THEF-NIJM 02.01
KVI 1564
nucl-th/0201068

Angle-dependent normalization of neutron-proton differential cross sections

J.J. de Swart[a] and R.G.E. Timmermans[b]

[a] *Institute for Theoretical Physics, University of Nijmegen, P.O. Box 9010, 6500 GL Nijmegen, The Netherlands*

[b] *Theory Group, KVI, University of Groningen, Zernikelaan 25, 9747 AA Groningen, The Netherlands*

Abstract

Systematic errors in the database of *np* differential cross sections below 350 MeV are studied. By applying angle-dependent normalizations the χ^2-values of some seriously flawed data sets can be reduced significantly at the expense of a few degrees of freedom. The renormalized data sets can be made statistically acceptable such that they do not have to be discarded any longer in partial-wave analyses of the two-nucleon scattering data.

Typeset using REVTEX

I. INTRODUCTION

A measurement of the differential cross section for elastic neutron-proton (np) scattering is notoriously difficult. It is even so difficult that almost none of the data sets measured at energies below the pion-production threshold is completely free of systematic flaws. In partial-wave analyses (PWA's) of the np scattering data [1,2] some of these flaws do not give rise to sizable systematic contributions to χ^2. Data sets with such minor flaws will not distort too much the statistics [3] in these PWA's, and therefore such sets can be included in the database. Examples are the LAMPF data [4] and the TRIUMF data [5]. Some flaws, on the other hand, are so serious that their contribution to χ^2 dominates over the statistical contribution to the extent that the standard rules of statistics no longer apply. Consequently, such data sets [6–10] were excluded from the databases used in PWA's. This is of course an unfortunate and undesirable situation, especially in view of the waste of investment and effort involved in these experiments.

In this paper we present the "adnorm" method. This is a method of angle-dependent normalization [11] to treat certain systematically-flawed data sets. We will show that the application of this method to certain, previously unacceptable, data sets can give impressive results. The values of χ^2 drop dramatically and can even become statistically acceptable. This implies that these data sets should from now on be included in the np database instead of discarding them. The salvation of these systematically-flawed data sets [6–10] is a major accomplishment of our adnorm method.

In recent publications [13–16] we pointed out that the Uppsala data at 162 MeV [8,9] contain large systematic errors. Also some other np differential cross section measurements appeared to have systematic errors similar (but not identical) to the Uppsala data. The Princeton [6] and Freiburg [7,10] data are prominent examples.

In the following we will compare the data in the standard way with the Nijmegen partial-wave analysis PWA93 [1]. Since we do not have any explanation for these systematic experimental errors, we will simply accept the fact that some of the data have such errors. Next we will apply the adnorm method and demonstrate that this method can correct for some of such systematic errors. In order to save two sizable data sets we changed moreover slightly the definition of an individual outlier. In this way we obtain normalized data sets that are statistically acceptable and that can be included henceforth in the np databases for PWA's.

II. NORMALIZATION

An np differential cross section $\sigma(\theta, expt)$, consisting of N_{data} data points, is called *experimentally* normalized, when for this data set the normalization has actually been measured. In that case, it has an experimental norm $N(expt) = 1.00$ with a corresponding experimental error $\delta N(expt)$. For backward np scattering this error is often of the order of 4% or larger. In the Nijmegen PWA's we also determine for each data set the normalization $N(pwa)$ with an error $\delta N(pwa)$. This is a *calculated* normalization (for a discussion of these points see Ref. [16]). This error $\delta N(pwa)$ is in most cases less than 1% [1]. When these two normalizations $N(expt)$ and $N(pwa)$ differ by more than 3 standard deviations (s.d.), we remove the experimental normalization and its error from the database. The data set is then "floated," which means in practice that a very large normalization error is assigned to the data set. A data set is also floated when its normalization has not been measured at all. In the case of floated data the normalization is determined solely by the angular distribution. The number of degrees of freedom N_{df} for a set with N_{data} data points is then $N_{df} = N_{data} - 1$. One could say that for the determination of the normalization $N(pwa)$ we have sacrificed one degree of freedom.

For seriously flawed data sets one cannot get a sufficiently low value of χ^2 by merely adjusting or floating the normalization. The main point of this paper is the observation that for some of such unacceptable data sets we can introduce an *angle-dependent* normalization $N(\theta)$ in such a way that we essentially sacrifice *two or more* degrees of freedom to obtain a statistically acceptable value for χ^2. Such a sacrifice is unfortunately necessary to save these data sets from being discarded otherwise.

