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Four-body structure of $^{7}_{\Lambda}$Li and ΛN spin-dependent interaction*

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Abstract

Two spin-doublet states of $^{7}_{\Lambda}$Li are studied on the basis of the $\alpha+\Lambda+n+p$ four-body model. We employ the two-body interactions which reproduce the observed properties of any subsystems composed of αN, $\alpha\Lambda$ and αNN, and $\alpha\Lambda N$. Furthermore, the ΛN interaction is adjusted so as to reproduce the 0^+-1^+ splitting of in $^{4}_{\Lambda}$H. The calculated energy splittings of $3/2^+$-$1/2^+$ and $7/2^+$-$5/2^+$ states in $^{7}_{\Lambda}$Li are 0.69 MeV and 0.46 MeV, which are in good agreement with the resent observed data. The spin-dependent components of the ΛN interaction are discussed.

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I. INTRODUCTION

It is a fundamental problem in hypernuclear physics to explore the features of underlying interactions between hyperons (Y) and nucleons (N) through analysis of many-body phenomena, because YN scattering data in free space are quite limited. Then, quantitative analyses for light Λ hypernuclei, where the features of ΛN interactions appear rather straightforwardly in observed level structures, are of a special significance. In this connection, it is very important that accurate measurements for γ-ray spectra have been performed systematically [1–5], which can be used to extract the spin-dependent components of ΛN interactions through the detailed analyses of hypernuclear structures. Although several shell-model calculations for light Λ hypernuclei have been performed with the restricted configuration of $(0s)^4(0p)^n 0s_\Lambda$ [6–9], their structures can be represented most excellently by the cluster models. Today, it is possible to perform fully microscopic calculations of three- and four-cluster systems with sufficient numerical accuracy. Such sophisticated calculations make it possible to study underlying ΛN interactions in comparison with the hypernuclear data observed in the γ-ray experiments [1, 5]. Because both short- and long-range correlations of the Λ in nuclei are treated very accurately in our approach, the characteristics of the free-space ΛN interactions can be studied very precisely.

The aim of this work is to analyze the ground $1/2^+$-$3/2^+$ and excited $5/2^+$-$7/2^+$ doublets in ${}^7_\Lambda$Li keeping the consistency with the $5/2^+$-$3/2^+$ doublet in ${}^9_\Lambda$Be, and to determine the ΛN spin-spin and spin-orbit interactions accurately based on the experimental data for ${}^9_\Lambda$Be, ${}^7_\Lambda$Li and ${}^4_\Lambda$H. The splitting energies for the ground $1/2^+$-$3/2^+$ and excited $5/2^+$-$7/2^+$ doublets in ${}^7_\Lambda$Li [1, 5] are related intimately to the spin-dependent potentials of the ΛN interaction. Considered naively, the former splitting is determined by the spin-spin interaction between the $0s$-Λ and the *deuteron* cluster, while the latter is related to both spin-spin and spin-orbit interactions. Thus, it is critical whether or not this $5/2^+$-$7/2^+$ splitting in ${}^7_\Lambda$Li is reproduced consistently with those of ${}^7_\Lambda$Li($1/2^+$-$3/2^+$) and ${}^9_\Lambda$Be($5/2^+$-$3/2^+$).

Before starting a realistic calculation with the microscopic four-body cluster model, we emphasize that the experimental data of the ${}^7_\Lambda$Li and ${}^9_\Lambda$Be energy levels are of a great value for the ΛN interaction study. First the low-lying state energies, ${}^7_\Lambda$Li$(1/2^+, 3/2^+)$, ${}^7_\Lambda$Li$(5/2^+, 7/2^+)$, ${}^9_\Lambda$Be$(1/2^+)$, and ${}^9_\Lambda$Be$(3/2^+, 5/2^+)$, are known recently with amazingly high resolution through the γ-ray measurements[1–5]. Secondly, the basic structure of these states are well understood on the basis of the symmetry consideration without assuming a specific form for the underlying ΛN interactions.

