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Soft-core meson-baryon interactions. I. One-hadron-exchange potentials*

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The Nijmegen soft-core model for the pseudoscalar-meson baryon interaction is derived, analogous to the Nijmegen NN and YN models. The interaction Hamiltonians are defined and the resulting amplitudes for one-meson-exchange and one-baryon-exchange in momentum space are given for the general mass case. The partial wave projection is carried through and explicit expressions for the momentum space partial wave meson-baryon potentials are presented.

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I. INTRODUCTION

The strong interactions between mesons and baryons, in particular for pion-nucleon, kaon-nucleon and antikaon-nucleon, have been the subject of investigation for some decades, experimentally as well as theoretically.

A large number of scattering experiment have been performed to investigate the pion-nucleon interaction. The empirical phase shifts are obtained from a partial wave (PW) analysis of the scattering observables, which analysis judges the consistency with general principles of the scattering data, and which provides a compact representation of these data. Although at first sight in principle an infinite number of phase shifts need to be determined from the data, the strong interactions are short-ranged and only the lower partial wave phase shifts will suffice. In constructing theoretical models for these interactions, it is usually much more economic to use the results of a PW-analysis than the scattering observables themselves. Different pion-nucleon PW-analyses [1–3] give quite accurate and consistent results. The most recent pion-nucleon PW-analysis has been performed by Arndt et al. [1], to which we refer for more information on the current pion-nucleon scattering data base.

However, the situation for the kaon-nucleon interaction is different from the pion-nucleon interaction. The kaon-nucleon scattering observables are known to less accuracy, especially at low energies, due to the relatively low flux of the kaon beams. Consequently, the different kaon-nucleon phase shift analyses do not give quite accurate and may be not totally consistent results. The most recent kaon-nucleon PW-analysis has been performed by Hyslop et al. [4], where much information on the kaon-nucleon scattering data base can be found.

This lack of empirical knowledge makes it impossible to construct realistic theoretical kaon-nucleon models, using as input only information from KN-data.

Recently there has been an increase of interest in the kaon-nucleon and antikaon-nucleon interaction. An exotic resonance, the so called "penta-quark", in the isospin zero kaon-nucleon system has been observed [5], this experiment, however, was not a simple scattering experiment and a resonance has never been seen in the present kaon-nucleon scattering data.

The construction of new *K-factories* at the Japan Proton Accelerator Research Complex (J-PARC), and at GSI (FAIR) in Germany, will hopefully change the experimental situation drastically. One of the major beams of these new accelerators will be kaon beams, having a much higher intensity (ca. ten times) than the presently available kaon beams, for example at Brookhaven National Laboratory and KEK. Therefore, in the near future many more and accurate experimental data on the kaon-nucleon and antikaon-nucleon interaction can therefore be expected. Other new scattering data could be delivered by the DAΦNE facility at Frascati [6]. These activities will give much stronger constraints on kaon-nucleon models and a better understanding of the role of $SU_f(3)$ in meson-baryon interactions. Akaishi and Yamazaki [7] have investigated the possibility of nuclear anti-kaon bound states in nuclei in the framework of the Brueckner-Hartree-Fock theory using a simple phenomenological antikaon-nucleon model. Such a state has indeed been observed experimentally [8].

In view of these experimental and theoretical developments it is rather timely to construct theoretical kaon-nucleon models as realistically as possible, and this work is an attempt to do so.

The subject of this work is the construction of a dynamical model for the pion-nucleon (πN) and kaon-nucleon (K^+N) interactions. In two papers we describe the so called Nijmegen soft-core meson-baryon model (NSC model) and report on the results, obtained so far. First a soft-core meson- and baryon-exchange model for the πN interaction is derived, showing that the soft-core approach of the Nijmegen group is not only successful for baryon-baryon (NN and YN) interactions but also for meson-baryon interactions. The rich and accurate πN

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scattering data base is used to determine the nonstrange coupling constants. Several other πN models already exist and the NSC πN-model, besides the value in its own right, mainly serves as a natural starting point for the construction of the NSC K^+N-model. This K^+N-model is an $SU_f(3)$ extension of the NSC πN-model, similar to the successful Nijmegen soft-core one-boson-exchange nucleon-nucleon and hyperon-nucleon models [9] and [10]. In this way many parameters in the NSC K^+N-model are determined by the NSC πN-model, and the lack of accurate K^+N-data can be overcome partially.

In concept, the approach for the strong low- and intermediate-energy hadron-hadron interactions [11–13], used by the Nijmegen group, is schematically outlined in Figure 1. The starting point is the Standard Model, in which strong interactions occur between the six quarks and the gluons, and integrating out the heavier quarks to arrive at an effective QCD for the light quarks (u,d,s) only. Generally accepted, the vacuum of QCD becomes unstable for momenta transfer $q^2 \leq \Lambda^2_{\chi SB} \simeq 1\text{GeV}^2$ and the chiral symmetry is broken spontaneously (χSB). The vacuum goes through a phase transition and generates constituent quark masses ($m_q \approx 300$ MeV) and reduces the strong coupling constant α_s. The pseudoscalar-mesons are viewed as the Nambu-Goldstone bosons originating from the χSB, which makes it natural to assume the presence of a meson-cloud around the constituent quarks.

This provides a natural basis for an approach to the interaction between mesons and baryons using effective baryon-meson Lagrangians. At low- and intermediate-energies we do not consider a mixed phase of hadrons and quarks, as is done by others using the resonating-group method (RGM) [14–16], but restrict ourselves to the hadronic phase only. Furthermore, heavy baryons and mesons can be viewed as being integrated out, using for example the renormalization method in the manner of Wilson [17], and an effective field theory, with meson and baryon masses with M$\leq$ 1.5 GeV, results. In this work, this general picture is appealed to in the construction of a soft-core meson-baryon model for low- and intermediate-energy interactions.

In the NSC model the one-meson-exchange and one-baryon-exchange potentials are obtained from field theoretical Feynman diagrams in momentum space using effective interaction Hamiltonians, together with the meson-baryon Green's function they constitute the kernel of the two-particle integral equation for the amplitude, which is a three-dimensional reduction of the fully covariant (four-dimensional) Bethe-Salpeter equation [18]. Alternatively, one could view this work in the framework of the covariant perturbation theory as formulated by Kadyshevsky [19–22]. Here the particles, also in the intermediate states, remain on the mass shell, and pair-suppression can be implemented in a covariant way. Moreover, the three-dimensional integral equation obtained in the Kadyshevsky-scheme has exactly the same form as used in this work.

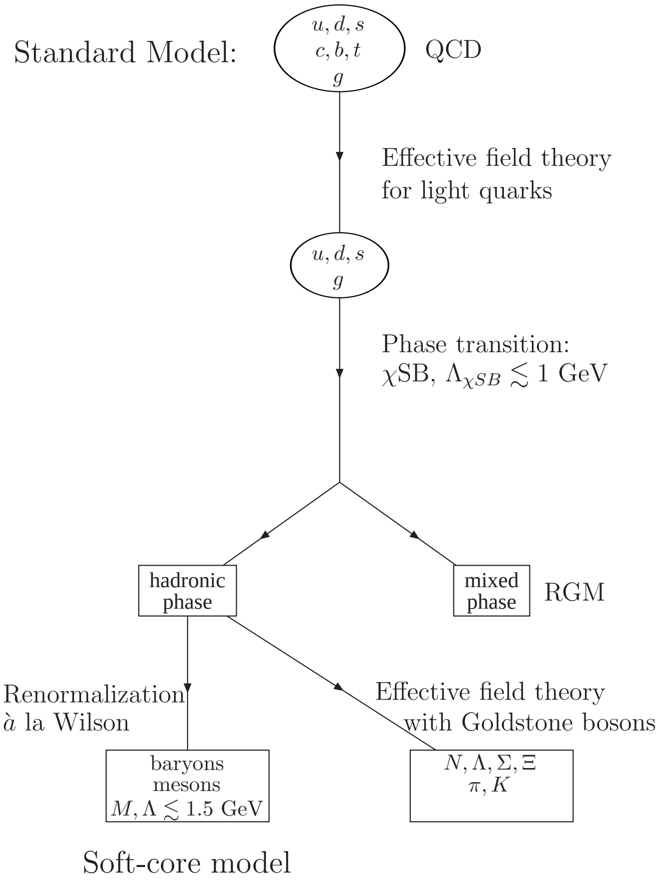

Figure 1: Overview of the theoretical basis for the soft-core meson-baryon interactions.

Form factors of the Gaussian type are introduced to take into account the extended size of the hadrons and to make the integral equation of the Fredholm type. The Coulomb interaction, playing a role at very low energies only and which is important in charge symmetry breaking (CSB) studies, will be neglected in this work. The integral equation for the amplitude is solved on the partial wave basis, in this way only one-dimensional integrals need to be performed to find the amplitude and the corresponding scattering observables or phase shifts for each partial wave.

We present this work in two papers. In general, this first paper, referred to as I, contains a description of the theory and the second paper, referred to as paper II [23], gives the results for πN and K^+N.

The contents of this first paper are as follows. The definition of the field theoretical one-meson-exchange and one-baryon-exchange potentials in the context of a three-dimensional integral equation, a relativistic generalization of the Lippmann-Schwinger equation, is reviewed in Sec. II. We introduce the usual potential forms in Pauli-spinor space, where we include the central (C) and the spin-orbit (SO) potentials, which are the only relevant potentials in case of spin-1 spin-$\frac{1}{2}$ interactions. And the relations between the relativistic and center-of-mass amplitudes are given.

The integral equation for the amplitude is solved on the partial wave basis in order to find the partial wave phase shifts, which are compared with the empirical phase shifts. Therefore we perform the basic partial wave projections, in particular those for the spinor invariants, in Sec. III. And the relations between the partial wave amplitudes, the phase shifts and the scattering observables, σ, $d\sigma/d\Omega$ and P, for spin-1 spin$\frac{1}{2}$ scattering are given. The partial wave basis is chosen according to the convention of [24].

The effective baryon-baryon-meson and meson-meson-meson interaction Hamiltonians from which the one-meson-exchange and one-baryon-exchange Feynman diagrams are derived, are given in Sec. IV. The explicit expressions for the momentum space Feynman diagrams for scalar-meson-, vector-meson-, tensor-meson- and baryon-exchanges for general baryon and meson masses as well as their partial wave projections are also listed in this section.

In the appendices added, details are given on the calculation of the partial wave matrix elements (Appendix A), the one-particle-exchange Feynman diagrams (Appendix B), and the expansion coefficients, X, Y and Z, of the partial wave potentials in $x = \cos\theta$ for the different exchanges (Appendix C).

For results and a discussion, we refer to paper II.

II. THE MESON-BARYON POTENTIAL AND AMPLITUDE

The field theoretical one-particle-exchange meson-baryon potentials in the context of a two-particle equation are defined in this section for the case of $J^{PC} = 0^{-+}$ mesons and $J^P = \frac{1}{2}$ baryons. We approximate the Bethe-Salpeter equation by assuming "dynamical pair suppression", hence neglecting the propagation of negative-energy states and by integrating out the time variable, we end up with a three-dimensional integral equation for the meson-baryon amplitude in the center of mass system. The relations between the center of mass and the relativistic amplitudes are given in the last part of this section.

A. Kinematics and relativistic amplitudes

We consider the meson-baryon or more specific the πN and K^+N reactions

$$\mathcal{P}_i(q) + \mathcal{B}_i(p,s) \rightarrow \mathcal{P}_f(q') + \mathcal{B}_f(p',s') \ , \tag{2.1}$$

where $\mathcal{P}$ stands for the pseudoscalar-mesons, and $\mathcal{B}$ stands for the baryons. We will refer to $\mathcal{P}_i$ and $\mathcal{P}_f$ as particles 1 and 3 and to $\mathcal{B}_i$ and $\mathcal{B}_f$ as particles 2 and 4. The four momentum of particle i is $p_i = (E_i, \mathbf{p}_i)$ where $E_i = \sqrt{\mathbf{p}_i^2 + M_i^2}$ is the energy and M_i is the mass of particle i. In our convention the transition amplitude matrix M is related to the S-matrix via

$$\langle f|S|i\rangle = \langle f|i\rangle - i(2\pi)^4\delta^4(P_f - P_i)\langle f|M|i\rangle \ , \tag{2.2}$$

in this convention a negative potential corresponds with attraction and a positive potential with repulsion. Here $P_i = p + q$ and $P_f = p' + q'$ represent the total four momentum for the initial state $|i\rangle$ and the final state $|f\rangle$. The latter refer to the two-particle states, which we normalize in the following way, see e.g. [25, 26],

$$\begin{aligned}\langle \mathbf{p}'_1, \mathbf{p}'_2|\mathbf{p}_1, \mathbf{p}_2\rangle \ &= \ (2\pi)^3 2E(\mathbf{p}_1)\delta^3(\mathbf{p}'_1 - \mathbf{p}_1) \times \\ & \qquad (2\pi)^3 2E(\mathbf{p}_2)\delta^3(\mathbf{p}'_2 - \mathbf{p}_2) \ .\end{aligned} \tag{2.3}$$

With this normalization, the unpolarized differential cross section in the center of mass (CM) system is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{p_f}{p_i}\frac{1}{2}\sum\left|\frac{\langle f|M|i\rangle}{8\pi\sqrt{s}}\right|^2 \ , \tag{2.4}$$

where $\sum$ stands for the summation over the spin of the final baryon.

Since in this work, the scattering particles are always on the mass-shell, i.e. $p_i^2 = m_i^2$, parity conservation and Lorentz invariance implies that the matrix elements of the M-operator for meson-baryon interactions, which is a 4×4-matrix sandwiched between Dirac spinors, can be written in terms of two independent amplitudes

$$\begin{aligned}\langle f|M|i\rangle \ &= \ \bar{u}_{B'}(\mathbf{p}', s_f)\left[A_{fi}(s,t,u) + \frac{\not{q}' + \not{q}}{2}B_{fi}(s,t,u)\right] \\ & \quad \times \ u_B(\mathbf{p}, s_i) \ ,\end{aligned} \tag{2.5}$$

where f and i stand for the two-particle channels $\pi N, K^+N$, etc.... In the Dirac spinors s_f, s_i are the magnetic spin variables, which will be specified later. The functions $A_{fi}(s,t,u)$ and $B_{fi}(s,t,u)$ are Lorentz scalars, and depend on the Mandelstam invariants

$$\begin{aligned} s \ &= \ (p+q)^2 = (p'+q')^2 \ , \\ t \ &= \ (q'-q)^2 = (p-p')^2 \ , \\ u \ &= \ (p-q')^2 = (p'-q)^2 \ , \end{aligned} \tag{2.6}$$

which satisfy the well-known (on-mass-shell) relation $s + t + u = \sum_{i=1}^4 m_i^2$. The total and relative four-momenta (P_c and k_c) of the initial, final, and intermediate channel ($c = i, f, n$) are defined by

$$P_c \ = \ p_c + q_c \ , \ k_c = \mu_{c,2}\ p_c - \mu_{c,1}\ q_c \ , \tag{2.7}$$

where the weights are arbitrary besides the condition $\mu_{c,1} + \mu_{c,2} = 1$. For each channel the four-momenta of the baryons and pseudoscalar-mesons (p_c and q_c) in terms of P_c and k_c are

$$p_c \ = \ \mu_{c,1}P_c + k_c \ , \ \ q_c = \mu_{c,2}\ P_c - k_c \ . \tag{2.8}$$

In this work we will use $\mu_{c,1}=\mu_{c,2}=\frac{1}{2}$. In the center-of-mass system (CM-system) we have for on-mass-shell momenta

$$\begin{aligned} p_c &= (E(\mathbf{p}_c), \mathbf{p}_c) \ , \ q_c = (\mathcal{E}(\mathbf{p}_c), -\mathbf{p}_c) \ , \\ P_c &= (W_c, \mathbf{0}) \ , \qquad k_c = (\mu_2 E(\mathbf{p}_c) - \mu_1 \mathcal{E}(\mathbf{p}_c), \mathbf{p}_c) \ , \end{aligned} \tag{2.9}$$

where the total energy is $W_c=\sqrt{s}=E(\mathbf{p}_c)+\mathcal{E}(\mathbf{p}_c)$. Obviously the relative three-momentum is equal to the center-of-mass three-momentum of the baryon.

In general Feynman diagrams, in particular in the Green's functions, the particles are off-mass-shell. In the following the three-momenta for the initial and the final states are denoted respectively by $\mathbf{q}_i$ and $\mathbf{q}_f$. Because of translation invariance $P_i=P_f$ and so $\sqrt{s}=W_i=W_f$. As introduced here, the total energies in the CM-system are $W_i=E(\mathbf{q}_i)+\mathcal{E}(\mathbf{q}_i)$, and $W_f=E(\mathbf{q}_f)+\mathcal{E}(\mathbf{q}_f)$.