In the adnorm method we have to make assumptions about the angular dependence of the normalization. Therefore we first map the experimental angular interval $[\theta_{min}, \theta_{max}]$ onto the interval $[-1, 1]$. This mapping can be done in many ways. We use

$$x = (\theta - \theta_+)/\theta_- \ , \tag{1}$$

with $\theta_\pm = (\theta_{max} \pm \theta_{min})/2$. Next we expand $N(\theta)$ in orthogonal polynomials $S_n(x)$ on the discrete set of data points [17], and write

$$N(\theta) = N_0 \left[1 + f(\theta) \sum_{n=1}^{p} c_n \, S_n(x) \right] \ . \tag{2}$$

We allow for the introduction of an extra function $f(\theta)$. In practical cases we make the simplest choice $f(\theta) \equiv 1$, but *e.g.* $f(\theta) = 1/\sigma(\theta, pwa)$ could also be a suitable choice. The normalization N_0 and the p adnorm parameters c_n $(n = 1, \ldots, p)$ and their errors are determined by the least-squares method, where the data are compared with PWA93. In an actual PWA such data sets contribute with $N_{df} = N_{data} - (1 + p)$ degrees of freedom. The χ^2 that results when p adnorm parameters are introduced is called χ^2_p. Therefore, when the standard angle-independent normalization (with zero adnorm parameters) is applied the χ^2 is called χ^2_0.

How many adnorm parameters c_n do we have to introduce? The basic rule is that each one should cause a *significant* drop in χ^2. We apply a 3 s.d. criterion: We introduce the parameter c_n, when $\chi^2_{n-1} - \chi^2_n \geq 9$. This procedure of introducing additional parameters stops when no significant drop in χ^2 can be achieved anymore.

III. UPPSALA DATA

Let us see how this procedure works out for the Uppsala data [9] at $T_L = 162$ MeV. At this neutron beam energy the np differential cross section was measured in five overlapping angular regions. We ordered these sets by increasing neutron scattering angles and called the sets 1 to 5, where set 1 contains the data at the most forward angles and set 5 at the most backward angles [16]. These data were then compared to PWA93. We removed the point at 93° from set 2 because it contributes more than 9 (3 s.d.) to χ^2.

In Table I we list the number of data N_{data} in each set, the value χ^2_0 obtained by applying the standard angle-independent normalization (with adnorm parameters $c_n \equiv 0$), the values of χ^2_p obtained by applying the adnorm method of Eq. 1. From Table I we see that the fits of the sets 1 and 4 improve significantly (a drop in χ^2_p of much more than 9) when introducing only one adnorm parameter c_1. For sets 2 and 3 this did not happen and therefore we take $c_1 = 0$ for these two sets. For set 5 we must use two adnorm parameters. After the adnorm method is applied with only one adnorm parameter the slope of the angle-dependent normalization $N(\theta)$ is called α. The value of this slope can be found in the next-to-last row of Table I. This slope is used to compare systematic errors in different experiments. It is important to note that the slope for set 1 is positive, while the slopes for the sets 4 and 5 are negative. In the last row is given the variation in % of the normalization $N(\theta)$ in the case p=1 over the interval $[\theta_{min}, \theta_{max}]$.

The combined data set has $N_{data} = 87$ and when normalized in the standard angle-independent way (no adnorm parameters) we obtain $\chi^2_0 = 243$. This value is 12 s.d. higher than the expectation value $\langle\chi^2_0\rangle = 82(13)$. The value for χ^2 drops to $\chi^2_p = 92$ after introducing four adnorm parameters (c_1 for each of the sets 1 and 4, and c_1 and c_2 for set 5). The conclusion is that the large systematic errors of unknown origin present in the Uppsala data can be corrected for by using the adnorm method. The drop 151 in χ^2 resulting from the introduction of only four adnorm parameters is impressive.

To present the data similar to the way as was done by the Uppsala group [9], we averaged the data in the overlap regions between the different sets. The difference $\Delta\sigma(\theta) = N(\theta)\ \sigma(\theta, expt) - \sigma(\theta, pwa)$, normalized in the various ways discussed, is presented in Fig. 1. In the top panel we show the data normalized in the Uppsala way and we get $\chi^2 = 393$ for the 54 data points. In the middle panel we show the data normalized in the standard angle-independent way; this leads to $\chi^2_0 = 135$. In the bottom panel we present the data normalized with the adnorm method; we obtain then $\chi^2_p = 59$. In Fig. 1 one clearly sees the difference between the various ways the data have been normalized, and the enormous improvement obtained with the adnorm method.