In order to verify level-energy consistency in the second point mentioned above, let us make a preliminary calculation based on the naive SU$_3$ wave functions. In other words, we check whether the use of the low-lying state energies known for ${}^7_\Lambda$Li ($1/2^+$, $3/2^+$, $5/2^+$) and ${}^9_\Lambda$Be ($1/2^+$, $3/2^+$, $5/2^+$) leads to the right position of the ${}^7_\Lambda$Li ($7/2^+$). Based on the nuclear core wave functions

$$\Phi_6({}^6\mathrm{Li}: 1_g^+, 3_1^+; T=0) = |[2](20)_{L=0,2}(S=1); J_c\rangle \tag{1}$$

$$\Phi_8({}^8\mathrm{Be}; 0_g^+, 2_1^+, T=0) = |[4](40)_{L=0,2}(S=0); J_c\rangle, \tag{2}$$

the hypernuclear states in ${}^7_\Lambda$Li and ${}^9_\Lambda$Be can be assumed to have the configurations with an s-state Λ weak coupling: $[\Phi_N(L,S;J_c,T)\otimes\Lambda(s_{1/2})]_{J_H^+}$. By using the Hamiltonian $\widetilde{H}_A = H_N + \epsilon_\Lambda(s_{1/2}) + \sum V_{N\Lambda}$, the hypernuclear level energies $\widetilde{E}_A(J_H)$ can be expressed straightforwardly as

$$
\begin{bmatrix} \widetilde{E}_7(1/2^+) \\ \widetilde{E}_7(3/2^+) \\ \widetilde{E}_7(5/2^+) \\ \widetilde{E}_9(3/2^+) \\ \widetilde{E}_9(5/2^+) \end{bmatrix} = \begin{bmatrix} E_6(1_g^+) \\ E_6(1_g^+) \\ E_6(3^+) \\ E_8(2^+) \\ E_8(2^+) \end{bmatrix} + \epsilon_\Lambda
+ \begin{bmatrix} \frac{5}{18} & \frac{19}{18} & \frac{11}{18} & \frac{1}{18} & \frac{-16\sqrt{2}}{27} \\ \frac{10}{9} & \frac{2}{9} & \frac{4}{9} & \frac{2}{9} & \frac{8\sqrt{2}}{27} \\ \frac{1}{4} & \frac{7}{4} & 0 & 0 & 0 \\ \frac{25}{24} & \frac{13}{8} & \frac{3}{4} & \frac{7}{12} & \frac{\sqrt{2}}{2} \\ \frac{25}{12} & \frac{7}{12} & \frac{7}{6} & \frac{1}{6} & \frac{-\sqrt{2}}{3} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ v_e \end{bmatrix} \qquad (3)
$$

where v_a, v_b, v_c, v_d, and v_e stand for the $N\Lambda$ interaction matrix elements $\langle p_{3/2}s^\Lambda_{1/2}|V_{N\Lambda}|p_{3/2}s^\Lambda_{1/2}\rangle_{2^-}$, $\langle p_{3/2}s^\Lambda_{1/2}|V_{N\Lambda}|p_{3/2}s^\Lambda_{1/2}\rangle_{1^-}$, $\langle p_{1/2}s^\Lambda_{1/2}|V_{N\Lambda}|p_{1/2}s^\Lambda_{1/2}\rangle_{1^-}$, $\langle p_{1/2}s^\Lambda_{1/2}|V_{N\Lambda}|p_{1/2}s^\Lambda_{1/2}\rangle_{0^-}$, and $\langle p_{3/2}s^\Lambda_{1/2}|V_{N\Lambda}|p_{1/2}s^\Lambda_{1/2}\rangle_{1^-}$, respectively. If we input the experimental energies for E_6, E_8, $\widetilde{E}_7$, and $\widetilde{E}_9$, a set of the interaction matrix elements $\{v'\text{s}\}$ are obtained. Then, one finds that the use of the solution $\{v'\text{s}\}$ leads to the theoretical result of 2.53 MeV for $\widetilde{E}_7(7/2^+)$ and also that this result is quite consistent with the experimental value 2.521 ± 0.04 MeV[5]. Thus these energy levels of ${}^7_\Lambda$Li and ${}^9_\Lambda$Be are surely based on the similar maximum spatial symmetry, and therefore a detailed realistic calculation with the microscopic cluster model should have an important value.