B. Relativistic two-particle equations

The Bethe-Salpeter equation, a full two-particle relativistic scattering equation, for the $\mathcal{M}$-amplitudes reads

$$\mathcal{M}_{fi}(q_f,q_i;P) = \mathcal{M}^{irr}_{fi}(q_f,q_i;P) + \sum_n \int d^4k_n \times \mathcal{M}^{irr}_{fn}(q_f,k_n;P)\, G(k_n,P)\, \mathcal{M}_{ni}(k_n,q_i;P) \ , \tag{2.10}$$

where the interaction kernel is denoted by $\mathcal{M}^{irr}$, and G is the two-particle Green's function. The contributions to the kernel $\mathcal{M}^{irr}$ come from the meson-baryon irreducible Feynman diagrams. The reducible diagrams are generated by the integral equation. In deriving Eq. (2.10) the integration over the momenta of the intermediate particles can be replaced by an integration over the total and relative momenta $\int\int d^4p_n d^4q_n \to \int\int d^4P_n d^4k_n$. Then, using the conservation of the total four-momentum, one can perform $\int d^4P_c$ and separate an overall $(2\pi)^4\delta^4(P_f-P_i)$ factor. The meson-baryon Green's function is given in terms of the one-particle Green's functions

$$\begin{aligned} G(k_n,P) &= \frac{i}{(2\pi)^4}\left[\frac{1}{\gamma\left(\mu_{n,1}P+k\right)-M_n}\right]^{(a)} \\ &\times \left[\frac{1}{\left(\mu_{n,2}P-k\right)^2-m_n^2}\right]^{(b)} . \end{aligned} \tag{2.11}$$

It is instructive to separate the positive- and the negative-energy components of the propagator. For that purpose, we rewrite the one-particle propagators as follows. For the spin-$\frac{1}{2}$ baryons the off-mass-shell propagator can be written in terms of the Dirac spinors as

$$\frac{\not p+M}{p^2-M^2+i\delta} = \frac{M}{E(\mathbf{p})}\left[\frac{\Lambda_+(\mathbf{p})}{p^0-E(\mathbf{p})+i\delta} - \frac{\Lambda_-(-\mathbf{p})}{p^0+E(\mathbf{p})-i\delta}\right] , \tag{2.12}$$

where the projection operators $\Lambda_+(\mathbf{p})$ and $\Lambda_-(\mathbf{p})$ on the positive- and negative-energy states are [27]

$$\begin{aligned} \Lambda_+(\mathbf{p}) &= \sum_s \frac{u(\mathbf{p},s)\otimes\bar{u}(\mathbf{p},s)}{2M} = \frac{\not p+M}{2M} \ , \\ \Lambda_-(\mathbf{p}) &= -\sum_s \frac{v(\mathbf{p},s)\otimes\bar{v}(\mathbf{p},s)}{2M} = \frac{-\not p+M}{2M} \ , \end{aligned} \tag{2.13}$$

and $u(\mathbf{p},s)$ and $v(\mathbf{p},s)$ are the Dirac spinors for spin-$\frac{1}{2}$ particles, which are on-mass-shell by definition. For the meson propagator similar to Eq. (2.12) one has the identity

$$\frac{1}{q^2-m^2+i\delta} = \frac{1}{2\mathcal{E}(\mathbf{q})}\left[\frac{1}{q^0-\mathcal{E}(\mathbf{q})+i\delta} - \frac{1}{q^0+\mathcal{E}(\mathbf{q})-i\delta}\right] . \tag{2.14}$$

Then, in the CM-system, where $\mathbf{P}=0$ and $P_0=W$, the meson-baryon Green's function can be written as

$$\begin{aligned} G(k_n,P) &= \frac{i}{(2\pi)^4}\left[\frac{M}{2E(\mathbf{k}_n)\mathcal{E}(\mathbf{k}_n)}\right] \\ &\times \left[\frac{\Lambda_+(\mathbf{k}_n)}{\mu_{n,1}W+k_n^0-E(\mathbf{k}_n)+i\delta} - \frac{\Lambda_-(-\mathbf{k}_n)}{\mu_{n,1}W+k_n^0+E(\mathbf{k}_n)-i\delta}\right] \\ &\times \left[\frac{1}{\mu_{n,2}W-k_n^0-\mathcal{E}(\mathbf{k}_n)+i\delta} - \frac{1}{\mu_{n,2}W-k_n^0+\mathcal{E}(\mathbf{k}_n)-i\delta}\right] . \end{aligned} \tag{2.15}$$

Multiplying out Eq. (2.15), writing the ensuing terms using an obvious short hand notation, the contribution of the different propagating components is displayed fully

$$G(k_n,P) = G^+(k_n,W) + G^-(k_n,W) \ , \tag{2.16}$$

where the superscripts $(+)$ and $(-)$ indicate the positive- and negative-energy baryon states. Considering similarly the amplitudes $M^{\beta,\alpha}_{ij}$:

$$\begin{aligned} M^{+,+}_{ij} &= \bar{u}_{B'}(\mathbf{p}_f,s_f)\mathcal{M}_{ij}u_B(\mathbf{p}_i,s_i) \ , \\ M^{+,-}_{ij} &= \bar{u}_{B'}(\mathbf{p}_f,s_f)\mathcal{M}_{ij}v_B(\mathbf{p}_i,s_i) \ , \\ &\text{etc.} \end{aligned} \tag{2.17}$$

where the subscripts i and j refer to the different two-particle channels, one obtains from Eqs. (2.10), (2.16) and (2.17) the full relativistic scattering equation

$$\begin{aligned} M^{\beta,\alpha}_{fi}(q_f,q_i;P) &= (M^{irr})^{\beta,\alpha}_{fi}(q_f,q_i;P) + \\ &\sum_n \int d^4k_n (M^{irr})^{\beta,\gamma}_{fn}(q_f,k_n;P) \times \\ &G^{\gamma}_n(k_n,P)\, M^{\gamma,\alpha}_{ni}(k_n,q_i;P) \ . \end{aligned} \tag{2.18}$$

In all we have $2^2=4$ amplitudes, which are coupled as illustrated in Eq. (2.18).

The complexity of the previous equation can be reduced considerably if we assume dynamical pair-suppression, i.e. if we neglect the contribution of negative-energy states. Then the full scattering equation Eq. (2.18), for $\alpha = +$ and $\beta = +$, reduces to the four-dimensional integral equation

$$\begin{aligned} M^{+,+}_{fi}(q_f, q_i; P) &= (M^{irr})^{+,+}_{fi}(q_f, q_i; P) + \\ &\sum_n \int d^4k_n (M^{irr})^{+,+}_{fn}(q_f, k_n; P) \\ &G^+_n(k_n, P)\ M^{+,+}_{ni}(k_n, q_i; P)\ , \end{aligned} \tag{2.19}$$

with the positive-energy Green's function

$$\begin{aligned} G^+(k_n, P) &\approx \frac{i}{(2\pi)^4}\left[\frac{1}{4E(\mathbf{k}_n)\mathcal{E}(\mathbf{k}_n)}\right] \\ &\times \frac{1}{[\mu_1 W + k^0_n - E(\mathbf{k}_n) + i\delta]} \\ &\times \frac{1}{[\mu_2 W - k^0_n - \mathcal{E}(\mathbf{k}_n) + i\delta]}\ . \end{aligned} \tag{2.20}$$

We note that this simplification in principle brings about a hopefully tolerable breach of relativistic invariance. On the other hand in Feynman diagrams particles go off-mass-shell, and the off-mass-shell behavior is not really known for mesons and baryons, certainly not if a truncated kernel is used, which is always the case. Then it might be better to allow positive-energy states only.

C. Three-dimensional two-particle equations

Three-dimensional integral equations for the amplitudes can be derived in various ways. The methods assume 2-particle unitarity as a basic ingredient. The derivation for the meson-baryon systems follows the same procedure as that for the baryon-baryon channels. For the latter see e.g. references [28–34]. In [35] the derivation is based entirely on two-particle unitarity and the analyticity properties of the amplitudes, using the N/D-formalism. In the latter approach the in essence Regge pole nature of meson-exchange can be apprehended most easily.

1. *On-mass-shell approximation*

The simplest way to reduce the four-dimensional integral equation, Eq. (2.19), to a three-dimensional one is to put the intermediate particles on the mass-shell, i.e. $p^0_n = E(\mathbf{k}_n) = \sqrt{\mathbf{k}^2_n + M^2_n}$, $q^0_n = \mathcal{E}(\mathbf{k}_n) = \sqrt{\mathbf{k}^2_n + m^2_n}$. It can readily be shown from Eq. (2.8) that the zero components of the relative and total momenta k_n and P_n are given by

$$\begin{aligned} k^0_n &= \mu_{n,2}E(\mathbf{k}_n) - \mu_{n,1}\mathcal{E}(\mathbf{k}_n)\ , \\ P^0_n &= E(\mathbf{k}_n) + \mathcal{E}(\mathbf{k}_n)\ . \end{aligned} \tag{2.21}$$

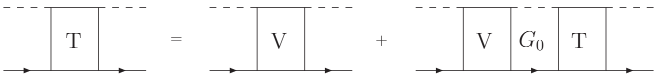

Figure 2: Diagrammatic representation of the meson-baryon scattering equation Eq. (2.24). The solid line denotes the baryon and the dashed line denotes the meson.

If we neglect the k^0_n-dependence of the amplitudes, and evaluate them at the value given by Eq. (2.21), the dependence of the four-dimensional equation on k^0_n only occurs in the Green's function, and the k^0_n-integration of the Green's function can be done. We can define the k^0_n-independent amplitudes

$$\begin{aligned} T_{ni}(\mathbf{k}_n, \mathbf{q}_i; W) &= M^{+,+}_{ni}(\tilde{k}_n, q_i; P)\ , \\ V_{fn}(\mathbf{q}_f, \mathbf{k}_n; W) &= \left(M^{irr}\right)^{+,+}_{fn}(q_f, \tilde{k}_n; P)\ , \end{aligned} \tag{2.22}$$

where $\tilde{k}^0_n = \mu_{n,2}E(\mathbf{k}_n) - \mu_{n,1}\mathcal{E}(\mathbf{k}_n)$. Now, the k^0_n-integration in Eq. (2.19) can be carried through, this leads to

$$\begin{aligned} G_0(\mathbf{k}_n; W) &= \int_{-\infty}^{\infty} dk_{n0}\ G^+_n(k_n; P) \\ &= \frac{1}{(2\pi)^3}\frac{1}{4E(\mathbf{k}_n)\mathcal{E}(\mathbf{k}_n)} \\ &\times \frac{1}{W - E(\mathbf{k}_n) - \mathcal{E}(\mathbf{k}_n) + i\delta}\ . \end{aligned} \tag{2.23}$$

The four-dimensional integral equation, Eq. (2.19), now results in the three-dimensional integral equation , which is also derived in [34],

$$\begin{aligned} T_{fi}(\mathbf{q}_f, \mathbf{q}_i; W) &= V_{fi}(\mathbf{q}_f, \mathbf{q}_i; W) + \sum_n \int \frac{d^3k_n}{(2\pi)^3} \times \\ &V_{fn}(\mathbf{q}_f, \mathbf{k}_n; W) G_0(\mathbf{k}_n, W) T_{ni}(\mathbf{k}_n, \mathbf{q}_i; W)\ . \end{aligned} \tag{2.24}$$

The integral equation for the T-matrix, Eq. (2.24), is schematically given in Figure 2.

We remark that the three-dimensional integral equation for the amplitude, Eq. (2.24), is here obtained as an approximation of the Bethe-Salpeter equation, but in the formulation of Quantum Field Theory (QFT) as developed by Kadyshevsky [19–22] this integral equation is obtained without making any approximation. In this formulation of QFT all particles, in particular the intermediate particles, are always on the mass-shell in contrast to the formalism of Feynman. Hence a covariant form of pair-suppression can be introduced phenomenologically.

Until this subsection the intermediate particles were in principle off-mass-shell and the total four momentum was conserved. Now we have put the intermediate particles on-mass-shell, but now in principle they are off-energy-shell, which means that $W \neq E(\mathbf{k}_n) + \mathcal{E}(\mathbf{k}_n)$. And the total four momentum in not conserved, but the total three momentum is conserved.

Note that if the intermediate state is on-energy-shell, i.e. $W = E(\mathbf{k}_n) + \mathcal{E}(\mathbf{k}_n)$, the two poles of the Green's function, Eq. (2.20), coincide. The value of k_n^0 at which this "pinching" occurs is given by the on-mass-shell value, Eq. (2.21),

$$k_n^0 = \mu_{n,2} E(\mathbf{k}_n) - \mu_{n,1}\mathcal{E}(\mathbf{k}_n) \pm i\delta \ . \tag{2.25}$$

The contribution to the integral around this value of k_n^0 will be dominant, due to the pinching of the poles. This is the rationale for the "on-mass-shell approximation".

2. Potentials for the 3-d integral equation

In order to calculate cross sections or phase shifts we need to solve Eq. (2.24), which is a complex integral equation for the T-matrix, even for physical momenta. It is possible to transform Eq. (2.24) into a Lippmann-Schwinger equation (which can be Fourier-transformed into coordinate space). However we do our calculations always in momentum space, so we do not need to solve the Lippmann-Schwinger equation but we will always solve Eq. (2.24).

Using rotational invariance and parity conservation we expand the T-matrix, which is a 2×2-matrix in Pauli-spinor space, into a complete set of Pauli-spinor invariants. Introducing the momentum vectors

$$\mathbf{q} = \frac{1}{2}(\mathbf{q}_f + \mathbf{q}_i) \ , \ \ \mathbf{k} = \mathbf{q}_f - \mathbf{q}_i \ , \ \ \hat{\mathbf{n}} = \hat{\mathbf{q}}_i \times \hat{\mathbf{q}}_f \ , \tag{2.26}$$

where $\mathbf{q}_f$ and $\mathbf{q}_i$ are the final and initial CM-three-momenta respectively, there are only two independent spinor invariants P_α, rotational invariant and invariant under parity transformations. We choose for the operators P_α in spin-space

$$P_1 = 1 \ \ , \ \ P_2 = \boldsymbol{\sigma}\cdot\mathbf{q}_i \times \mathbf{q}_f \ , \tag{2.27}$$

corresponding to the central and spin-orbit piece of the amplitude, now the expansion of the T-matrix in spinor invariants reads

$$\begin{aligned} T &= \sum_1^2 T_\alpha(\mathbf{q}_f^2, \mathbf{q}_i^2, \mathbf{q}_f\cdot\mathbf{q}_i) \ P_\alpha \\ &= f(\mathbf{q}_f, \mathbf{q}_i) + i\ g(\mathbf{q}_f, \mathbf{q}_i)\ (\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}) \ . \end{aligned} \tag{2.28}$$

For the partial wave projection we found it convenient to rewrite the T-matrix in terms of the amplitudes F and G

$$T = F\,(\mathbf{q}_f, \mathbf{q}_i) + (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_f)\ G\,(\mathbf{q}_f, \mathbf{q}_i)\ (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_i) \ . \tag{2.29}$$

The relation between the "spin-nonflip" and "spin-flip" amplitudes f, g and the amplitudes F, G is readily found to be

$$F = f + (\hat{\mathbf{q}}_f\cdot\hat{\mathbf{q}}_i)\ g \ \ , \ \ G = -g \ . \tag{2.30}$$

The connection between the non-relativistic amplitudes F and G defined in Eq. (2.29) and the relativistic amplitudes A and B defined in Eq. (2.5) can be obtained in a straightforward way using the explicit representation of the Dirac spinors, as will be shown in Sec. II D. Similar to Eq. (2.28) we expand the potentials V, so

$$\begin{aligned} V &= \sum_1^2 V_\alpha(\mathbf{q}_f^2, \mathbf{q}_i^2, \mathbf{q}_f\cdot\mathbf{q}_i)\ P_\alpha \\ &= V_C(\mathbf{q}_f, \mathbf{q}_i) + i\ V_{SO}(\mathbf{q}_f, \mathbf{q}_i)\ (\boldsymbol{\sigma}\cdot\mathbf{n}) \ . \end{aligned} \tag{2.31}$$