IV. FREIBURG DATA

Another place where the angle-dependent normalization procedure works impressively is the abundant Freiburg data [7,10]. This data set consists of four different measurements (labeled expt I to expt IV) of np differential cross sections, each at twenty beam energies between $T_L = 199.9$ and 580.0 MeV, with a spacing of about 20 MeV. Because we compare with PWA93 we can only study data with energies less than 350 MeV, *i.e.* the eight energies from 199.9 MeV to 340.0 MeV. Because of their too high individual contribution to χ^2 (more than 3 s.d.) we remove from the database the four data points (expt, T_L, θ) = (II, 261.9 MeV, 154.96°), (II, 300.2 MeV, 148.34°), (II, 340.0 MeV, 148.07°), and (III, 199.9 MeV, 144.32°). We are left then with a total of 859 data points.

In Table II we present the χ^2_0/N_{data} values for these four experiments at eight energies after the standard angle-independent normalization. It is clear that χ^2_0 for most of these 32 data sets is much too high. For the total data set of 859 points we find $\chi^2_0 = 2139$, which is 32 s.d. higher than the expectation value $\langle\chi^2_0\rangle = 827(41)$. Therefore, the total Freiburg data set would normally be discarded in PWA's. However, we can try the adnorm method. The results of applying the adnorm method are presented in Table III. The first striking observation is the enormous drop in χ^2, from 2139 to 831, for the 859 data points. This drop was achieved by introducing next to the original 32 normalizations N_0 also 45 adnorm parameters. This implies on the average a drop of no less than 29 per adnorm parameter.

Looking at the four experiments separately, one sees that expt I and expt IV have values for χ^2_p that are -0.8 s.d. lower than their expectation values, that expt II has a χ^2_p that is 0.6 s.d. higher than its expectation value, and that expt III has a χ^2_p that is 3.3 s.d. higher than its expectation value. Unfortunately, this ordinarily means that expt III should be excluded from the database for PWA93. For the remaining 647 points of expt I, II, and IV we expect $\langle\chi^2_p\rangle = 588(34)$. We get $\chi^2_p = 571$, which is an excellent result. We have checked explicitly that the χ^2 distribution of the renormalized Freiburg data is in very good agreement with the theoretical expectation [3]. In Table III we also present the values of the slope α of the data at $T_L = 199.9$ MeV after applying the adnorm method with $p = 1$.

Expt III can also be saved when we are willing to bend a little our rule for individual outliers. When for expt III a 2.5 s.d. rule is used instead of a 3 s.d. rule, we must remove also the three data points (III, 199.9 MeV, 133.95°), (III, 240.2 MeV, 149.25°), and (III, 340.0 MeV, 130.47°) as 2.5 s.d. outliers. In Table III the most right column of expt III contains the relevant information for this case. For the 209 data points left from expt III we have the expectation value $\langle\chi^2_p\rangle = 191(20)$, and we find $\chi^2_p = 238$, which is 2.3 s.d. higher than expected. Therefore, expt III is now also statistically acceptable. The conclusion is that the four Freiburg experiments, consisting of 856 data points, can be made statistically acceptable with the adnorm method.

V. PRINCETON DATA

Finally, we consider the Princeton data [6]. This relatively old data set is generally not included in PWA's and these data were *e.g.* also discarded in the final version of PWA93. We can, however, revisit these data with the adnorm method. The results are given in Table IV. After we have removed two data points ($T_L = 313$ MeV, $\theta = 168.1°$ and $170.3°$) as 3 s.d. outliers, the total set contains 156 data points, divided over nine energies below $T_L = 350$ MeV. When we normalize these data in the standard manner we get $\chi^2_0 = 582$. This is about 25 s.d. higher than the expectation value. Next we applied the adnorm method. This required 14 additional adnorm parameters. The expectation value is then $\langle\chi^2_p\rangle = 133(16)$. We obtain $\chi^2_p = 195$, which is still 3.9 s.d. too high. Therefore the Princeton data, unfortunately, cannot be saved by the adnorm method alone, despite the enormous improvement in χ^2 from 582 to 195, which amounts on the average to a drop of 28 per adnorm parameter. However, when we are again willing to bend our rule for individual outliers a little, we can also save these data. According to the 2.5 s.d. rule the three data points $(T_L, \theta) = $ (224 MeV, 131.6°), (239 MeV, 139.7°), and (257 MeV, 178.6°) must also be omitted. There are then 153 data points left, which leads to the expectation value $\langle\chi^2_p\rangle = 130(16)$. The second entries in Table IV give the relevant information for this case. We obtain $\chi^2_p = 169$, which is 2.4 s.d. higher than expected.