In the past, two types of cluster model calculations have been performed on the basis of the $\alpha+d+\Lambda$ and ${}^5_\Lambda$He$+n+p$ configurations. Using the $\alpha+d+\Lambda$ cluster model[10–12], it has been discussed that the ground $1/2^+_1$-$3/2^+_1$ doublet in ${}^7_\Lambda$Li is an important candidate to extract the ΛN spin-spin interaction. In Ref. [13], we proposed the ${}^5_\Lambda\text{He} + n + p$ model, where the full $n - p$ correlation is taken into account without the frozen-deuteron cluster approximation. The splitting energies of the ground and excited doublets in ${}^7_\Lambda$Li were calculated to be 0.87 MeV and 0.81 MeV, respectively.

As for the spin-orbit splittings in ${}^9_\Lambda$Be and ${}^{13}_\Lambda$C, the three-body $(2\alpha + \Lambda)$ model and the four-body $(3\alpha + \Lambda)$ model have been applied, respectively [14]. The calculated splitting energies of ${}^9_\Lambda$Be$(5/2^+_1$-$3/2^+_1)$ were 80-200 keV when we adopted the symmetric (SLS) and anti-symmetric (ALS) spin-orbit interactions derived from the Nijmegen OBE models. These theoretical values are considerably larger than the experimental value 43 ± 5 keV [3]. At the same time[14], however, we tried to enlarge the ALS potential to be 85 % of the SLS as inspired by the a quark-model ΛN interaction, we predicted the smaller splitting of 35-40 keV before the experiment. The similar discussion was given also to explain the small spin-orbit splitting of the Λ p-state observed in ${}^{13}_\Lambda$C[2, 3].

In this work, we extend these cluster models to the four-body treatment of $\alpha+n+p+\Lambda$ so as to take account of the full correlations among all the constituent particles. Such an extended calculation has been tried once in Ref. [15]. Here we focus our attention especially to two spin-doublets in ${}^7_\Lambda$Li, keeping the consistency with that in ${}^9_\Lambda$Be. In order to extract the dynamical information on the underlying ΛN spin-spin and spin-orbit interactions, two-body interactions among constituent particles (α, n, p, Λ) are chosen so as to reproduce accurately the observed properties of all the subsystems composed of αN, $\alpha\Lambda$ and αNN, and $\alpha\Lambda N$.

II. FOUR-BODY CLUSTER MODEL AND METHOD

In this work, the hypernucleus ${}^{7}_{\Lambda}$Li is considered to be composed of α cluster, Λ particle and two nucleons (N). The core nucleus α is considered to be an inert core and to have the $(0s)^4$ configuration, $\Psi(\alpha)$. The Pauli principle between the valence nucleon and the core nucleons is taken into account by the orthogonality condition model (OCM) [16], as the valence nucleon's wavefunction should be orthogonal to that the core nucleon.

Nine set of the Jacobian coordinates of the four-body system of ${}^{7}_{\Lambda}$Li are illustrated in Fig. 1 in which we further take into account the antisymmetrization between the two nucleons. The

