; for the diagonal element in the $^3S_1$ channel we use two parameters, for the off-diagonal element one parameter, and for the diagonal element in the $^3D_1$ channel we take the free P-matrix value at $T_{\rm lab} = 0$ MeV [17]: $l + 1 = 3$. The other lower partial waves ($l \leq 2$) are either parametrized in a similar way or the phase shifts are, after adapting them to np scattering [18], taken from our analysis of the low-energy pp data [17]. The higher partial waves with $l \geq 3$ are taken to be pure OPE.

In our phase-shift analysis we have a total number of 478 degrees of freedom and seven model parameters. We therefore expect $\langle\chi^2_{\rm min}\rangle = 478 \pm 31$. Data that were more than 3 standard deviations off are rejected, decreasing the expected $\langle\chi^2_{\rm min}\rangle$ by an additional 13. In our analysis we actually reach $\chi^2 = 527.3$. The errors are found by the variation of all parameters in such a way that χ^2 does not rise by more than 1, so the quoted errors are purely statistical. A more complete discussion of the phase-shift analysis will be given in a forthcoming paper.

To check our ability to extrapolate from the scattering region to the deuteron pole, we decided first to predict the location of the deuteron pole and its residue from the scattering data alone. We found

$$\begin{aligned} &B = 2.211 \pm 0.011 \text{ MeV} \ , \quad 10^3 N_p^2 = 777.1 \pm 3.6 \text{ fm}^{-1} \ , \\ &10^3\eta_p = 27.12 \pm 0.15 \ , \quad 10^3 A_{S,p} = 881.2 \pm 2.0 \text{ fm}^{-1/2} \ . \end{aligned}$$

We see that the predicted binding energy of the deuteron is a little more than 1 standard deviation off. The values for N_p^2 and $A_{S,p}$ are dependent on the binding energy of the bound state. Therefore, the values of N_p^2 and $A_{S,p}$ will not be totally accurate. A better procedure will be to predict the effective range $\rho_0(-B,-B)$ at the bound state for the eigenphase shift δ_0 from (for definitions of the various effective ranges see Hulthén and Sugawara [19])

$$\rho_0(-B,-B) = R(B) - 2/N_p^2 \ . \tag{7}$$

We then find $\rho_0(-B,-B) = 1.7591 \pm 0.0025$ fm. Using this value of $\rho_0(-B,-B)$, the correct binding energy, and Eq. (7), we predict

$$
\begin{aligned}
10^3 N_p^2 &= 781.3 \pm 0.8 \text{ fm}^{-1} \ , \\
10^3 A_{S,p} &= 883.5 \pm 0.5 \text{ fm}^{-1/2} \ .
\end{aligned}
$$

In the next step of our analysis the deuteron binding energy of Ref. [16] is included with its error and the P-matrix parameters are redetermined. The changes in the parameters are only small and χ^2 rises to 528.9. The residue at the deuteron pole is calculated again and we obtain

$$
\begin{aligned}
10^3 \eta_p &= 27.12 \pm 0.22 \ , \\
10^3 N_p^2 &= 781.6 \pm 0.7 \text{ fm}^{-1} \ .
\end{aligned}
$$

This implies then

$$
\begin{aligned}
10^3 A_{S,p} &= 883.8 \pm 0.4 \text{ fm}^{-1/2} \ , \\
10^3 A_{D,p} &= 24.0 \pm 0.2 \text{ fm}^{-1/2} \ , \\
\rho_0(-B,-B) &= 1.7602 \pm 0.0023 \text{ fm} \ .
\end{aligned}
$$

We have also made an effective-range expansion for the ${}^3S_1$ channel. For the scattering length a_t we find $a_t = 5.4193(20)$ fm and for the effective range r_t we find $r_t = \rho_0(0,0) = 1.7571(27)$ fm. This results in $\rho_0(0,-B) = 2R(1 - R/a_t) = 1.7539(25)$ fm.

Let us now compare our results with other determinations. The quantity $A_{S,p}$ can be determined by the analysis of nd (Ref. [13]) or pd (Refs. [13] and [14]) unpolarized differential cross sections. In this way Berthold and Zankel [13] find $10^3 A_{S,p} = 884.7 \pm 32.6$ fm$^{-1/2}$, whereas Borbély *et al.* [14] find the value $10^3 A_{S,p} = 878.1 \pm 4.4$ fm$^{-1/2}$. Kermode and co-workers [20, 21] make use of the effective-range expansion and some model input and obtain the value $10^3 A_{S,p} = 888.3 \pm 4.4$ fm$^{-1/2}$. Comparing our value of $A_{S,p}$ with these determinations we note a close agreement, where our result is the most accurate.