VI. DISCUSSION AND CONCLUSIONS

About the adnorm method that we proposed here, the question could be raised: "Is it successful because it corrects for experimental errors, or perhaps because it corrects for unknown biases in the PWA's?" We claim that we correct for unknown systematic *experimental* errors. To demonstrate this we defined the slope parameter $\alpha = (1/N_0)(dN(\theta)/d\theta)$ for the case $p = 1$. Looking at the different problematic experiments in about the same angular region and at about the same energy, we note significant differences. In the backward direction $[150°, 180°]$ there are several experiments at about the same energy. These are the Uppsala set 5 at 162 MeV, the Freiburg expt's I and II at 199.9 MeV, and the Princeton data at 182 MeV. The values of $10^3\,\alpha$ are $-4.9(5)$, $-2.8(4)$, $-2.5(4)$, and $-0.7(4)$, respectively. These slopes do not agree! The disagreement between the Uppsala sets 1 and 2 and the Freiburg expt IV is worse. Uppsala set 1 covers the angular region $[73°, 107°]$ and has $\alpha = 3.7(8)\,10^{-3}$. Uppsala set 2 covers $[89°, 129°]$ and has $\alpha = 0.3(7)\,10^{-3}$. The Freiburg expt IV covers $[81°, 124°]$ and has $\alpha = -1.9(3)\,10^{-3}$, which is even of the *opposite* sign as the values for the Uppsala sets 1 and 2. When one wants to blame PWA93 for the discrepancy and claim that the Uppsala and Freiburg data are in agreement, then one should at least find the same values for α. Because the α values for these experiments are significantly different, we can conclude that the Uppsala and Freiburg data are not in agreement with each other. It is then also clear that the discrepancies are of experimental origin.

In conclusion, we have shown that an angle-dependent normalization of *np* differential cross sections can give rise to impressive drops in the χ^2 values for certain seriously flawed data sets. These data sets, that previously could not be included in PWA's, become statistically acceptable after application of the adnorm method. Some of the sets required also a slight change in the definition of outlier. In this manner, several *np* data sets [8–10,6] can be saved from oblivion.

ACKNOWLEDGMENTS

We thank M.C.M. Rentmeester, Th.A. Rijken, and R.A. Bryan for helpful discussions. The research of R.G.E.T. was made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences (KNAW).

REFERENCES

[1] V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, and J.J. de Swart, Phys. Rev. C **48**, 792 (1993); `http://NN-OnLine.sci.kun.nl`.

[2] R.A. Arndt, I.I. Strakovsky, and R.L. Workman, Phys. Rev. C **50**, 2731 (1994); Phys. Rev. C **62**, 034005 (2000); `http://gwdac.phys.gwu.edu`.

[3] J.R. Bergervoet, P.C. van Campen, W.A. van der Sanden, and J.J. de Swart, Phys. Rev. C **38**, 15 (1988).

[4] B.E. Bonner, J.E. Simmons, C.L. Hollas, C.R. Newsom, P.J. Riley, G. Glass, and M. Jain, Phys. Rev. Lett. **41**, 1200 (1978).

[5] R.K. Keeler, R. Dubois, E.G. Auld, D.A. Axen, M. Comyn, G. Ludgate, L.P. Robertson, J.R. Richardson, D.V. Bugg, J.A. Edgington, W.R. Gibson, A.S. Clough, N.M. Stewart, and B. Dieterle, Nucl. Phys. **A377**, 529 (1982).

[6] P.F. Shepard, T.J. Devlin, R.E. Mischke, and J. Solomon, Phys. Rev. D **10**, 2735 (1974).

[7] W. Hürster, Th. Fisher, G. Hammel, K. Kern, M. Kleinschmidt, I. Lehmann, H.Schmitt, L. Schmitt, and D.M. Sheppard, Phys. Lett. **B90**, 367 (1980).