3. Lippmann-Schwinger equation

In order to arrive at a Lippmann-Schwinger equation, one chooses a new Green's function $g(\mathbf{k}, W)$ which satisfies a dispersion relation in $\mathbf{p}^2(s)$ rather than in s [28]. Then one obtains

$$g(\mathbf{k}_n, W) = \frac{-1}{2[E(\mathbf{k}_n) + \mathcal{E}(\mathbf{k}_n)]}(\mathbf{k}_n^2 - \mathbf{q}_n^2 - i\delta)^{-1} \ , \tag{2.32}$$

where $\mathbf{q}_n$ is the on-energy-shell momentum. This Green's function is then used in the integral equation Eq. (2.24) instead of the Green's function $G_0(\mathbf{k}_n, W)$. So the corrections to $\langle f|W|i\rangle$ due to the transformation of the Green's functions are neglected here, they are of higher order in the couplings and are usually discarded in an OBE-approach. With the substitution of g for G_0, Eq. (2.23) becomes identical to Eq. (2.19) of [28]. From now on we follow Sec. II of [28] in detail, the transformation to the non-relativistic normalization of the two-particle states leads to states with

$$\begin{aligned} (\mathbf{p}_1', s_1'; \mathbf{p}_2', s_2'|\mathbf{p}_1, s_1; \mathbf{p}_2, s_2) &= (2\pi)^6\delta^3(\mathbf{p}_1' - \mathbf{p}_1)\delta_{s_1', s_1} \\ &\quad \times\delta^3(\mathbf{p}_2' - \mathbf{p}_2)\delta_{s_2', s_2} \end{aligned} \tag{2.33}$$

For these states we define the non-relativistic $\mathcal{T}$-matrix

$$(f|\mathcal{T}|i) = \frac{1}{\sqrt{4\mu_{34}(E_3+E_4)}}\langle f|T|i\rangle\frac{1}{\sqrt{4\mu_{12}(E_1+E_2)}} \ , \tag{2.34}$$

where μ_{12} and μ_{34} are the reduced masses for respectively the initial and final state. Then we get from Eq. (2.24) the Lippmann-Schwinger equation

$$\begin{aligned} (3,4|\mathcal{T}|1,2) &= (3,4|\mathcal{V}|1,2) + \sum_n \int \frac{d^3k_n}{(2\pi)^3}(3,4|\mathcal{V}|n_1,n_2) \\ &\quad \times \frac{2\mu_{n_1,n_2}}{\mathbf{q}_n^2 - \mathbf{k}_n^2 + i\delta}(n_1,n_2|\mathcal{T}|1,2) \ , \end{aligned} \tag{2.35}$$

where the potential $\mathcal{V}$ is defined analogously to the $\mathcal{T}$-matrix, Eq. (2.34). If in the low-energy approximation, the energies are expanded in terms of the momenta squared, the Lippmann-Schwinger equation in momentum space can in principle be Fourier-transformed into the equivalent Schrödinger equation in configuration space. However our calculations are always in momentum space, so we always solve Eq. (2.24).

D. Relation between relativistic- and CM-amplitudes

The relation between the relativistic amplitudes A and B and the non-relativistic amplitudes F and G is found making use of the representation of the Dirac spinors [27]. Since in the three-dimensional integral equation, Eq. (2.24), off-energy shell amplitudes appear, we now distinguish between the CM-energies of the final and initial states, defined by

$$W_f^2 \equiv s_f = (p' + q')^2 \quad , \quad W_i^2 \equiv s_i = (p+q)^2 \ . \quad (2.36)$$

Then, a straightforward calculation of the operators 1 and $\not{Q}$ between Dirac spinors gives the corresponding operators between Pauli spinors.

$$\begin{aligned} \bar{u}(\mathbf{p}_f, s_f)\, u(\mathbf{p}_i, s_i) \ &= \ \sqrt{(E_f+M_f)(E_i+M_i)}\ \chi_f^\dagger \times \\ & \left[1 - \frac{\boldsymbol{\sigma}\cdot\mathbf{p}_f\ \boldsymbol{\sigma}\cdot\mathbf{p}_i}{(E_f+M_f)(E_i+M_i)}\right]\chi_i \ , \\ \bar{u}(\mathbf{p}_f, s_f)\, \not{Q}\, u(\mathbf{p}_i, s_i) \ &= \ \sqrt{(E_f+M_f)(E_i+M_i)}\ \chi_f^\dagger \times \\ & \left[\frac{1}{2}\left[(W_f - M_f) + (W_i - M_i)\right] + \right. \\ & \frac{1}{2}\left[(W_f + M_f) + (W_i + M_i)\right] \ \times \\ & \left. \frac{\boldsymbol{\sigma}\cdot\mathbf{p}_f\ \boldsymbol{\sigma}\cdot\mathbf{p}_i}{(E_f+M_f)(E_i+M_i)}\right]\chi_i \quad (2.37) \end{aligned}$$

with

$$Q^\mu = \frac{1}{2}\left(q_f + q_i\right)^\mu \ . \quad (2.38)$$

In Eq. (2.37) we used the shorthand notations $E_f = E(\mathbf{p}_f)$ etc. for the baryon variables. The meson variables were eliminated using $q^0 = W - E$ etc. From the expressions in Eqs. (2.37), (2.5) and (2.29) we immediately obtain the relations between the amplitudes F, G and A,B

$$\begin{aligned} F(\mathbf{p}_f, \mathbf{p}_i) \ &= \ \sqrt{(E_f+M_f)(E_i+M_i)}\left[A(s,t,u) + \right. \\ & \left. \frac{W_f - M_f + W_i - M_i}{2}\, B(s,t,u)\right] \ , \\ G(\mathbf{p}_f, \mathbf{p}_i) \ &= \ \sqrt{(E_f-M_f)(E_i-M_i)}\left[-A(s,t,u) + \right. \\ & \left. \frac{W_f + M_f + W_i + M_i}{2}\, B(s,t,u)\right] \ . \quad (2.39) \end{aligned}$$

III. THE PARTIAL WAVE EQUATION

The NSC model is fitted to the partial wave analyses of the πN and K^+N scattering data, for this purpose the integral equation for the meson-baryon amplitude must be solved on the partial wave basis. This section deals with the transformation of the integral equation on the plane wave basis to the integral equation on the partial wave (LSJ) basis. From the unitarity of the scattering matrix, the relation between the partial wave amplitude and the partial wave phase shifts is derived.

A. Partial wave analysis

[1] The states for the meson-baryon system are characterized by J, L, where J is the total angular momentum and L the orbital angular momentum. The latter, for fixed J-value, can assume the values $L = J \mp \frac{1}{2}$, since the spin of the baryons is $S = \frac{1}{2}$. Distinguishing between the partial waves with parity $P = (-)^{J-1/2}$ and $P = (-)^{J+1/2}$, using rotational invariance, we can write the potential matrix elements on the LSJ-basis in the following way

(i) $P = (-)^{L_+}, L_+ = J - 1/2$:

$$\begin{aligned} & (q_f; L'J'M'|\ V\ |q_i; LJM) = \\ & \quad 4\pi\ V^{J,L_+}(L', L)\ \delta_{J'J}\delta_{M'M}\delta_{L'L} \ , \end{aligned} \quad (3.1)$$

(ii) $P = (-)^{L_-}, L_- = J + 1/2$:

$$\begin{aligned} & (q_f; L'J'M'|\ V\ |q_i; LJM) = \\ & \quad 4\pi\ V^{J,L_-}(L', L)\ \delta_{J'J}\delta_{M'M}\delta_{L'L} \ . \end{aligned} \quad (3.2)$$

Because of parity conservation in strong interactions, the $L_+ = J - 1/2$ and the $L_- = J + 1/2$ waves obviously are decoupled. So mixing between states with different angular momentum never occurs.

The spherical wave functions in momentum space with quantum numbers $J, M, L, S = 1/2$ are

$$\mathcal{Y}_{JL}^M(\hat{p}, s) = \sum_{m,\mu} C^{L\ \frac{1}{2}\ J}_{m\ \mu\ M}\ Y_m^L(\hat{p})\ \chi_\mu^{(\frac{1}{2})}(s) \ , \quad (3.3)$$

where s is a spin variable for the baryons. For example, s denotes the helicity of the baryon, or the projection of the spin along the normal $\hat{\mathbf{n}}$ to the scattering plane, or the projection of the spin along the z-axis. The latter spin variable we will use in this work. Then, in Eq. (3.3) we have $\chi_\mu^{(1/2)}(s) = \delta_{s,\mu}$. The central and non-central potential matrix elements on the LSJ-basis are derived in detail in Appendix A 1, the results are

1. *Central* $P_1 = 1$:

$$\begin{aligned} & (q_f; L'J'M'|F(\mathbf{q}_f, \mathbf{q}_i)|q_i; LJM) = \\ & \quad 4\pi F_L\,(q_f, q_i)\ \delta_{L'L}\delta_{J',J}\delta_{M',M} \ , \end{aligned} \quad (3.4)$$

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[1] In this section we use the non-relativistic normalization Eq. (2.33) of the two-particle states.

2. *Non-central* $P_2' = (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_f)(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_i)$:

$$(q_f;L'J'M'|G(\mathbf{q}_f,\mathbf{q}_i)|q_i;LJM) = 4\pi\sum_{L''} a_{L',L''}G_{L''}(q_f,q_i)\,a_{L'',L}\;\delta_{J',J}\delta_{M',M} \quad (3.5)$$

where the partial wave projections F_L and G_L as well as the matrix $a_{L',L}$ are defined in Appendix A. The partial wave potentials V^{J,L_+} and V^{J,L_-} in Eqs. (3.1) and (3.2) can be expressed in terms of the partial wave expansions of F and G. As expected from parity conservation, the partial wave potentials are diagonal in the $L=(J\pm1/2)$-space

$$V^{J,L_\pm} = F_{L_\pm} + G_{L_\pm\pm1} \;. \quad (3.6)$$

The partial wave potentials can also be expressed in terms of the partial wave projections of the central and spin-orbit potential. The relation between F and G and the central and spin-orbit potentials is given by Eq. (2.30),

$$F = V^C + \cos\theta\, V^{SO} \;,\; G = -V^{SO} \;. \quad (3.7)$$

The partial wave potentials in terms of the partial wave projections of the central and spin-orbit potentials becomes

$$\begin{aligned} V^{J,L_+} &= V^C_{L_+} \;; \qquad\qquad L_+ = 0 \;, \\ V^{J,L_\pm} &= V^C_{L_\pm} + \frac{L_\pm+1}{2L_\pm+1}V^{SO}_{L_\pm+1} + \\ &\quad \frac{L_\pm}{2L_\pm+1}V^{SO}_{L_\pm-1} - V^{SO}_{L_\pm\pm1} \;;\; L_\pm \geq 1 \;. \end{aligned} \quad (3.8)$$

B. Partial wave integral equations and the unitarity relations

1. *Partial wave integral equations*

The integral equation Eq. (2.24) we write first explicitly in terms of the plane wave states

$$\begin{aligned} \left(\mathbf{q}_f,s_f|T(\sqrt{s})|\mathbf{q}_i,s_i\right) &= \left(\mathbf{q}_f,s_f|V(\sqrt{s})|\mathbf{q}_i,s\right) \\ &+\sum_n \int\frac{d^3k_n}{(2\pi)^3}\left(\mathbf{q}_f,s_f|V(\sqrt{s})|\mathbf{k}_n,s_n\right)\; G_0(\mathbf{k}_n,\sqrt{s}) \\ &\times\left(\mathbf{k}_n,s_n|T(\sqrt{s})|\mathbf{q}_i,s_i\right) \;, \end{aligned} \quad (3.9)$$

where, apart from spin-space, the amplitude T, the Green's function G_0 and the potential V are matrices in the two-particle channel space. The partial wave T-matrix for $L=L_i=L_f$ defined by

$$T_{J,L}(q_f,q_i;\sqrt{s}) = \left(q_f;LJM|T(\sqrt{s})|q_i;LJM\right) \;, \quad (3.10)$$

which is independent of $J_z=M$ due to rotation invariance, is related to the T-matrix on the plane wave basis by

$$\begin{aligned} T_{J,L}(q_f,q_i;\sqrt{s}) &= \sum_{s_f,s_i}\int\frac{d^3q_f'}{(2\pi)^3}\int\frac{d^3q_i'}{(2\pi)^3}\,(q_f;LJM|\mathbf{q}_f',s_f) \\ &\times\left(\mathbf{q'}_f,s_f|T(\sqrt{s})|\mathbf{q'}_i,s_i\right)\left(\mathbf{q}_i',s_i|q_i;LJM\right) \;. \end{aligned} \quad (3.11)$$

The integral equation for the partial wave amplitude now becomes

$$\begin{aligned} T_{J,L}(q_f,q_i;\sqrt{s}) &= V_{J,L}(q_f,q_i;\sqrt{s}) + \sum_n\int_0^\infty\frac{k_n^2dk_n}{(2\pi)^3}\times \\ &V_{J,L}(q_f,q_n;\sqrt{s})\; G_0(k_n,\sqrt{s})T_{J,L}(q_n,q_i;\sqrt{s}) \;. \end{aligned} \quad (3.12)$$

2. *Partial wave unitarity relations, phase shifts*

From the unitarity of the S-matrix, $S^\dagger\,S=1$, the M-matrix in Eq. (2.2) satisfies the condition

$$\begin{aligned} 2\Im\langle f|M|i\rangle &= -(2\pi)^4\sum_n\delta^4(P_f-P_n) \\ &\times\langle f|M^\dagger|n\rangle\langle n|M|i\rangle \;. \end{aligned} \quad (3.13)$$

In deriving Eq. (3.13) one factors out a $\delta^4(P_f-P_i)$. The previous equation for the CM-amplitudes can be written more explicitly, see for example Eq. (II.1.14) of [34], as

$$\begin{aligned} 2\Im\left(\mathbf{q}_f,s_f|T|\mathbf{q}_i,s_i\right) &= -\frac{1}{(2\pi)^2}\sum_n\int\frac{d^3k_n}{4E(\mathbf{k}_n)\mathcal{E}(\mathbf{k}_n)} \\ &\times\left(\mathbf{q}_f,s_f|T^\dagger|\mathbf{k}_n,s_n\right)\delta(\sqrt{s}-E(\mathbf{k}_n)-\mathcal{E}(\mathbf{k}_n)) \\ &\times\left(\mathbf{k}_n,s_n|T|\mathbf{q}_i,s_i\right) \;, \end{aligned} \quad (3.14)$$

where the summation $\sum_n$ is over all intermediate two-particle channels coupled to the initial and final state. Here, s_i, s_f, and s_n are the spin labels for the initial, final, and intermediate states. The momentum of the intermediate state k_n is such that $E(\mathbf{k}_n)+\mathcal{E}(\mathbf{k}_n)=\sqrt{s}$. The unitarity relation for the partial wave amplitude then becomes

$$\begin{aligned} 2\Im T_{J,L}(q_f,q_i) &= -\sum_n\frac{q_n}{16\pi^2\sqrt{s}}T^\dagger_{J,L}(q_f,q_n) \\ &\times T_{J,L}(q_n,q_i) \;, \end{aligned} \quad (3.15)$$

where q_n is the on-energy-shell momentum of the intermediate state. Introducing the partial wave amplitudes $F_{J,L}$ by the definition

$$T_{J,L} = -32\pi^2\sqrt{s}\;F_{J,L} \;, \quad (3.16)$$

we find the simple unitarity relation for these amplitudes

$$\Im F_{J,L}(q_f,q_i) = \sum_c q_c\;F_{J,L}(q_f,q_c)^\dagger F_{J,L}(q_c,q_i) \;. \quad (3.17)$$

For the single channel case $q_c=q_i=q_f=q$, phase shifts can be defined for the partial wave amplitude $F_{J,L}$ in the usual way

$$F_{J,L} = \frac{1}{q}\;\sin\delta_{J,L}(q)\exp\left(i\delta_{J,L}(q)\right) \;. \quad (3.18)$$

The relation of $F_{J,L}$ with the partial wave S-matrix is

$$S_{J,L} = e^{2i\delta_{J,L}} = 1 + 2iq\ F_{J,L}\ . \tag{3.19}$$

Now the expression for the differential cross section becomes

$$\frac{d\sigma}{d\Omega} = \left|\tilde{f}\right|^2 + |\tilde{g}|^2\ , \tag{3.20}$$

where the commonly used spin-nonflip and spin-flip amplitudes $\tilde{f}$ and $\tilde{g}$ are given by

$$\begin{aligned}\tilde{f} &= \frac{f}{8\pi\sqrt{s}} = \sum_L \left[(L+1)\,F_{L+\frac{1}{2},L} + LF_{L-\frac{1}{2},L}\right] P_L(\cos\theta)\ ,\\ \tilde{g} &= \frac{g}{8\pi\sqrt{s}} = \sum_L \left[F_{L+\frac{1}{2},L} - F_{L-\frac{1}{2},L}\right]\sin\theta\ \frac{dP_L(\cos\theta)}{d\cos\theta}\ .\end{aligned} \tag{3.21}$$

The expressions for the total cross section, which is found by integrating the differential cross section, and the polarization are

$$\begin{aligned}\sigma &= 4\pi \sum_J \frac{2J+1}{2}\left(\left|F_{J,L_+}\right|^2 + \left|F_{J,L_-}\right|^2\right)\ ,\\ P(\theta) &= \frac{2\Im\left(\tilde{f}\tilde{g}^*\right)}{\left|\tilde{f}\right|^2 + |\tilde{g}|^2}\ .\end{aligned} \tag{3.22}$$

IV. BARYON- AND MESON-EXCHANGE POTENTIALS

The effective local interaction Hamiltonians that are used to calculate the one-hadron-exchange potentials are defined in this section. The Lorentz structure of the interaction is given and the $SU_f(3)$ structure is reviewed, since we extend the NSC πN-model to the NSC K^+N-model. The amplitudes of the one-hadron-exchange Feynman diagrams are given and a partial wave projection is made to find the partial wave potentials.