After observing in various potential models a linear relation between A_S (presumably $A_{S,d}$) and the deuteron radius r_d, Ericson [22] used the experimental values of r_d to predict $A_{S,d}$. Taking a weighted average of the r_d meaurements [23, 24], he recommends $10^3 A_{S,d} = 880.2 \pm 2.0$ fm$^{-1/2}$. Recently, Klarsfeld *et al.* [25] made a much more careful determination of r_d and they conclude that $10^3 A_{S,d} = 875.1 \pm 1.7$ fm$^{-1/2}$. When we compare this value of $A_{S,d}$ with our determination of $A_{S,p}$ we see that these values are significantly different. This presumably indicates the admixture of channels other than NN in the deuteron and/or energy-dependent potentials.

The direct experimental determination of η can be classified into three methods. The first method is based on the extraction of η_p from an angular extrapolation of the tensor polarized cross section σT_{22} in (d,p) elastic scattering to the neutron-exchange pole [1, 2]. The extrapolation is beset with difficulties because there is a nearby singularity due to the Coulomb interaction. A number of groups have made use of this method, the difference mainly lying in the way they do the extrapolation. We mention here the determination of Londergan, Price, and Stephenson [4] with $10^3\eta_p = 26.7 \pm 1.4$, of Horáček *et al.* [5] with $10^3\eta_p = 27.0 \pm 0.6$, and of Borbély *et al.* [6] with $10^3\eta_p = 26.7 \pm 0.4$.

In the second method, the same technique is used as in the first method, but now applied only to the ^{2}H$(d,p)^3$H data. The difficulties with the extrapolation procedure are now no longer present. This method has been used by Borbély *et al.* [6, 7], leading to $10^3\eta_p = 27.2 \pm 0.3$.

The third method is based on the fact that the distorted-wave Born-approximation calculations for the tensor analyzing powers T_{20}, T_{21}, and T_{22} of (d,p) sub-Coulomb stripping reactions ^{208}Pb$(d,p)^{209}$Pb turn out to have a strong dependence on η [8, 9]. We think these experiments measure η_d. From these tensor analyzing powers one can determine [8] the parameter D_2, which can be approximated very well by $D_2 \approx \eta_d R^2$. This results [8, 10, 11] in $10^3\eta_d = 27.1 \pm 0.8$. A more recent analysis [12], using lower energy ^{208}Pb data and including ^{136}Xe data, quotes $10^3\eta_d = 25.6 \pm 0.4$.

For the quantity η there are some indirect determinations, too. These theoretical estimates are based on the relation between η_d and the rms radius and the quadrupole moment, and on the assumption of a strong OPE dominance. Klarsfeld, Martorell, and Sprung [26, 27] arrive this way at $10^3\eta_d = 26.8 \pm 0.7$. On the other hand, Ericson and Rosa-Clot [28] claim that η_d can be determined to a high accuracy and nearly model independently as $10^3\eta_d = 26.33 \pm 0.35$.

When we compare the results for η_p with our value we observe a good agreement, where our result is the most accurate. However, the determinations of η_d lead to values that are somewhat lower than our value of η_p. Especially the very accurate result for η_d given by Rodning and Knutson [12] is significantly different from our value for η_p. We are not aware of any simple mechanism that makes η_p different from η_d. We checked explicitly that, with energy-dependent potentials and also with closed isobar channels, we still have $\eta_p = \eta_d$.

In summary, in an *np* phase-shift analysis of the scattering data below $T_{\rm lab} = 30$ MeV the deuteron binding energy B was included as a datum. The residue of S at the deuteron pole was calculated, leading to $10^3 A_{S,p} = 883.8 \pm 0.4$ fm$^{-1/2}$ and $10^3\eta_p = 27.12 \pm 0.22$, in close agreement with other determinations of these pole observables, but our determination is more accurate. We note a difference with the value of $A_{S,d}$ as determined by Klarsfeld *et al.* This difference is an indication either for energy-dependent potentials, or that the picture of the deuteron as a pure NN state is too simple and that one must allow for the admixture in the deuteron of closed isobar channels. There is also a difference between the value for η_d as determined by Rodning and Knutson and our value for η_p.

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