[8] T.E.O. Ericson, B. Loiseau, J. Nilsson, N. Olsson, J. Blomgren, H. Condé, K. Elmgren, O. Jonsson, L. Nilsson, P.-U. Renberg, A. Ringbom, T. Rönnqvist, C. Tibell, and R. Zorro, Phys. Rev. Lett. **75**, 1046 (1995).

[9] J. Rahm, J. Blomgren, H. Condé, S. Dangtip, K. Elmgren, N. Olsson, T. Rönnqvist, R. Zorro, A. Ringbom, G. Tibell, O. Jonsson, L. Nilsson, P.-U. Renberg, T.E.O. Ericson, and B. Loiseau, Phys. Rev. C **57**, 1077 (1998).

[10] J. Franz, E. Rössle, H. Schmitt, and L. Schmitt, Physica Scripta **T87**, 14 (2000).

[11] Angle-dependent normalization has been proposed previously by the Zürich group [12] for the analysis of their *pp* cross sections measured at very low energies.

[12] Ch. Thomann, J.E. Benn, and S. Münch, Nucl. Phys. **A303**, 457 (1978).

[13] M.C.M. Rentmeester, R.A.M. Klomp, and J.J. de Swart, Phys. Rev. Lett. **81**, 5253 (1998).

[14] J.J. de Swart, M.C.M. Rentmeester, and R.G.E. Timmermans, πN Newsletter **13**, 96 (1997); `nucl-th/9802084`.

[15] M.C.M. Rentmeester, Nucl. Phys. **A631**, 431c (1998).

[16] M.C.M. Rentmeester, R.G.E. Timmermans, and J.J. de Swart, Phys. Rev. C **64**, 034004 (2001).

[17] For data points x_i ($i = 1, N_{data}$) on the x-interval $[-1, 1]$, we define the inner product $(x, y) = (1/N_{data}) \sum_i x_i\, y_i$. The polynomials $S_n(x)$ are orthogonal with respect to this inner product and normalized such that $S_n(1) = 1$.

TABLES

set	1	2	3	4	5	total
N_{data}	18	20	18	16	15	87
χ^2_0	38	35	18	35	117	243
χ^2_p/p	15/1	35/0	18/0	15/1	9/2	92
$10^3\,\alpha$	3.7(8)	0.3(7)	−1.5(6)	−2.2(5)	−4.9(5)	
var in %	12.8	3.2	5.4	7.0	12.2	

TABLE I. Results for the Uppsala data at 162 MeV.

T_L (MeV)	expt I	expt II	expt III	expt IV
199.9	71/27	58/27	44/25	57/22
219.8	66/27	42/28	71/27	64/22
240.2	76/27	50/30	53/27	73/23
261.9	82/27	40/30	65/27	122/23
280.0	68/27	56/32	57/26	132/24
300.2	65/27	102/32	74/27	62/24
320.1	47/27	63/33	82/26	70/24
340.0	47/27	60/33	70/27	50/24
total	522/216	471/245	516/212	630/186

TABLE II. The values of χ^2_0/N_{data} for the 32 Freiburg data sets [T_L,expt] and the totals per experiment.

T_L (MeV)	expt I	expt II	expt III	expt IV
199.9	29/1	21/1	44/0/37	13/2
219.8	26/2	23/1	31/1/31	16/2
240.2	23/2	30/1	28/1/21	18/2
261.9	24/2	16/2	27/2/27	32/1
280.0	15/2	36/1	26/1/26	19/1
300.2	24/2	44/1	31/2/31	25/1
320.1	20/1	36/2	33/2/33	16/1
340.0	18/2	33/1	40/1/32	14/1
total	179/14	239/10	260/10/238	153/11
$\langle\chi^2_p\rangle$	194(20)	227(21)	194(20)191	167(18)
s.d.	−0.8	0.6	3.3 / 2.3	−0.8
$10^3\,\alpha$	−2.8(4)	−2.5(4)	−1.1(5)	−1.9(3)

TABLE III. The values of χ^2_p/p (p is the number of adnorm parameters) for the 32 Freiburg data sets [T_L,expt], the totals per experiment, their expectation value, and the slope α. The last column for expt III gives χ^2_p after three additional data points were removed.

T_L(MeV)	N_{data}	χ_0^2	p	χ_p^2	$10^3\,\alpha$
182	14	11	0	11	$-0.7(4)$
196	16	49	2	27	$-1.2(3)$
210	16	43	1	14	$-1.9(4)$
224	16/15	71/67	1	24/16	$-2.6(4)$
239	18/17	46/37	1	22/14	$-1.9(4)$
257	19/18	90/61	2	32/22	$-1.7(3)$
284	19	114	2	21	$-2.4(3)$
313	17	76	2	20	$-2.2(3)$
344	21	82	3	24	$-2.1(4)$
total	156/153	582/540	14	195/169	

TABLE IV. Results for the Princeton data.