A. The interaction Hamiltonians

The potentials we use are obtained from the t-channel one-meson-exchange (OBE) and the u- and s-channel baryon-exchange Feynman diagrams. In the t-channel we consider the exchange of vector- and scalar-mesons and in the u- and s-channel we consider the exchange of $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$-baryons. In this work we also include Pomeron-exchange diagrams, where the physical nature of the Pomeron can be understood in the light of QCD as a two-gluon-exchange effect, see [36, 37]. The contribution of the Pomeron will almost completely cancel the contribution of the isoscalar scalar-meson σ.

The OBE Feynman diagrams for meson-baryon interactions contain a meson-baryon-baryon vertex and a meson-meson-meson vertex. These vertices are determined by the effective local interaction Hamiltonian densities. The Lorentz structure of the local interaction densities for the meson-baryon-baryon (MBB) vertices we use are listed below

a. $J^{PC} = 0^{-+}$ Pseudoscalar-mesons:
For the pseudoscalar-mesons we use the pseudovector interaction Hamiltonian

$$\mathcal{H}_{PV} = \frac{f}{m_{\pi^+}}\ \bar{\psi}_f\gamma_5\gamma_\mu\psi_i\ \partial^\mu\phi_P\ , \tag{4.1}$$

which is scaled with the charged-pion mass in order to have a dimensionless pseudovector coupling constant.

b. $J^{PC} = 1^{--}$ Vector-mesons:
The interaction Hamiltonian is given in terms of the electric and magnetic interaction

$$\begin{aligned}\mathcal{H}_V &= g_V\ \bar{\psi}_f\gamma_\mu\psi_i\ \phi_V^\mu\\ &\quad + \frac{f_V}{4\mathcal{M}}\bar{\psi}_f\sigma_{\mu\nu}\psi_i\left(\partial^\mu\phi_V^\nu - \partial^\nu\phi_V^\mu\right)\ ,\end{aligned} \tag{4.2}$$

where usually the proton mass is used for $\mathcal{M}$ to scale the magnetic part of the interaction Hamiltonian. The antisymmetric tensor operator used here, is defined as $\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]$.

c. $J^{PC} = 0^{++}$ Scalar-mesons:

$$\mathcal{H}_S = g_S\ \bar{\psi}_f\psi_i\ \phi_S\ . \tag{4.3}$$

Since we include Pomeron-exchange in the NSC model, the scalar-meson-exchange is canceled for the greater part, hence it is possible to satisfy the soft-pion theorem while including scalar-meson-exchange.

d. $J^{PC} = 2^{++}$ Tensor-mesons:
For the tensor-mesons we use the interaction Hamiltonian

$$\begin{aligned}\mathcal{H}_T &= \left[\frac{i}{4}\bar{\psi}_f\left(\gamma_\mu\overleftrightarrow{\partial}_\nu + \gamma_\nu\overleftrightarrow{\partial}_\mu\right)\psi_i\ F_1 - \right.\\ &\quad \left.\frac{1}{4}\left(\bar{\psi}_f\ \overleftrightarrow{\partial}^\mu\overleftrightarrow{\partial}^\nu\ \psi_i\right)\ F_2\right]\ \phi_T^{\mu\nu}\ ,\end{aligned} \tag{4.4}$$

where the coupling constants F_1 and F_2 are related to the dimensionless Pauli coupling constants by $G_{T,1} = \mathcal{M}F_1$ and $G_{T,2} = \mathcal{M}^2F_2$. Using the Gordon decomposition the Pauli coupling constants are related to the Dirac coupling constants by $g_T = G_{T,1} + G_{T,2}$ and $f_T = -G_{T,2}$.

e. $J^P = \frac{3}{2}^+$ Resonance-Baryon-Pseudoscalar-meson:
The local interaction density for the $J^P = \frac{3}{2}^+$ Resonance -Nucleon-Pseudoscalar-meson (Y^*NP) interaction is

$$\mathcal{H}_{Y^*NP} = -i\frac{f^*}{m_{\pi^+}}\ \bar{\psi}_N\ \psi_{Y^*,\mu}\ \partial^\mu\phi_P\ , \tag{4.5}$$

where the charged-pion mass makes the coupling dimensionless. We use the Rarita-Schwinger formalism for the spin-3/2 resonances, see e.g. [26, 38].

f. $J^P = \frac{1}{2}^-$ Resonance-Baryon-Pseudoscalar-meson:
The local interaction Hamiltonian for the $J^P = \frac{1}{2}^-$ Resonance-Nucleon-Pseudoscalar-meson (RNP) interaction is

$$\mathcal{H}_{RNP} = \frac{f^{*(v)}}{m_{\pi^+}} \, \bar{\psi}_N \gamma_\mu \psi_R \, \partial^\mu \phi_P \ , \tag{4.6}$$

where ψ_R denotes the $J^P = \frac{1}{2}^-$ resonance, which has opposite parity to the nucleon. The $J^P = \frac{1}{2}^-$ resonances we consider in this work are the $S_{11}(1555)$ in the πN system and the $\Lambda(1405)$ in the KN system.

Here ϕ denotes the pseudoscalar-, vector-, scalar-, and tensor-meson fields respectively and ψ denotes the baryon fields. The Pomeron-baryon-baryon interaction density we use, has the same Lorentz structure as the scalar-mesons.

We note that, making use of the Dirac equation $(\gamma^\mu \partial_\mu + M)\,\psi = 0$, the pseudovector interaction Hamiltonian density in Eq. (4.1) is "equivalent" to the pseudoscalar density $\mathcal{H}_{PS} = ig \ \bar{\psi}_f \gamma_5 \psi_i \ \phi_P$ for on-mass-shell particles. The coupling constants are then related according to $g/(M_{B_f} + M_{B_i}) = f/m_{\pi^+}$. Analogous we find that the vector coupling Hamiltonian density in Eq. (4.6) is "equivalent" to the scalar density $\mathcal{H}_S = ig^{*(s)} \bar{\psi}_N \psi_R \ \phi_P$ for on-mass-shell particles. The coupling constants are in this case related according to $g^{*(s)}/(M_N - M_R) = f^{*(v)}/m_{\pi^+}$.

The Lorentz structure of the local interaction density for triple-meson (MMM) vertices is schematically given below, they are discussed in more detail in paper II.

(i) $J^{PC} = 1^{--}$ Vector-mesons:

$$\mathcal{H}_{PPV} \ = \ g_{PPV} \ \phi_V^\mu \ \left(\phi_P \ \overleftrightarrow{\partial}_\mu \ \phi_P\right) \ . \tag{4.7}$$

(ii) $J^{PC} = 0^{++}$ Scalar-mesons:

$$\mathcal{H}_{PPS} \ = \ g_{PPS} \ \phi_S \ \left(\phi_P \ \phi_P\right) \ . \tag{4.8}$$

(iii) $J^{PC} = 2^{++}$ Tensor-mesons:

$$\mathcal{H}_{PPT} \ = \ \frac{2g_{PPT}}{m_{\pi^+}} \ \phi_T^{\mu\nu} \left(\partial_\mu \phi_P\right) \left(\partial_\nu \phi_P\right) \ . \tag{4.9}$$

Concerning the flavor structure of the interaction densities, we assume that the coupling constants are related via $SU_f(3)$ symmetry, as outlined in paper II , here the relevant isoscalar and isospin factors are given. However the potentials will break the $SU_f(3)$ symmetry dynamically, since we use the physical masses of the particles.

B. The relativistic invariant amplitudes

Using the previously defined interaction Hamiltonians, we give, besides the isospin and isoscalar factors, the contributions to the relativistic invariant amplitudes $A(s,t,u)$ and $B(s,t,u)$ in Eq. (2.5) for the elastic (e.g. πN and K^+N) channels, i.e. $M_i = M_f \equiv M$, $m_i = m_f = m$, where M_f and M_I are the final and initial baryon masses and m_f and m_i are the final and initial pseudoscalar-meson masses respectively. Amplitudes for the general mass case are listed in Appendix B.

1. Baryon-exchange amplitudes

For $J^P = \frac{1}{2}^+$ baryon-exchange the relativistic amplitudes are

$$\begin{aligned}
A_{ps}(s,t,u) \ &= \ -\frac{g_{14}g_{23}}{u - M_B^2 + i\epsilon} \left[M_B - M\right], \\
B_{ps}(s,t,u) \ &= \ -\frac{g_{14}g_{23}}{u - M_B^2 + i\epsilon} \ , \\
A_{pv}(s,t,u) \ &= \ -\frac{f_{14}f_{23}/m_{\pi^+}^2}{u - M_B^2 + i\epsilon} \left[u\,(M + M_B)\right. \\
&\quad \left. -M^3 - M^2 M_B\right] \ , \\
B_{pv}(s,t,u) \ &= \ -\frac{f_{14}f_{23}/m_{\pi^+}^2}{u - M_B^2 + i\epsilon} \left[u + 2MM_B\right. \\
&\quad \left. +M^2\right] \ ,
\end{aligned} \tag{4.10}$$

for pseudoscalar (ps) and pseudovector (pv) coupling respectively, M_B is the mass of the exchanged baryon. The $J^P = \frac{1}{2}^+$ baryon direct pole gives rise to the relativistic amplitudes

$$\begin{aligned}
A_{ps}(s,t,u) \ &= \ -\frac{g_{12}g_{34}}{s - M_B^2 + i\epsilon} \left[M_B - M\right], \\
B_{ps}(s,t,u) \ &= \ \frac{g_{12}g_{34}}{s - M_B^2 + i\epsilon} \ , \\
A_{pv}(s,t,u) \ &= \ -\frac{f_{12}f_{34}/m_{\pi^+}^2}{s - M_B^2 + i\epsilon} \left[s\,(M + M_B)\right. \\
&\quad \left. -M^3 - M^2 M_B\right] \ , \\
B_{pv}(s,t,u) \ &= \ \frac{f_{12}f_{34}/m_{\pi^+}^2}{s - M_B^2 + i\epsilon} \left[s + 2MM_B\right. \\
&\quad \left. +M^2\right] \ ,
\end{aligned} \tag{4.11}$$

for pseudoscalar and pseudovector coupling respectively. For $J^P = \frac{1}{2}^-$ baryon-exchange the relativistic amplitudes are

$$\begin{aligned}
A_s(s,t,u) \ &= \ \frac{g_{14}^{*(s)} g_{23}^{*(s)}}{u - M_B^2 + i\epsilon} \left[M_B + M\right], \\
B_s(s,t,u) \ &= \ -\frac{g_{14}^{*(s)} g_{23}^{*(s)}}{u - M_B^2 + i\epsilon} \ ,
\end{aligned}$$

$$
\begin{aligned}
A_v(s,t,u) &= \frac{f_{14}^{*(v)} f_{23}^{*(v)}/m_{\pi^+}^2}{u-M_B^2+i\epsilon}\left[u\left(-M+M_B\right)\right.\\
&\left.+M^3-M^2M_B\right] \;, \\
B_v(s,t,u) &= \frac{f_{14}^{*(v)} f_{23}^{*(v)}/m_{\pi^+}^2}{u-M_B^2+i\epsilon}\left[-u+2MM_B\right.\\
&\left.-M^2\right] \;, \qquad (4.12)
\end{aligned}
$$

for scalar (s) and vector (v) coupling respectively, M_B is the mass of the exchanged baryon. The $J^P = \frac{1}{2}^-$ baryon direct pole gives rise to the relativistic amplitudes

$$
\begin{aligned}
A_s(s,t,u) &= \frac{g_{12}^{*(s)} g_{34}^{*(s)}}{s-M_B^2+i\epsilon}\left[M_B+M\right], \\
B_s(s,t,u) &= \frac{g_{12}^{*(s)} g_{34}^{*(s)}}{s-M_B^2+i\epsilon} \;, \\
A_v(s,t,u) &= \frac{f_{12}^{*(v)} f_{34}^{*(v)}/m_{\pi^+}^2}{s-M_B^2+i\epsilon}\left[s\left(-M+M_B\right)\right.\\
&\left.+M^3-M^2M_B\right] \;, \\
B_v(s,t,u) &= -\frac{f_{12}^{*(v)} f_{34}^{*(v)}/m_{\pi^+}^2}{s-M_B^2+i\epsilon}\left[-s+2MM_B\right.\\
&\left.-M^2\right] \;, \qquad (4.13)
\end{aligned}
$$

for scalar and vector coupling respectively. The $J^P = \frac{3}{2}^+$ resonance-exchange relativistic amplitudes are more complicated

$$
\begin{aligned}
A_{Y^*}(s,t,u) &= \frac{f_{14}^* f_{23}^*/m_{\pi^+}^2}{u-M_{Y^*}^2+i\epsilon}\left[\frac{\left[t-2m^2\right]}{2}\left(M+M_{Y^*}\right)\right.\\
&+\frac{M_{Y^*}}{3}\left[u-M^2\right]\\
&+\frac{1}{6M_{Y^*}}\left[-u^2+2Mu\left(M+M_{Y^*}\right)\right.\\
&\left.-2M^3M_{Y^*}-M^4+m^4\right]\\
&\left.+\frac{1}{6M_{Y^*}^2}\left[M^2-m^2-u\right]^2\left(M+M_{Y^*}\right)\right], \\
B_{Y^*}(s,t,u) &= \frac{f_{14}^* f_{23}^*/m_{\pi^+}^2}{u-M_{Y^*}^2+i\epsilon}\left[-\frac{\left[t-2m^2\right]}{2}\right.\\
&+\frac{1}{3M_{Y^*}}\left[\left(M+M_{Y^*}\right)\left(2MM_{Y^*}-m^2\right)\right.\\
&\left.-uM+M^3\right]\\
&\left.-\frac{1}{6M_{Y^*}^2}\left[u-M^2+m^2\right]^2\right] \;, \qquad (4.14)
\end{aligned}
$$

where M_{Y^*} is the mass of the exchanged resonance. The $J^P = \frac{3}{2}^+$ resonance direct pole gives rise to the relativistic amplitudes

$$
\begin{aligned}
A_{Y^*}(s,t,u) &= \frac{f_{12}^* f_{34}^*/m_{\pi^+}^2}{s-M_{Y^*}^2+i\epsilon}\left[\frac{\left[t-2m^2\right]}{2}\left(M+M_{Y^*}\right)\right.\\
&+\frac{M_{Y^*}}{3}\left[s-M^2\right]\\
&+\frac{1}{6M_{Y^*}}\left[-s^2+2Ms\left(M+M_{Y^*}\right)\right.\\
&\left.-2M^3M_{Y^*}-M^4+m^4\right]\\
&\left.+\frac{1}{6M_{Y^*}^2}\left[M^2-m^2-s\right]^2\left(M+M_{Y^*}\right)\right] \;, \\
B_{Y^*}(s,t,u) &= -\frac{f_{12}^* f_{34}^*/m_{\pi^+}^2}{s-M_{Y^*}^2+i\epsilon}\left[-\frac{\left[t-2m^2\right]}{2}\right.\\
&+\frac{1}{3M_{Y^*}}\left[\left(M+M_{Y^*}\right)\left(2MM_{Y^*}-m^2\right)\right.\\
&\left.-sM+M^3\right]\\
&\left.-\frac{1}{6M_{Y^*}^2}\left[s-M^2+m^2\right]^2\right] \;. \qquad (4.15)
\end{aligned}
$$

2. *Meson- and Pomeron-exchange amplitudes*

The relativistic amplitudes for the t-channel Pomeron-exchange, scalar-meson-exchange, vector-meson-exchange and tensor-meson-exchange are

$$
\begin{aligned}
A_P(s,t,u) &= \frac{g_{PPP}\; g_P}{\mathcal{M}} \quad , \quad B_P(s,t,u) = 0 \;, \\
A_S(s,t,u) &= \frac{g_{PPS}\; g_S}{t-m_S^2+i\epsilon} \quad , \quad B_S(s,t,u) = 0 \;, \\
A_V(s,t,u) &= \frac{g_{PPV}}{t-m_V^2+i\epsilon}\frac{f_V}{2\mathcal{M}}\left[s-u\right] \;, \\
B_V(s,t,u) &= -2\frac{g_{PPV}}{t-m_V^2+i\epsilon}\left[g_V+\frac{M}{\mathcal{M}}\; f_V\right] \;, \\
A_T(s,t,u) &= \frac{g_{PPT}/m_{\pi^+}}{t-m_T^2+i\epsilon}\left[\frac{1}{4}\left(s-u\right)^2 F_2-\frac{1}{6}\left[4m^2-t\right]\right.\\
&\left.\times\left[2MF_1+\frac{1}{2}\left(4M^2-t\right)F_2\right]\right] \;, \\
B_T(s,t,u) &= \frac{g_{PPT}/m_{\pi^+}}{t-m_T^2+i\epsilon}\left[s-u\right]\; F_1 \;, \qquad (4.16)
\end{aligned}
$$

where m_S, m_V and m_T are the masses of the exchanged scalar-meson, vector-meson and tensor-meson respectively.

Here we notice that for the meson-exchange and Pomeron-exchange amplitudes an extra factor 2 must be added to the amplitudes if both the initial and final state contain a π or η, this is not the case for any other combination of pseudoscalar-mesons in the initial and final state. For elastic πN scattering for example, an extra factor 2 is added to the ρ-exchange, Pomeron-exchange and σ-exchange amplitudes.

C. Partial wave potentials

As discussed in Sec. III we solve the integral equation for the T-matrix on the partial wave basis, Eq. (3.12). And in paper II we fit the NSC model to the πN partial wave analysis [1] and the K^+N partial wave analysis [4]. For this purpose we need to calculate the partial wave projection of the potentials, Eq. (3.6). In our approximation, the potentials are given by the invariant amplitudes A and B, Eqs. (4.10)–(4.16), of the one-meson-exchange and one-baryon-exchange Feynman diagrams.

Until this point, we did not mention the need for form factors to regulate the high energy behavior, i.e. the short distance behavior, of the potentials, but in fact the kernel of the integral equation without form factors does not satisfy the Fredholm condition, $\int\int\ dp\ dk\ \ |K(p,k)|^2 < \infty$, in general. Furthermore we have derived our one-meson-exchange and one-baryon-exchange potentials from Quantum Field Theory, which is in principle only valid for point particles, while mesons and baryons have an internal structure. Therefore we need to take into account the extended size of the mesons and baryons by means of a form factor. Since the ground state wave functions of the quarks are Gaussian, form factors of the Gaussian type are used in the NSC model. For t-channel exchanges we multiply the potentials by the form factor

$$F(\Lambda) = e^{-(\mathbf{p}_f-\mathbf{p}_i)^2/\Lambda^2}\ , \tag{4.17}$$

where $\mathbf{p}_i$ and $\mathbf{p}_f$ are the CM three-momenta for the initial and final state respectively, i.e. at both vertices we have used the difference between the final and initial three-momenta. Λ is a cutoff mass, which will be determined in the fit to the experimental phases.

For u- and s-channel exchanges, the difference between the final and initial three-momenta of the baryon is used, giving the form factor

$$F(\Lambda) = e^{-(\mathbf{p}_f^2+\mathbf{p}_i^2)/2\Lambda^2}\ . \tag{4.18}$$

This form factor obviously does not depend on the scattering angle θ, which makes the partial wave projection easier. For the u- and t-channel we rewrite the denominators of the potentials in the form

$$\begin{aligned}
\frac{1}{t-m^2} &= \frac{-1}{2p_fp_i}\frac{1}{z_t-x}\ ,\\
\frac{1}{u-m^2} &= \frac{-1}{2p_fp_i}\frac{1}{z_u+x}\ ,
\end{aligned}\tag{4.19}$$

where $x=\cos(\theta)$ and θ is the angle between the final and initial three-momenta $\mathbf{p}_f$ and $\mathbf{p}_i$. Here we have defined the z_t and z_u factors

$$\begin{aligned}
z_t &= \frac{1}{2p_fp_i}\left[m^2+p_f^2+p_i^2-\frac{1}{4}\left[E_i-E_f-\omega_i+\omega_f\right]^2\right]\ ,\\
z_u &= \frac{1}{2p_fp_i}\left[m^2+p_f^2+p_i^2-\frac{1}{4}\left[E_i+E_f-\omega_i-\omega_f\right]^2\right]\ ,
\end{aligned}\tag{4.20}$$

where $E_{f,i}$ are the baryon energies, $\omega_{f,i}$ are the meson energies and m the mass of the exchanged particle. For positive and real momenta, i.e. for open channels, we have $z>1$. Now it is clear that the potentials $V^{(\alpha)}$ of Eq. (2.31), where α stands for central or spin-orbit, can be expanded in x as [2]

$$\begin{aligned}
V^{(\alpha)}(\mathbf{p}_f,\mathbf{p}_i) &= \frac{1}{2p_fp_i}\left[X^{(\alpha)}+xY^{(\alpha)}+x^2Z^{(\alpha)}\right]\frac{F(\Lambda_t)}{z_t-x}\ ,\\
V^{(\alpha)}(\mathbf{p}_f,\mathbf{p}_i) &= \frac{1}{2p_fp_i}\left[X^{(\alpha)}+xY^{(\alpha)}+x^2Z^{(\alpha)}\right]\frac{F(\Lambda_u)}{z_u+x}\ ,\\
V^{(\alpha)}(\mathbf{p}_f,\mathbf{p}_i) &= \left[X^{(\alpha)}+xY^{(\alpha)}\right]\frac{F(\Lambda_s)}{s-M_B^2}\ ,
\end{aligned}\tag{4.21}$$

for t-, u- and s-channel exchanges respectively, for all particles that are exchanged. The coefficients $X^{(\alpha)}$, $Y^{(\alpha)}$ and $Z^{(\alpha)}$ can be found easily by writing out the x-dependence of the invariant amplitudes A and B, they are listed in Appendix C for each type of exchange.

The partial wave potentials $V_L^{(\alpha)}$ are found by inverting the partial wave expansion Eq. (A2), giving

$$V_L^{(\alpha)}(p_f,p_i) = \frac{1}{2}\int_{-1}^{1} dx P_L(x) V^{(\alpha)}(\mathbf{p}_f,\mathbf{p}_i)\ . \tag{4.22}$$

The partial wave potentials now take the form

$$\begin{aligned}
V_L^{(\alpha)}(p_f,p_i) &= \left[X^{(\alpha)}\frac{F(\Lambda_s)}{s-M_B^2}\delta_{L,0}+Y^{(\alpha)}\frac{F(\Lambda_s)}{s-M_B^2}\frac{\delta_{L,1}}{3}\right]\\
V_L^{(\alpha)}(p_f,p_i) &= \frac{1}{2p_fp_i}\left[\left(X^{(\alpha)}+z_tY^{(\alpha)}+z_t^2Z^{(\alpha)}\right)\right.\\
&\quad\times U_L(\Lambda_t,z_t)-\left(Y^{(\alpha)}+z_tZ^{(\alpha)}\right)R_L(\Lambda_t,z_t)\\
&\quad\left.-Z^{(\alpha)}S_L(\Lambda_t,z_t)\right]\ ,\\
V_L^{(\alpha)}(p_f,p_i) &= \frac{(-1)^L}{2p_fp_i}\left[\left(X^{(\alpha)}-z_uY^{(\alpha)}+z_u^2Z^{(\alpha)}\right)\right.\\
&\quad\times U_L(\Lambda_u,z_u)-\left(-Y^{(\alpha)}+z_uZ^{(\alpha)}\right)R_L(\Lambda_u,z_u)\\
&\quad\left.-Z^{(\alpha)}S_L(\Lambda_u,z_u)\right]\ ,
\end{aligned}\tag{4.23}$$

for s-, t-, and u-channel exchanges respectively. We have defined the basic partial wave projections U_L, R_L, S_L and T_L in terms of the Legendre polynomials $P_L(x)$ and the form factors

$$U_L(\Lambda,z) = \frac{1}{2}\int_{-1}^{1} dx\frac{P_L(x)F(\Lambda)}{z-x}\ ,$$

---

[2] In case of more complicated exchanges, e.g. $J^{PC}=\frac{3}{2}^+$-resonance, the expansions of the potentials have an additional term of higher order in x, for the t- and u-channel $x^3U^{(\alpha)}$, and for the s-channel $x^2Z^{(\alpha)}$.
We notice that a similar expansion for F and G instead of $V^{(C)}$ and $V^{(SO)}$ would be a little simpler. However we will use the central and spin-orbit potentials in light of a momentum space version of the NSC model.

$$R_L(\Lambda, z) = \frac{1}{2}\int_{-1}^{1} dx P_L(x) F(\Lambda) ,$$
$$S_L(\Lambda, z) = \frac{1}{2}\int_{-1}^{1} dx P_L(x) x F(\Lambda) ,$$
$$T_L(\Lambda, z) = \frac{1}{2}\int_{-1}^{1} dx P_L(x) x^2 F(\Lambda) . \qquad (4.24)$$

The factor $(-)^L$ appearing in the u-channel partial wave potentials, which is the result of changing the integration variable $x \to -x$ in the Legendre polynomial, is typical for exchange forces. In this way it can be seen that the total partial wave potential is a linear combination of a direct and an exchange potential, $V^{(\pm)} = V_d \pm V_e$ and the corresponding T-matrix is $T^{(\pm)} = T_d \pm T_e$. The amplitudes T_d and T_e do not satisfy an integral equation, but the two linear combinations $T^{(\pm)} = T_d \pm T_e$ do.

We notice that if the form factor does not depend on x (in case of u-channel potentials) or if we consider the limit $\Lambda \to \infty$, i.e. $F(\Lambda) \to 1$, the basic partial wave projections defined in Eq. (4.24) are proportional to the simple functions

$$U_L(\Lambda, z) \propto Q_L(z) ,$$
$$R_L(\Lambda, z) \propto \delta_{L,0} ,$$
$$S_L(\Lambda, z) \propto \frac{1}{3}\delta_{L,1} ,$$
$$T_L(\Lambda, z) \propto \frac{1}{3}\delta_{L,0} + \frac{2}{15}\delta_{L,2} , \qquad (4.25)$$

where $Q_L(z)$ is the Legendre function of the second kind, which is an analytic function of its argument except for a cut on the real axis running from -1 to 1, as is clear from Eq. (4.24). In view of Eq. (4.20), the cut is entered only for on-energy-shell potentials below threshold, but we always calculate the on-energy-shell potentials above threshold, so we will never reach the cut.

In the NSC πN-model we will include s-channel baryon-exchange diagrams, which are in principle separable diagrams, having the form

$$V(p_f, p_i) = \frac{\Gamma(p_f)\ \Gamma(p_i)}{\sqrt{s} \pm M_0} . \qquad (4.26)$$

Writing out the partial wave potential for the Δ pole (P_{33}-wave) explicitly, using Eq. (3.6) and Eq. (4.23),

$$V_{33} = \frac{f^2_{\pi N \Delta}}{m^2_{\pi^+}}\ \frac{1}{3}\ \sqrt{(E_i + M_i)(E_f + M_f)}$$
$$\times\ p_f p_i\ \frac{1}{\sqrt{s} - M_0} , \qquad (4.27)$$

we see that this potential is of the separable kind indeed.

We need to be careful in including the s-channel diagrams in a model that has been renormalized, i.e. in which (renormalized) physical coupling constants and masses are used. It is not possible to simply add the s-channel diagrams to the other ones, because iterations of s-channel diagrams will give contributions to the vertex and self-energy. The way these diagrams are included in the NSC model is described in paper II, here we show that bare masses and coupling constants should be used in the s-channel diagrams and that these bare parameters are determined by requiring that (i) the T-matrix has a pole at the physical mass $\sqrt{s} = M_e$, (ii) the residue at the pole is given by the physical coupling constant.

V. SUMMARY

Analogous to the Nijmegen soft-core one-boson-exchange NN and YN models, we have derived the NSC model for the interaction between pseudoscalar-mesons and baryons (πN, K^+N, etc.).

For the general mass case the meson-baryon potentials in the context of a relativistic two-particle equation, the Bethe-Salpeter equation, are defined in Sec. II. The potentials consist of one-meson and one-baryon-exchange Feynman diagrams. The Bethe-Salpeter equation is approximated by assuming dynamical pair-suppression, hence neglecting the propagation of negative-energy states, and by integrating over the time variable, giving a three-dimensional integral equation for the scattering amplitude, which is a generalization of the Lippmann-Schwinger equation.

A transformation of this equation on the plane wave basis to the partial wave (LSJ) basis is described in Sec. III. A one-dimensional integral equation for the partial wave scattering amplitude is derived, which is decoupled for each partial wave, because of parity conservation in strong interactions.

In Sec. IV the interaction Hamiltonians are given and the resulting one-baryon-exchange and one-meson-exchange invariant amplitudes have been derived, these amplitudes define the partial wave potentials used in the calculations.

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Appendix A: MATRIX ELEMENTS ON THE LSJ-BASIS

1. Partial wave amplitudes

Here we derive the central and non-central potential matrix elements on the LSJ-basis in Eqs. (3.4) and (3.5)

1. *Central* $P_1 = 1$:

$$(q_f; L'J'M'|F(\mathbf{q}_f, \mathbf{q}_i)|q_i; LJM) = \sum_{s_f,s_i} \int \frac{d^3q'_f}{(2\pi)^3} \int \frac{d^3q'_i}{(2\pi)^3} \times \left(q_f; L'J'M'|\mathbf{q'}_f, s_f\right) \left(\mathbf{q'}_f, s_f|F_{op}|\mathbf{q'}_i, s_i\right)\ (\mathbf{q'}_i, s_i|q_i, LJM)\,. \tag{A1}$$

We now use the matrix elements

$$\begin{aligned}\left(\mathbf{q'}_f, s_f|F_{op}|\mathbf{q'}_i, s_i\right) &= F(\mathbf{q'}_f, \mathbf{q'}_i)\delta_{s_f,s_i} = 4\pi \sum_{l=0}^{\infty} F_l(q'_f, q'_i) \sum_{n=-l}^{l} Y^l_n(\hat{\mathbf{q}}'_f) Y^l_n(\hat{\mathbf{q}}'_i)^*\ \delta_{s_f,s_i}\ , \\ (\mathbf{q}', s|q, LJM) &= (2\pi)^3\ \frac{\delta(q'-q)}{q^2}\ \mathcal{Y}^M_{JL}(\hat{\mathbf{q}}', s)\ .\end{aligned} \tag{A2}$$

Then, substituting Eq. (A2) into Eq. (A1) and performing the momentum and angular integrals and summations, we find

$$(q_f; L'J'M'|F(\mathbf{q}_f, \mathbf{q}_i)|q_i; LJM) = 4\pi F_L\,(q_f, q_i)\ \delta_{L',L}\delta_{J',J}\ \delta_{M',M}\ . \tag{A3}$$

2. *Non-central* $P'_2 = (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_f)(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_i)$:

$$(q_f; L'J'M'|G(\mathbf{q}_f, \mathbf{q}_i)|q_i; LJM) = \sum_{s_f,s_i} \int \frac{d^3q'_f}{(2\pi)^3} \int \frac{d^3q'_i}{(2\pi)^3} \times \left(q'_f; L'J'M'|\mathbf{q'}_f, s_f\right) \left(\mathbf{q'}_f, s_f|G_{op}|\mathbf{q'}_i, s_i\right)\ (\mathbf{q'}_i, s_i|q_i, LJM)\ , \tag{A4}$$

where analogously to Eq. (A2),

$$\left(\mathbf{q'}_f, s_f|G_{op}|\mathbf{q'}_i, s_i\right) = 4\pi \sum_{l=0}^{\infty} G_l(q'_f, q'_i) \sum_{n=-l}^{l} Y^l_n(\hat{\mathbf{q}}'_f) Y^l_n(\hat{\mathbf{q}}'_i)^* \cdot \sum_s \left(s_f|(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}'_f)|s\right)\ (s|(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}'_i)|s_i)\ . \tag{A5}$$

Using Eq. (A2) and substituting Eq. (A5) into Eq. (A4), and performing the momentum integrals, we find

$$\begin{aligned}(q_f; L'J'M'|G_{op}|q_i; LJM) &= 4\pi \sum_{s_f,s_i,s} \sum_l G_l\,(q_f, q_i) \sum_{n=-l}^{l} \\ &\times \int d\hat{\Omega}_{q_f}\ \mathcal{Y}^{M'}_{J'L'}(\hat{\mathbf{q}}_f, s_f)^*(s_f|\,(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_f)\,|s)\ Y^l_n(\hat{\mathbf{q}}_f) \\ &\times \int d\hat{\Omega}_{q_i}\ Y^l_n(\hat{\mathbf{q}}_i)^*(s|\,(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_i)\,|s_i)\ \mathcal{Y}^M_{JL}(\hat{\mathbf{q}}_i, s_i)\ .\end{aligned} \tag{A6}$$