FIGURES

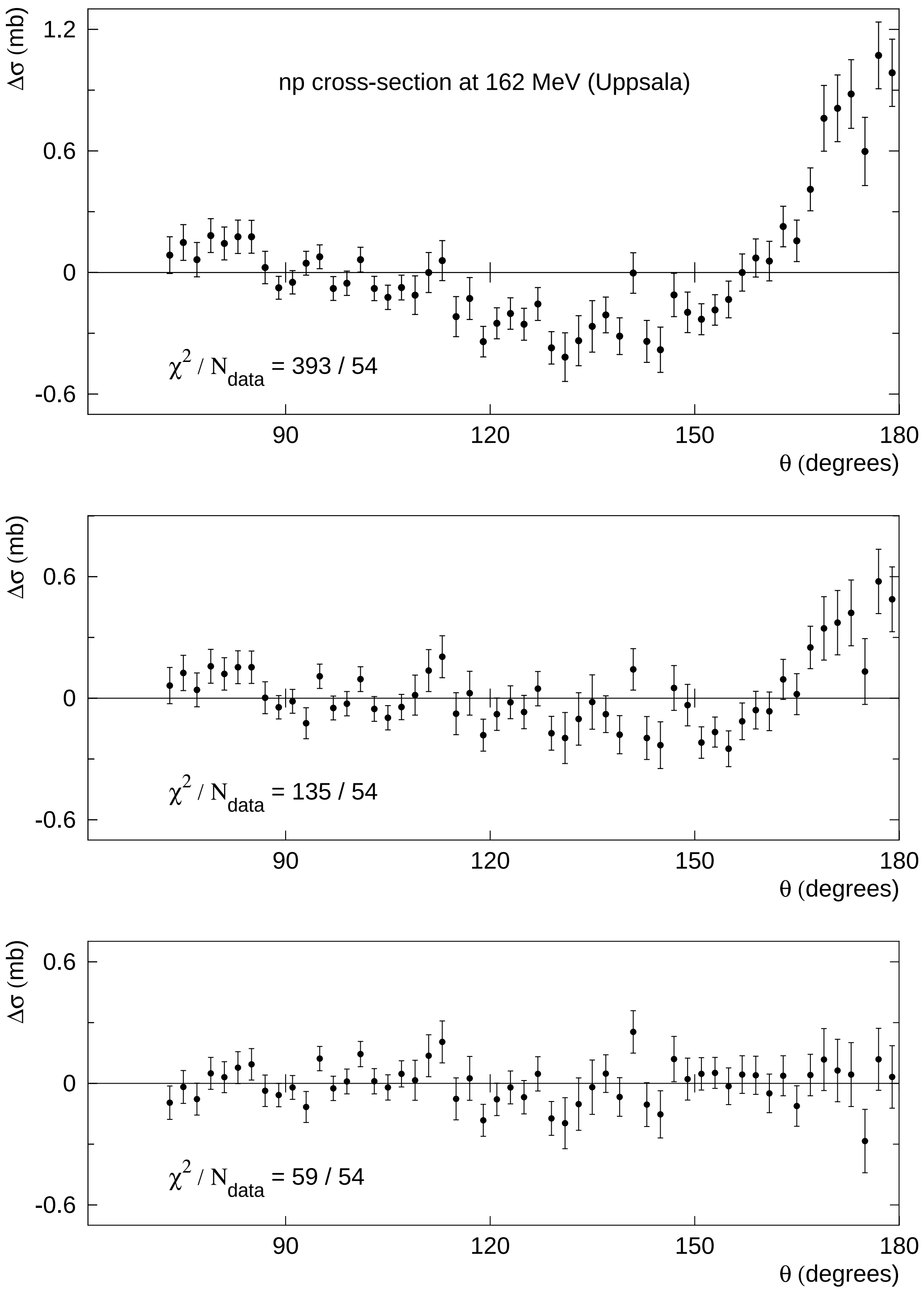

FIG. 1. Uppsala data at 162 MeV. Top panel: Uppsala's normalization. Middle panel: standard normalization using PWA93. Bottom panel: normalized using the adnorm method.