In the two-dimensional $L = J \mp \frac{1}{2}$-space, the $(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})$-operator has the matrix elements, see Appendix A 2,

$$\sum_{s'} (s|\,(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\,|s')\ \mathcal{Y}^M_{JL}(\hat{\mathbf{q}}, s') = \sum_{L'} \mathcal{Y}^M_{JL'}(\hat{\mathbf{q}}, s)\ a_{L',L} \quad \text{where } a_{L',L} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\ . \tag{A7}$$

The angular integrals in Eq. (A6) can now be performed easily. Then, the result for the non-central amplitude is

$$(q_f; L'J'M'|G|q_i; LJM) = 4\pi \sum_{L''} a_{L',L''} G_{L''}\,(q_f, q_i)\ a_{L'',L}\delta_{J',J}\delta_{M',M}\ . \tag{A8}$$

2. LSJ representation operator

In this appendix we derive Eq. (A7). The spherical wave functions in momentum space with quantum numbers J, L, S, are for spin-0 spin-1/2 given by [39]

$$\mathcal{Y}^M_{JL}(\hat{\mathbf{p}},s)=\sum_{m,\mu} C^{L\ \frac{1}{2}\ J}_{m\ \mu\ M}\ Y^L_m(\hat{\mathbf{p}})\ \chi^{(\frac{1}{2})}_{\mu}(s)\ , \tag{A9}$$

where χ is the baryon spin wave function. Using the definition for $\mathcal{Y}^M_{JL}$, Eq. (A9), we have

$$\begin{aligned}\sum_s (s'|\,(\boldsymbol{\sigma}\cdot\hat{\mathbf{p}})\,|s)\ \mathcal{Y}^M_{JL}(\hat{\mathbf{p}},s) &= (-)^m\hat{\mathbf{p}}_m\ (s'|\sigma_{-m}|s)\ C^{L\ \frac{1}{2}\ J}_{m_l\ \mu\ M}\ Y^L_{m_l}(\hat{\mathbf{p}})\ \chi^{(\frac{1}{2})}_{\mu}(s)\\ &= (-)^m\hat{\mathbf{p}}_m\ (s'|\sigma_{-m}|s)\ C^{L\ \frac{1}{2}\ J}_{m_l\ s\ M}\ Y^L_{m_l}(\hat{\mathbf{p}})\ ,\end{aligned} \tag{A10}$$

where we used the convention of summation over repeated indices, and quantization along the z-axis, which defines the spin variables s, s'. Now, we use the expressions

$$\begin{aligned}\hat{\mathbf{p}}_m\ Y^L_{m_l}(\hat{\mathbf{p}}) &= \sqrt{\frac{4\pi}{3}}Y^1_m(\hat{\mathbf{p}})\ Y^L_{m_l}(\hat{\mathbf{p}})\\ &= \sqrt{\frac{4\pi}{3}}\left[\frac{3(2L+1)}{4\pi(2L'+1)}\right]^{1/2} C^{L\ \ 1\ L'}_{0\ \ 0\ 0}\ C^{L\ \ 1\ L'}_{m_l\ m\ m_l'}\ Y^{L'}_{m_l'}(\hat{\mathbf{p}})\ ,\\ (s'|\sigma_{-m}|s) &= \sqrt{3}\ C^{\frac{1}{2}\ \ 1\ \ \frac{1}{2}}_{s\ -m\ s'}\ ,\\ (-)^m &= -\sqrt{3}\ C^{1\ \ \ 1\ \ 0}_{m\ -m\ 0}\ .\end{aligned} \tag{A11}$$

From the definition of the $9j$-coefficient [40], these formulas give for Eq. (A10) the result

$$\sum_s (s'|\,(\boldsymbol{\sigma}\cdot\hat{\mathbf{p}})\,|s)\ \mathcal{Y}^M_{JL}(\hat{\mathbf{p}},s) = -3\sum_{L'}\left[\frac{(2L+1)}{(2L'+1)}\right]^{1/2} C^{L\ \ 1\ L'}_{0\ \ 0\ 0}\begin{bmatrix} L & \frac{1}{2} & J\\ 1 & 1 & 0\\ L' & \frac{1}{2} & J\end{bmatrix}\mathcal{Y}^M_{JL'}(\hat{\mathbf{p}},s')\ . \tag{A12}$$

Evaluating Eq. (A12), one finds for the matrix a in Eq. (A7), $a=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$.

Appendix B: RELATIVISTIC INVARIANT AMPLITUDES

In this appendix the contributions from the various Feynman diagrams to the relativistic invariant amplitudes $A_{fi}(s,t,u)$ and $B_{fi}(s,t,u)$, defined in Eq. (2.5), are given. The results are valid for elastic as well as inelastic reactions. For details of the derivation we refer to [41].

1. Momentum space baryon-exchange diagrams

(i) $J^P=\frac{1}{2}^+$ baryon-exchange

(i) pseudoscalar coupling:

$$\begin{aligned}A_{ps} &= -\frac{g_{14}g_{23}}{u-M_B^2+i\epsilon}\left[-\frac{M_f+M_i}{2}+M_B\right]\ ,\\ B_{ps} &= -\frac{g_{14}g_{23}}{u-M_B^2+i\epsilon}\ .\end{aligned} \tag{B1}$$

(ii) pseudovector coupling:

$$\begin{aligned}A_{pv} &= -\frac{f_{14}f_{23}/m^2_{\pi^+}}{u-M_B^2+i\epsilon}\left[u\left(\frac{M_f+M_i}{2}+M_B\right)\right.\\ &\quad\left.-\frac{M_f+M_i}{2}M_fM_i-\frac{M_f^2+M_i^2}{2}M_B\right]\ ,\\ B_{pv} &= -\frac{f_{14}f_{23}/m^2_{\pi^+}}{u-M_B^2+i\epsilon}\left[u+(M_f+M_i)\,M_B\right.\\ &\quad\left.+M_fM_i\right]\ .\end{aligned} \tag{B2}$$

(ii) $J^P=\frac{1}{2}^+$ pole diagram

Using crossing symmetry [42], we can cross the results of the u-channel baryon-exchange into the s-channel and obtain the invariant amplitudes $A_{fi}(s,t,u)$ and $B_{fi}(s,t,u)$ for the pole diagram. We have to replace

$q \to -q'$ and $q' \to -q$, which means that we have to make the substitutions $u \leftrightarrow s$, $m_f^2 \leftrightarrow m_i^2$ and add a minus sign to the amplitude B because of Eq. (2.5). The $J^P = \frac{1}{2}^+$ pole amplitudes are

(i) pseudoscalar coupling:

$$\begin{aligned} A_{ps} &= -\frac{g_{12}g_{34}}{s - M_B^2 + i\epsilon}\left[-\frac{M_f + M_i}{2} + M_B\right] , \\ B_{ps} &= \frac{g_{12}g_{34}}{s - M_B^2 + i\epsilon} . \end{aligned} \tag{B3}$$

(ii) pseudovector coupling:

$$\begin{aligned} A_{pv} &= -\frac{f_{12}f_{34}/m_{\pi^+}^2}{s - M_B^2 + i\epsilon}\left[s\left(\frac{M_f + M_i}{2} + M_B\right)\right. \\ &\left. -\frac{M_f + M_i}{2}M_f M_i - \frac{M_f^2 + M_i^2}{2}M_B\right] , \\ B_{pv} &= \frac{f_{12}f_{34}/m_{\pi^+}^2}{s - M_B^2 + i\epsilon}\left[s + (M_f + M_i)M_B\right. \\ &\left. +M_f M_i\right] . \end{aligned} \tag{B4}$$

(iii) $J^P = \frac{1}{2}^-$ baryon-exchange

(i) scalar coupling:

$$\begin{aligned} A_s &= -\frac{g_{14}^{*(s)}g_{23}^{*(s)}}{u - M_B^2 + i\epsilon}\left[-\frac{M_f + M_i}{2} - M_B\right] , \\ B_s &= -\frac{g_{14}^{*(s)}g_{23}^{*(s)}}{u - M_B^2 + i\epsilon} . \end{aligned} \tag{B5}$$

(ii) vector coupling:

$$\begin{aligned} A_v &= -\frac{f_{14}^{*(v)}f_{23}^{*(v)}/m_{\pi^+}^2}{u - M_B^2 + i\epsilon}\left[u\left(\frac{M_f + M_i}{2} - M_B\right)\right. \\ &\left. -\frac{M_f + M_i}{2}M_f M_i + \frac{M_f^2 + M_i^2}{2}M_B\right] , \\ B_v &= -\frac{f_{14}^{*(v)}f_{23}^{*(v)}/m_{\pi^+}^2}{u - M_B^2 + i\epsilon}\left[u - (M_f + M_i)M_B\right. \\ &\left. +M_f M_i\right] . \end{aligned} \tag{B6}$$

(iii) $J^P = \frac{1}{2}^-$ pole diagram

Applying crossing symmetry again we find, similar to the $J^P = \frac{1}{2}^+$ baryon pole diagram, the invariant amplitudes for the $J^P = \frac{1}{2}^-$ baryon pole diagram.

(i) scalar coupling:

$$\begin{aligned} A_s &= -\frac{g_{12}^{*(s)}g_{34}^{*(s)}}{s - M_B^2 + i\epsilon}\left[-\frac{M_f + M_i}{2} - M_B\right] , \\ B_s &= \frac{g_{12}^{*(s)}g_{34}^{*(s)}}{s - M_B^2 + i\epsilon} . \end{aligned} \tag{B7}$$

(ii) vector coupling:

$$\begin{aligned} A_v &= -\frac{f_{12}^{*(v)}f_{34}^{*(v)}/m_{\pi^+}^2}{s - M_B^2 + i\epsilon}\left[s\left(\frac{M_f + M_i}{2} - M_B\right)\right. \\ &\left. -\frac{M_f + M_i}{2}M_f M_i + \frac{M_f^2 + M_i^2}{2}M_B\right] , \\ B_v &= \frac{f_{12}^{*(v)}f_{34}^{*(v)}/m_{\pi^+}^2}{s - M_B^2 + i\epsilon}\left[s - (M_f + M_i)M_B\right. \\ &\left. +M_f M_i\right] . \end{aligned} \tag{B8}$$

(v) $J^P = \frac{3}{2}^+$ baryon-exchange

$$\begin{aligned} A_{Y^*} &= \frac{f_{14}^* f_{23}^*/m_{\pi^+}^2}{u - M_{Y^*}^2 + i\epsilon}\left[\frac{t - m_f^2 - m_i^2}{2}\right. \\ &\times\left[\frac{M_f + M_i}{2} + M_{Y^*}\right] \\ &+\frac{1}{6M_{Y^*}^2}\left[M_f^2 - m_i^2 - u\right]\left[M_i^2 - m_f^2 - u\right] \\ &\times\frac{M_f + M_i}{2} + \frac{M_{Y^*}}{3}\left[u - \frac{M_f^2 + M_i^2}{2}\right] \\ &+\frac{1}{3}\left[\frac{M_f + M_i}{2}u - \frac{\left(M_f^2 + M_i^2\right)(M_f + M_i)}{4}\right. \\ &\left.+\frac{\left(m_i^2 - m_f^2\right)(M_f - M_i)}{4}\right] + \frac{1}{6M_{Y^*}} \times \\ &\left[\left(M_f^2 - m_i^2 - u\right)\left(\frac{M_i}{2}(M_i - M_f) - m_f^2\right) - \right. \\ &\left.\left.\left(M_i^2 - m_f^2 - u\right)\left(\frac{M_f}{2}(M_i - M_f) + m_i^2\right)\right]\right] , \end{aligned}$$

$$\begin{aligned} B_{Y^*} &= \frac{f_{14}^* f_{23}^*/m_{\pi^+}^2}{u - M_{Y^*}^2 + i\epsilon}\left[-\frac{t - m_f^2 - m_i^2}{2}\right. \\ &-\frac{1}{6M_{Y^*}^2}\left[u - M_f^2 + m_i^2\right]\left[u - M_i^2 + m_f^2\right] \\ &+\frac{M_{Y^*}}{3}(M_f + M_i) - \frac{m_f^2 + m_i^2 - (M_f + M_i)^2}{6} \end{aligned}$$

$$+\frac{1}{6M_{Y^*}}\left[M_f\left(M_i^2-m_f^2-u\right)\right.$$
$$\left.\left.+M_i\left(M_f^2-m_i^2-u\right)\right]\right] . \tag{B9}$$

(vi) $J^P=\frac{3}{2}^+$ pole diagram

Applying crossing symmetry again we find, similar to the $J^P=\frac{1}{2}^+$ baryon pole diagram, the invariant amplitudes for the $J^P=\frac{3}{2}^+$ baryon pole diagram

$$\begin{aligned}
A_{Y^*} &= \frac{f_{12}^* f_{34}^*/m_{\pi^+}^2}{s-M_{Y^*}^2+i\epsilon}\left[\frac{t-m_f^2-m_i^2}{2}\right.\\
&\times\left[\frac{M_f+M_i}{2}+M_{Y^*}\right]\\
&+\frac{1}{6M_{Y^*}^2}\left[M_f^2-m_f^2-s\right]\left[M_i^2-m_i^2-s\right]\\
&\times\frac{M_f+M_i}{2}+\frac{M_{Y^*}}{3}\left[s-\frac{M_f^2+M_i^2}{2}\right]\\
&+\frac{1}{3}\left[\frac{M_f+M_i}{2}s-\frac{\left(M_f^2+M_i^2\right)\left(M_f+M_i\right)}{4}\right.\\
&\left.+\frac{\left(m_f^2-m_i^2\right)\left(M_f-M_i\right)}{4}\right]+\frac{1}{6M_{Y^*}}\times\\
&\left[\left(M_f^2-m_f^2-s\right)\left(\frac{M_i}{2}\left(M_i-M_f\right)-m_i^2\right)-\right.\\
&\left.\left.\left(M_i^2-m_i^2-s\right)\left(\frac{M_f}{2}\left(M_i-M_f\right)+m_f^2\right)\right]\right] ,\\
B_{Y^*} &= -\frac{f_{12}^* f_{34}^*/m_{\pi^+}^2}{s-M_{Y^*}^2+i\epsilon}\left[-\frac{t-m_f^2-m_i^2}{2}\right.\\
&-\frac{1}{6M_Y^2}\left[s-M_f^2+m_f^2\right]\left[s-M_i^2+m_i^2\right]\\
&+\frac{M_{Y^*}}{3}\left(M_f+M_i\right)-\frac{m_f^2+m_i^2-\left(M_f+M_i\right)^2}{6}\\
&+\frac{1}{6M_{Y^*}}\left[M_f\left(M_i^2-m_i^2-s\right)\right.\\
&\left.\left.+M_i\left(M_f^2-m_f^2-s\right)\right]\right] .
\end{aligned} \tag{B10}$$

2. Momentum space meson-exchange diagrams

(i) $J^P=0^{++}$ scalar-meson-exchange

$$\begin{aligned}
A_S &= \frac{g_{PPS}\,g_S}{t-m_S^2+i\epsilon} ,\\
B_S &= 0 .
\end{aligned} \tag{B11}$$

(ii) $J^P=1^{--}$ vector-meson-exchange

$$\begin{aligned}
A_V &= \frac{g_{PPV}}{t-m_V^2+i\epsilon}\left[g_V\,\frac{m_f^2-m_i^2}{m_V^2}\left(M_i-M_f\right)\right.\\
&\left.+\frac{s-u}{2\mathcal{M}}f_V\right] ,\\
B_V &= -2\frac{g_{PPV}}{t-m_V^2+i\epsilon}\left[f_V\frac{M_f+M_i}{2\mathcal{M}}+g_V\right] .
\end{aligned} \tag{B12}$$

(iii) $J^P=2^{++}$ tensor-meson-exchange

$$\begin{aligned}
A_T &= \frac{g_{PPT}/m_{\pi^+}}{\Delta^2-m_T^2+i\epsilon}\left[\left(\frac{s-u}{2}\right)^2\ F_2\right.\\
&-\frac{1}{2m_T^2}\left(m_f^2-m_i^2\right)(s-u)\left[(M_i-M_f)\ F_1\right.\\
&\left.+\left(M_i^2-M_f^2\right)\ F_2\right]+\frac{1}{2m_T^4}\left(m_f^2-m_i^2\right)^2\times\\
&\left(M_i^2-M_f^2\right)\left[(M_i-M_f)\,F_1+\frac{M_i^2-M_f^2}{2}F_2\right]\\
&-\frac{4}{3}\left[-Q^2+\frac{1}{4m_T^2}\left(m_f^2-m_i^2\right)^2\right]\times\\
&\left[-\frac{1}{2}\left((M_f+M_i)\,F_1+\frac{1}{2}\left(p'+p\right)^2F_2\right)\right.\\
&+\frac{M_i^2-M_f^2}{2m_T^2}\left((M_i-M_f)F_1\right.\\
&\left.\left.\left.+\frac{1}{2}\left(M_i^2-M_f^2\right)F_2\right)\right]\right] ,\\
B_T &= \frac{g_{PPT}/m_{\pi^+}}{\Delta^2-m_T^2+i\epsilon}\left[(s-u)\ F_1\right.\\
&\left.-\frac{\left(m_f^2-m_i^2\right)\left(M_i^2-M_f^2\right)}{m_T^2}\ F_1\right] .
\end{aligned} \tag{B13}$$

3. Momentum space Pomeron-exchange

$$\begin{aligned}
A_P &= \frac{g_{PPP}\,g_P}{\mathcal{M}} ,\\
B_P &= 0 .
\end{aligned} \tag{B14}$$

Appendix C: X,Y,Z-COEFFICIENTS

Here we list the explicit expressions for the expansion coefficients $X^{(\alpha)}$, $Y^{(\alpha)}$, $Z^{(\alpha)}$ and $U^{(\alpha)}$ of the partial wave potentials, Eq. (4.23), for each type of exchange in the s-, u- and t-channel. We have introduced the notation $N^{\pm}_{fi} = \sqrt{(E_i \pm M_i)(E_f \pm M_f)}$.

1. Baryon-exchange

(i) $J^P = \frac{1}{2}^+$ baryon-exchange

(i) pseudoscalar coupling:

$$
\begin{aligned}
X_B^{(C)} &= g_{14}g_{23}N^+_{fi}\left[M_B + \frac{W_f+W_i}{2} - M_f - M_i\right] ,\\
Y_B^{(C)} &= g_{14}g_{23}\ N^-_{fi}\left[-M_B + \frac{W_f+W_i}{2} + M_f + M_i\right] ,\\
X_B^{(SO)} &= -g_{14}g_{23}\ N^-_{fi}\left[-M_B + \frac{W_f+W_i}{2} + M_f + M_i\right] .
\end{aligned}
\tag{C1}
$$

(ii) pseudovector coupling:

$$
\begin{aligned}
X_B^{(C)} &= -\frac{f_{14}f_{23}}{m^2_{\pi^+}}\ N^+_{fi}\Bigg[\left(-\frac{M_f+M_i}{2} - M_B\right)\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2\right) + \frac{M_f+M_i}{2}M_fM_i \\
&+\frac{M_f^2+M_i^2}{2}M_B - \frac{W_f+W_i-M_f-M_i}{2}\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2 + (M_f+M_i)\,M_B\right. \\
&\left.+M_fM_i\right)\Bigg] ,\\
Y_B^{(C)} &= -\frac{f_{14}f_{23}}{m^2_{\pi^+}}\Bigg[N^+_{fi}\left[-\left(-\frac{M_f+M_i}{2} - M_B\right) + \frac{W_f+W_i-M_f-M_i}{2}\right]2p_fp_i + N^-_{fi}\Bigg[-\Bigg(-\frac{M_f+M_i}{2} \\
&-M_B)\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2\right) - \frac{M_f+M_i}{2}M_fM_i - \frac{M_f^2+M_i^2}{2}M_B \\
&-\frac{W_f+W_i+M_f+M_i}{2}\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2 + (M_f+M_i)\,M_B + M_fM_i\right)\Bigg]\Bigg] ,\\
Z_B^{(C)} &= -\frac{f_{14}f_{23}}{m^2_{\pi^+}}\ N^-_{fi}\left[-\frac{M_f+M_i}{2} - M_B + \frac{W_f+W_i+M_f+M_i}{2}\right]2p_fp_i ,\\
X_B^{(SO)} &= \frac{f_{14}f_{23}}{m^2_{\pi^+}}\ N^-_{fi}\Bigg[-\left(-\frac{M_f+M_i}{2} - M_B\right)\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2\right) - \frac{M_f+M_i}{2}M_fM_i \\
&-\frac{M_f^2+M_i^2}{2}M_B\ - \frac{W_f+W_i+M_f+M_i}{2}\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2 + (M_f+M_i)\,M_B\right. \\
&\left.+M_fM_i\right)\Bigg] ,\\
Y_B^{(SO)} &= \frac{f_{14}f_{23}}{m^2_{\pi^+}}\ N^-_{fi}\left[-\frac{M_f+M_i}{2} - M_B + \frac{W_f+W_i+M_f+M_i}{2}\right]2p_fp_i .
\end{aligned}
\tag{C2}
$$

(ii) $J^P = \frac{1}{2}^+$ pole term

(i) pseudoscalar coupling:

$$\begin{aligned}
X_B^{(C)} &= -g_{12}g_{34}\ N_{fi}^{+}\left[M_B - \frac{W_f+W_i}{2}\right] \ , \\
Y_B^{(C)} &= g_{12}g_{34}\ N_{fi}^{-}\left[M_B + \frac{W_f+W_i}{2}\right] \ , \\
X_B^{(SO)} &= -g_{12}g_{34}\ N_{fi}^{-}\left[M_B + \frac{W_f+W_i}{2}\right] \ .
\end{aligned} \tag{C3}$$

(ii) pseudovector coupling:

$$\begin{aligned}
X_B^{(C)} &= \frac{f_{12}f_{34}}{m_{\pi^+}^2}\ N_{fi}^{+}\left[\left(-\frac{M_f+M_i}{2} - M_B\right)s + \frac{M_f+M_i}{2}M_fM_i + \frac{W_f+W_i-M_f-M_i}{2}\left[s+(M_f+\right.\right. \\
&\quad \left.\left. M_i)\,M_B + M_fM_i\right] + \frac{M_f^2+M_i^2}{2}M_B\right] \ , \\
Y_B^{(C)} &= \frac{f_{12}f_{34}}{m_{\pi^+}^2}\ N_{fi}^{-}\left[-\left(-\frac{M_f+M_i}{2} - M_B\right)s - \frac{M_f+M_i}{2}M_fM_i + \frac{W_f+W_i-M_f-M_i}{2}\left[s+(M_f+\right.\right. \\
&\quad \left.\left. M_i)\,M_B + M_fM_i\right] - \frac{M_f^2+M_i^2}{2}M_B\right] \ , \\
X_B^{(SO)} &= -\frac{f_{12}f_{34}}{m_{\pi^+}^2}\ N_{fi}^{-}\left[-\left(-\frac{M_f+M_i}{2} - M_B\right)s - \frac{M_f+M_i}{2}M_fM_i + \frac{W_f+W_i-M_f-M_i}{2}\left[s+(M_f+\right.\right. \\
&\quad \left.\left. M_i)\,M_B + M_fM_i\right] - \frac{M_f^2+M_i^2}{2}M_B\right] \ .
\end{aligned} \tag{C4}$$

(iii) $J^P = \frac{1}{2}^-$ baryon-exchange

(i) scalar coupling:

$$\begin{aligned}
X_B^{(C)} &= g_{14}^{*(s)}g_{23}^{*(s)}N_{fi}^{+}\left[-M_B + \frac{W_f+W_i}{2} - M_f - M_i\right] \ , \\
Y_B^{(C)} &= g_{14}^{*(s)}g_{23}^{*(s)}\ N_{fi}^{-}\left[M_B + \frac{W_f+W_i}{2} + M_f + M_i\right] \ , \\
X_B^{(SO)} &= -g_{14}^{*(s)}g_{23}^{*(s)}\ N_{fi}^{-}\left[M_B + \frac{W_f+W_i}{2} + M_f + M_i\right] .
\end{aligned} \tag{C5}$$

(ii) vector coupling:

$$\begin{aligned}
X_B^{(C)} &= -\frac{f_{14}^{*(v)}f_{23}^{*(v)}}{m_{\pi^+}^2}\ N_{fi}^{+}\left[\left(-\frac{M_f+M_i}{2} + M_B\right)\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2\right) + \frac{M_f+M_i}{2}M_fM_i\right. \\
&\quad -\frac{M_f^2+M_i^2}{2}M_B - \frac{W_f+W_i-M_f-M_i}{2}\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2 - (M_f+M_i)\,M_B\right. \\
&\quad \left.\left. +M_fM_i\right)\right] \ , \\
Y_B^{(C)} &= -\frac{f_{14}^{*(v)}f_{23}^{*(v)}}{m_{\pi^+}^2}\ \left[N_{fi}^{+}\left[-\left(-\frac{M_f+M_i}{2} + M_B\right) + \frac{W_f+W_i-M_f-M_i}{2}\right]2p_fp_i + N_{fi}^{-}\left[-\left(-\frac{M_f+M_i}{2}\right.\right.\right. \\
&\quad \left. +M_B\right)\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2 - p_f^2 - p_i^2\right) - \frac{M_f+M_i}{2}M_fM_i + \frac{M_f^2+M_i^2}{2}M_B
\end{aligned}$$

$$
\begin{aligned}
& -\frac{W_f+W_i+M_f+M_i}{2}\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2-p_f^2-p_i^2-\left(M_f+M_i\right)M_B+M_fM_i\right)\Bigg]\Bigg] \ , \\
Z_B^{(C)} &= -\frac{f_{14}^{*(v)}f_{23}^{*(v)}}{m_{\pi^+}^2}\ N_{fi}^-\left[-\frac{M_f+M_i}{2}+M_B+\frac{W_f+W_i+M_f+M_i}{2}\right]2p_fp_i \ , \\
X_B^{(SO)} &= \frac{f_{14}^{*(v)}f_{23}^{*(v)}}{m_{\pi^+}^2}\ N_{fi}^-\Bigg[-\left(-\frac{M_f+M_i}{2}+M_B\right)\left(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2-p_f^2-p_i^2\right)-\frac{M_f+M_i}{2}M_fM_i \\
& +\frac{M_f^2+M_i^2}{2}M_B \ -\frac{W_f+W_i+M_f+M_i}{2}\Bigg(\left(\frac{E_f+E_i-\omega_f-\omega_i}{2}\right)^2-p_f^2-p_i^2-\left(M_f+M_i\right)M_B \\
& +M_fM_i\Bigg)\Bigg] \ , \\
Y_B^{(SO)} &= \frac{f_{14}^{*(v)}f_{23}^{*(v)}}{m_{\pi^+}^2}\ N_{fi}^-\left[-\frac{M_f+M_i}{2}+M_B+\frac{W_f+W_i+M_f+M_i}{2}\right]2p_fp_i \ . \qquad \text{(C6)}
\end{aligned}
$$

(iv) $J^P=\frac{1}{2}^-$ pole term

(i) scalar coupling:

$$
\begin{aligned}
X_B^{(C)} &= g_{12}^{*(v)}g_{34}^{*(v)}\ N_{fi}^+\left[M_B+\frac{W_f+W_i}{2}\right] \ , \\
Y_B^{(C)} &= g_{12}^{*(v)}g_{34}^{*(v)}\ N_{fi}^-\left[-M_B+\frac{W_f+W_i}{2}\right] \ , \\
X_B^{(SO)} &= -g_{12}^{*(v)}g_{34}^{*(v)}\ N_{fi}^-\left[-M_B+\frac{W_f+W_i}{2}\right] \ . \qquad \text{(C7)}
\end{aligned}
$$

(ii) vector coupling:

$$
\begin{aligned}
X_B^{(C)} &= \frac{f_{12}^{*(v)}f_{34}^{*(v)}}{m_{\pi^+}^2}\ N_{fi}^+\Bigg[\left(-\frac{M_f+M_i}{2}+M_B\right)s+\frac{M_f+M_i}{2}M_fM_i+\frac{W_f+W_i-M_f-M_i}{2}\left[s-(M_f+\right. \\
& \left.M_i)\,M_B+M_fM_i\right]-\frac{M_f^2+M_i^2}{2}M_B\Bigg] \ , \\
Y_B^{(C)} &= \frac{f_{12}^{*(v)}f_{34}^{*(v)}}{m_{\pi^+}^2}\ N_{fi}^-\Bigg[-\left(-\frac{M_f+M_i}{2}+M_B\right)s-\frac{M_f+M_i}{2}M_fM_i+\frac{W_f+W_i-M_f-M_i}{2}\left[s-(M_f+\right. \\
& \left.M_i)\,M_B+M_fM_i\right]+\frac{M_f^2+M_i^2}{2}M_B\Bigg] \ , \\
X_B^{(SO)} &= -\frac{f_{12}^{*(v)}f_{34}^{*(v)}}{m_{\pi^+}^2}\ N_{fi}^-\Bigg[-\left(-\frac{M_f+M_i}{2}+M_B\right)s-\frac{M_f+M_i}{2}M_fM_i+\frac{W_f+W_i-M_f-M_i}{2}\left[s-(M_f+\right. \\
& \left.M_i)\,M_B+M_fM_i\right]+\frac{M_f^2+M_i^2}{2}M_B\Bigg] \ . \qquad \text{(C8)}
\end{aligned}
$$

(v) $J^P=\frac{3}{2}^+$ baryon-exchange

$$
\begin{aligned}
X_{Y^*}^{(C)} &= -\frac{f_{14}^*f_{23}^*}{m_{\pi^+}^2}N_{fi}^+\left[A_0+\frac{B_0}{2}\left(W_f+W_i-M_i-M_f\right)\right] \ , \\
Y_{Y^*}^{(C)} &= -\frac{f_{14}^*f_{23}^*}{m_{\pi^+}^2}\left[N_{fi}^+\left[A_1+\frac{B_1}{2}\left(W_f+W_i-M_f-M_i\right)\right]+N_{fi}^-\left[-A_0+\frac{B_0}{2}\left(W_f+W_i+M_f+M_i\right)\right]\right] \ , \\
Z_{Y^*}^{(C)} &= -\frac{f_{14}^*f_{23}^*}{m_{\pi^+}^2}\left[N_{fi}^+\left[A_2+\frac{B_2}{2}\left(W_f+W_i-M_f-M_i\right)\right]+N_{fi}^-\left[-A_1+\frac{B_1}{2}\left(W_f+W_i+M_f+M_i\right)\right]\right] \ ,
\end{aligned}
$$

$$
\begin{aligned}
U_{Y^*}^{(C)} &= \frac{f_{14}^* f_{23}^*}{m_{\pi^+}^2} N_{fi}^- \left[-A_2 + \frac{B_2}{2}\left(W_f + W_i + M_f + M_i\right)\right] \ , \\
X_{Y^*}^{(SO)} &= \frac{f_{14}^* f_{23}^*}{m_{\pi^+}^2} N_{fi}^- \left[-A_0 + \frac{B_0}{2}\left(W_f + W_i + M_i + M_f\right)\right] \ , \\
Y_{Y^*}^{(SO)} &= \frac{f_{14}^* f_{23}^*}{m_{\pi^+}^2} N_{fi}^- \left[-A_1 + \frac{B_1}{2}\left(W_f + W_i + M_i + M_f\right)\right] \ , \\
Z_{Y^*}^{(SO)} &= \frac{f_{14}^* f_{23}^*}{m_{\pi^+}^2} N_{fi}^- \left[-A_2 + \frac{B_0}{2}\left(W_f + W_i - M_f - M_i\right)\right] .
\end{aligned} \tag{C9}
$$

Where A_0, A_1, A_2, B_0, B_1 and B_2 depend on the mass and momentum of the particles as follows.

$$
\begin{aligned}
A_0 \ = \ & \frac{1}{12M_{Y^*}^2}\left(M_f+M_i\right)\left(-2p_f p_i z_u + M_{Y^*}^2\right)^2 + \left(-\frac{1}{6M_{Y^*}^2}\left(M_f^2+M_i^2-m_f^2-m_i^2\right)\frac{M_f+M_i}{2} + \frac{M_{Y^*}}{3} + \right. \\
& \left. \frac{M_f+M_i}{6} - \frac{1}{6M_{Y^*}}\left(\frac{M_i}{2}\left(M_i-M_f\right) - m_f^2 + \frac{M_f}{2}\left(M_f-M_i\right) - m_i^2\right)\right)\left(-2p_f p_i z_u + M_{Y^*}^2\right) \\
& + \frac{1}{2}\left(\frac{M_f+M_i}{2} + M_{Y^*}\right)\left(\frac{\left(E_f-E_i\right)^2 + \left(\omega_f-\omega_i\right)^2}{2} - p_f^2 - p_i^2\right) - \frac{m_f^2+m_i^2}{2}\left(\frac{M_f+M_i}{2} + M_{Y^*}\right) \\
& + \frac{1}{12M_{Y^*}^2}\left(M_f+M_i\right)\left(M_f^2-m_i^2\right)\left(M_i^2-m_f^2\right) - \frac{M_{Y^*}}{6}\left(M_f^2+M_i^2\right) + \frac{1}{12}\left(\left(m_i^2-m_f^2\right)\left(M_f-M_i\right)\right. \\
& \left. - \left(M_f^2+M_i^2\right)\left(M_f+M_i\right)\right) + \frac{1}{6M_{Y^*}}\left(\left(M_f^2-m_i^2\right)\left(\frac{M_i}{2}\left(M_i-M_f\right) - m_f^2\right) + \left(M_i^2-m_f^2\right)\times\right. \\
& \left. \left(\frac{M_f}{2}\left(M_f-M_i\right) - m_i^2\right)\right) \ ,
\end{aligned}
$$

$$
\begin{aligned}
A_1 \ &= \ \left[-\frac{1}{12M_{Y^*}^2}\left(M_f+M_i\right)2\left(-2p_f p_i z_u + M_{Y^*}^2\right) + \frac{1}{6M_{Y^*}^2}\left(M_f^2+M_i^2-m_f^2-m_i^2\right)\frac{M_f+M_i}{2} - \frac{M_{Y^*}}{3} - \right. \\
& \left. \frac{M_f+M_i}{6} + \frac{1}{6M_{Y^*}}\left(\frac{M_i}{2}\left(M_i-M_f\right) - m_f^2 + \frac{M_f}{2}\left(M_f-M_i\right) - m_i^2\right) + \frac{1}{2}\left(\frac{M_f+M_i}{2} + M_{Y^*}\right)\right] 2p_f p_i \ , \\
A_2 \ &= \ \frac{1}{12M_{Y^*}^2}\left(M_f+M_i\right)\left(2p_f p_i\right)^2 \ , \\
B_0 \ &= \ \frac{m_f^2+m_i^2}{2} - \frac{1}{6M_{Y^*}^2}\left(M_f^2-m_i^2\right)\left(M_i^2-m_f^2\right) + \frac{M_{Y^*}}{3}\left(M_f+M_i\right) - \frac{m_f^2+m_i^2-\left(M_f+M_i\right)^2}{6} \\
& + \frac{1}{6M_{Y^*}}\left(M_f\left(M_i^2-m_f^2\right) + M_i\left(M_f^2-m_i^2\right)\right) - \frac{1}{2}\left(\frac{\left(E_f-E_i\right)^2 + \left(\omega_f-\omega_i\right)^2}{2} - p_f^2 - p_i^2\right) \\
& - \frac{1}{6M_{Y^*}^2}\left(-2p_f p_i z_u + M_{Y^*}^2\right)^2 + \left(\frac{1}{6M_{Y^*}^2}\left(M_f^2+M_i^2-m_f^2-m_i^2\right) - \frac{1}{6M_{Y^*}}\left(M_f+M_i\right)\right)\times \\
& \left(-2p_f p_i z_u + M_{Y^*}^2\right) \ , \\
B_1 \ &= \ \left[\frac{1}{3M_{Y^*}^2}\left(-2p_f p_i z_u + M_{Y^*}^2\right) - \frac{1}{6M_{Y^*}}\left(\frac{1}{M_{Y^*}}\left(M_f^2+M_i^2-m_f^2-m_i^2\right) - \left(M_f+M_i\right)\right) - \frac{1}{2}\right] 2p_f p_i \ , \\
B_2 \ &= \ -\frac{1}{6M_{Y^*}^2}\left(2p_f p_i\right)^2 \ .
\end{aligned} \tag{C10}
$$

(vi) $J^P = \frac{3}{2}^+$ pole term

$$
\begin{aligned}
X_{Y^*}^{(C)} &= \frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2} N_{fi}^+ \left[A_0 + \frac{B_0}{2}\left(W_f + W_i - M_i - M_f\right)\right] \ , \\
Y_{Y^*}^{(C)} &= \frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2} \left[N_{fi}^+ \left[A_1 + \frac{B_1}{2}\left(W_f + W_i - M_f - M_i\right)\right] + N_{fi}^- \left[-A_0 + \frac{B_0}{2}\left(W_f + W_i + M_f + M_i\right)\right]\right] \ ,
\end{aligned}
$$

$$
\begin{aligned}
Z_{Y^*}^{(C)} &= \frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2} N_{fi}^- \left[-A_1 + \frac{B_1}{2} \left(W_f + W_i + M_f + M_i \right) \right] \ , \\
X_{Y^*}^{(SO)} &= -\frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2} N_{fi}^- \left[-A_0 + \frac{B_0}{2} \left(W_f + W_i + M_i + M_f \right) \right] \ , \\
Y_{Y^*}^{(SO)} &= -\frac{f_{12}^* f_{34}^*}{m_{\pi^+}^2} N_{fi}^- \left[-A_1 + \frac{B_1}{2} \left(W_f + W_i + M_i + M_f \right) \right] \ .
\end{aligned}
\tag{C11}
$$

Where A_0, A_1, B_0 and B_1 depend on the mass and momentum of the particles as follows.

$$
\begin{aligned}
A_0 &= \frac{1}{2} \left(\frac{(E_f - E_i)^2 + (\omega_f - \omega_i)^2}{2} - p_f^2 - p_i^2 - m_f^2 - m_i^2 \right) \left(\frac{M_f + M_i}{2} + M_{Y^*} \right) + \frac{1}{6M_{Y^*}^2} \left(M_f^2 - m_f^2 - s \right) \times \\
&\quad \left(M_i^2 - m_i^2 - s \right) \frac{M_f + M_i}{2} + \frac{1}{3M_{Y^*}} \left(s - \frac{1}{2} \left(M_f^2 + M_i^2 \right) \right) + \frac{1}{3} \left(\frac{M_f + M_i}{2} s - \frac{\left(M_f^2 + M_i^2 \right) (M_f + M_i)}{4} \right. \\
&\quad \left. + \frac{m_f^2 - m_i^2}{4} (M_f - M_i) \right) + \frac{1}{6M_{Y^*}} \left(\left(M_f^2 - m_f^2 - s \right) \left(\frac{M_i}{2} (M_i - M_f) - m_i^2 \right) - \left(M_i^2 - m_i^2 - s \right) \times \right. \\
&\quad \left. \left(\frac{M_f}{2} (M_i - M_f) + m_f^2 \right) \right) \ , \\
A_1 &= \frac{1}{2} \left[\frac{M_f + M_i}{2} + M_{Y^*} \right] 2 p_f p_i \ , \\
B_0 &= \frac{1}{2} \left(\frac{(E_f - E_i)^2 + (\omega_f - \omega_i)^2}{2} - p_f^2 - p_i^2 - m_f^2 - m_i^2 \right) - \frac{1}{6M_{Y^*}^2} \left(s - M_f^2 + m_f^2 \right) \left(s - M_i^2 + m_i^2 \right) \\
&\quad - \frac{M_{Y^*}}{3} (M_f + M_i) + \frac{m_f^2 + m_i^2 - (M_f + M_i)^2}{6} - \frac{1}{6M_{Y^*}} \left(M_f \left(M_i^2 - m_i^2 - s \right) + M_i \left(M_f^2 - m_f^2 - s \right) \right) \ , \\
B_1 &= p_f p_i \ .
\end{aligned}
\tag{C12}
$$

2. Meson-exchange

(i) $J^P = 0^{++}$ scalar-meson-exchange

$$
\begin{aligned}
X_S^{(C)} &= -g_{PPS} g_S \ N_{fi}^+ \ , \\
Y_S^{(C)} &= g_{PPS} g_S \ N_{fi}^- \ , \\
X_S^{(SO)} &= -g_{PPS} g_S \ N_{fi}^- \ .
\end{aligned}
\tag{C13}
$$

(ii) $J^P = 1^{--}$ vector-meson-exchange

$$
\begin{aligned}
X_V^{(C)} &= -g_{PPV} g_V \ N_{fi}^+ \left[\frac{\left(m_f^2 - m_i^2 \right) (M_i - M_f)}{m_V^2} - (W_f + W_i - M_i - M_f) \right] - g_{PPV} f_V \ N_{fi}^+ \left[- \frac{M_f + M_i}{2\mathcal{M}} \right. \\
&\quad \left. \times (W_f + W_i - M_i - M_f) + \frac{(\omega_f + \omega_i)(E_f + E_i) + p_f^2 + p_i^2}{2\mathcal{M}} \right] \ , \\
Y_V^{(C)} &= -g_{PPV} g_V \ N_{fi}^- \left[- \frac{\left(m_f^2 - m_i^2 \right) (M_i - M_f)}{m_V^2} - (W_f + W_i + M_i + M_f) \right] - g_{PPV} f_V \ \left[N_{fi}^+ \frac{p_f p_i}{\mathcal{M}} \right. \\
&\quad \left. + N_{fi}^- \left[- \frac{M_f + M_i}{2\mathcal{M}} (W_f + W_i + M_i + M_f) - \frac{(\omega_f + \omega_i)(E_f + E_i) + p_f^2 + p_i^2}{2\mathcal{M}} \right] \right] \ ,
\end{aligned}
$$

$$
\begin{aligned}
Z_V^{(C)} &= g_{PPV} f_V \; N_{fi}^- \; \frac{p_f p_i}{\mathcal{M}} \; , \\
X_V^{(SO)} &= g_{PPV} g_V \; N_{fi}^- \left[-\frac{\left(m_f^2 - m_i^2\right)\left(M_i - M_f\right)}{m_V^2} - \left(W_f + W_i + M_f + M_i\right) \right] + g_{PPV} f_V \; N_{fi}^- \left[-\frac{M_f + M_i}{2\mathcal{M}} \right. \\
&\quad \left. \times \left(W_f + W_i + M_f + M_i\right) - \frac{\left(\omega_f + \omega_i\right)\left(E_f + E_i\right) + p_f^2 + p_i^2}{2\mathcal{M}} \right] \; , \\
Y_V^{(SO)} &= -g_{PPV} f_V \; N_{fi}^- \; \frac{p_f p_i}{\mathcal{M}} \; .
\end{aligned} \tag{C14}
$$

(iii) $J^P = 2^{++}$ tensor-meson-exchange

$$
\begin{aligned}
X_T^{(C)} &= -\frac{g_{PPT} F_1}{m_{\pi^+}} N_{fi}^+ \left[A_0 + \frac{B_0}{2}\left(W_f + W_i - M_i - M_f\right) \right] \; , \\
Y_T^{(C)} &= -\frac{g_{PPT} F_1}{m_{\pi^+}} \left[N_{fi}^+ \left[A_1 + \frac{B_1}{2}\left(W_f + W_i - M_f - M_i\right) \right] + N_{fi}^- \left[-A_0 + \frac{B_0}{2}\left(W_f + W_i + M_f + M_i\right) \right] \right] \; , \\
Z_T^{(C)} &= -\frac{g_{PPT} F_1}{m_{\pi^+}} \left[N_{fi}^+ A_2 + N_{fi}^- \left[-A_1 + \frac{B_1}{2}\left(W_f + W_i + M_f + M_i\right) \right] \right] \; , \\
U_T^{(C)} &= \frac{g_{PPT} F_1}{m_{\pi^+}} N_{fi}^- A_2 \; , \\
X_T^{(SO)} &= \frac{g_{PPT} F_1}{m_{\pi^+}} N_{fi}^- \left[-A_0 + \frac{B_0}{2}\left(W_f + W_i + M_i + M_f\right) \right] \; , \\
Y_T^{(SO)} &= \frac{g_{PPT} F_1}{m_{\pi^+}} N_{fi}^- \left[-A_1 + \frac{B_1}{2}\left(W_f + W_i + M_i + M_f\right) \right] \; , \\
Z_T^{(SO)} &= -\frac{g_{PPT} F_1}{m_{\pi^+}} N_{fi}^- A_2 \; .
\end{aligned} \tag{C15}
$$

Where A_0, A_1, A_2, B_0 and B_1 depend on the mass and momentum of the particles as follows.

$$
\begin{aligned}
A_0 &= \frac{F_2}{F_1} \left[\left(\frac{\left(\omega_i + \omega_f\right)\left(E_f + E_i\right) + p_f^2 + p_i^2}{2} \right)^2 + \frac{1}{4 m_T^4} \left(m_f^2 - m_i^2\right)^2 \left(M_i^2 - M_f^2\right)^2 \right. \\
&\quad - \frac{\left(\omega_i + \omega_f\right)\left(E_f + E_i\right) + p_f^2 + p_i^2}{2 m_T^2} \left(m_f^2 - m_i^2\right)\left(M_i^2 - M_f^2\right) - \frac{1}{3} \left(-2\left(m_f^2 + m_i^2\right) + \frac{\left(m_f^2 - m_i^2\right)^2}{m_T^2} \right. \\
&\quad \left. + \frac{\left(E_f - E_i\right)^2 + \left(\omega_f - \omega_i\right)^2}{2} - p_f^2 - p_i^2 \right) \left(\frac{\left(M_i^2 - M_f^2\right)^2}{4 m_T^2} - \frac{M_f^2 + M_i^2}{2} + \frac{1}{4} \left(-p_f^2 - p_i^2 + \right. \right. \\
&\quad \left. \left. \left. \frac{\left(E_f - E_i\right)^2 + \left(\omega_f - \omega_i\right)^2}{2} \right) \right) \right] \\
&\quad + \left[-\frac{\left(\omega_i + \omega_f\right)\left(E_f + E_i\right) + p_f^2 + p_i^2}{2 m_T^2} \left(m_f^2 - m_i^2\right)\left(M_i - M_f\right) \right. \\
&\quad + \frac{1}{2 m_T^4} \left(m_f^2 - m_i^2\right)^2 \left(M_i^2 - M_f^2\right)\left(M_i - M_f\right) - \frac{1}{3} \left(-2\left(m_f^2 + m_i^2\right) + \frac{\left(m_f^2 - m_i^2\right)^2}{m_T^2} - p_f^2 - p_i^2 \right. \\
&\quad \left. \left. + \frac{\left(E_f - E_i\right)^2 + \left(\omega_f - \omega_i\right)^2}{2} \right) \left(\frac{\left(M_i^2 - M_f^2\right)}{2 m_T^2} \left(M_i - M_f\right) - \frac{1}{2}\left(M_f + M_i\right) \right) \right] \; ,
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= \frac{F_2}{F_1}\left[(\omega_i+\omega_f)(E_f+E_i)+p_f^2+p_i^2-\frac{\left(m_f^2-m_i^2\right)\left(M_i^2-M_f^2\right)}{m_T^2}-\frac{1}{6}\left(-2\left(m_f^2+m_i^2\right)+\frac{\left(m_f^2-m_i^2\right)^2}{m_T^2}\right.\right.\\
&\left.+\frac{(E_f-E_i)^2+(\omega_f-\omega_i)^2}{2}-p_f^2-p_i^2\right)-\frac{\left(M_i^2-M_f^2\right)^2}{6m_T^2}+\frac{M_f^2+M_i^2}{3}-\frac{1}{6}\left(\frac{(E_f-E_i)^2+(\omega_f-\omega_i)^2}{2}\right.\\
&\left.\left.-p_f^2-p_i^2\right)\right]p_fp_i\\
&+\left[-\frac{\left(m_f^2-m_i^2\right)(M_i-M_f)}{m_T^2}-\frac{\left(M_i^2-M_f^2\right)(M_i-M_f)}{3m_T^2}+\frac{M_f+M_i}{3}\right]p_fp_i\ ,\\
A_2 &= -\frac{1}{3}p_f^2p_i^2\frac{F_2}{F_1}\ ,\\
B_0 &= (\omega_i+\omega_f)(E_f+E_i)+p_f^2+p_i^2-\frac{\left(m_f^2-m_i^2\right)\left(M_i^2-M_f^2\right)}{m_T^2}\ ,\\
B_1 &= 2p_fp_i\ . \qquad \text{(C16)}
\end{aligned}
$$

3. Pomeron-exchange

$$
\begin{aligned}
X_P^{(C)} &= g_{PPP}g_P\ N_{fi}^+\ ,\\
Y_P^{(C)} &= -g_{PPP}g_P\ N_{fi}^-\ ,\\
X_P^{(SO)} &= g_{PPP}g_P\ N_{fi}^-\ . \qquad \text{(C17)}
\end{aligned}
$$

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$$\begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix} = [(2j_{13}+1)(2j_{31}+1)(2j_{23}+1) \times(2j_{32}+1)]^{1/2} \begin{Bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{Bmatrix} .$$

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