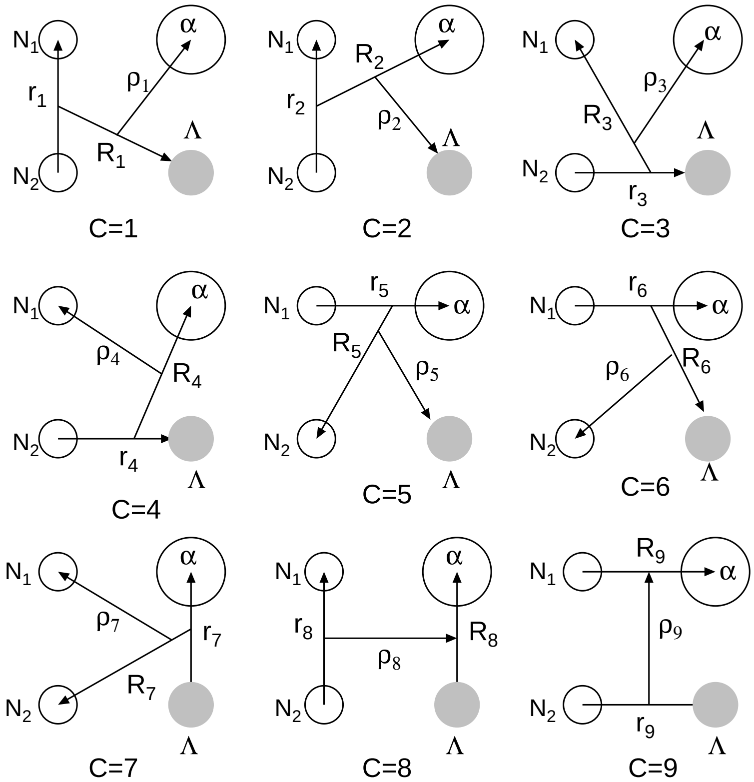

FIG. 1: Jacobian coordinates for all the rearrangement channels ($c = 1 \sim 9$) of the $\alpha + \Lambda + n + p$ four-body model. Two nucleons are to be antisymmetrized.

Schrödinger equation are given by

$$(H - E)\, \Psi_{JM}({}^{7}_{\Lambda}\mathrm{Li}) = 0 \;, \tag{4}$$

$$H = T + \sum_{a,b} V_{ab} + V_{\mathrm{Pauli}} \;, \tag{5}$$

where T is the kinetic-energy operator and V_{ab} is the interaction between the constituent particles a and b. The Pauli principle between the α particle and two nucleons is taken into account by the Pauli projection operator V_{Pauli} which is explained in the next section as well as V_{ab}. The total wavefunction is described as a sum of amplitudes of the rearrangement channels Fig. 1 in the LS coupling scheme:

$$\Psi_{JM}\ ({}^{A}_{\Lambda}\mathrm{Li}) = \sum_{c=1}^{9} \sum_{n,N,\nu} \sum_{l,L,\lambda} \sum_{S,\Sigma,I,K} C^{(c)}_{nlNL\nu\lambda S\Sigma IK}$$

$$
\begin{aligned}
&\times\ \mathcal{A}_N\Big[\Phi(\alpha)[\chi_s(\Lambda)[\chi_{\frac{1}{2}}(N_1)\chi_{\frac{1}{2}}(N_2)]_S]_\Sigma \\
&\times\ [[\phi^{(c)}_{nl}(\mathbf{r}_c)\psi^{(c)}_{NL}(\mathbf{R}_c)]_I\xi^{(c)}_{\nu\lambda}(\boldsymbol{\rho}_c)]_K\Big]_{JM} .
\end{aligned} \tag{6}
$$

Here the operator $\mathcal{A}_N$ stands for antisymmetrization between the two nucleons. $\chi_{\frac{1}{2}}(N_i)$ and $\chi_{\frac{1}{2}}(\Lambda)$ are the spin functions of the i-th nucleon and Λ particle. Following the Gaussian Expansion Method (GEM) [17–19], we take the functional form of $\phi_{nlm}(\mathbf{r})$, $\psi_{NLM}(\mathbf{R})$ and $\xi^{(c)}_{\nu\lambda\mu}(\boldsymbol{\rho}_c)$ as

$$
\begin{aligned}
\phi_{nlm}(\mathbf{r}) &= r^l\, e^{-(r/r_n)^2} Y_{lm}(\widehat{\mathbf{r}}) , \\
\psi_{NLM}(\mathbf{R}) &= R^L\, e^{-(R/R_N)^2} Y_{LM}(\widehat{\mathbf{R}}) , \\
\xi_{\nu\lambda\mu}(\boldsymbol{\rho}) &= \rho^\lambda\, e^{-(\rho/\rho_\nu)^2} Y_{\lambda\mu}(\widehat{\boldsymbol{\rho}}) ,
\end{aligned} \tag{7}
$$

where the Gaussian range parameters are chosen to lie in geometrical progressions:

$$
\begin{aligned}
r_n &= r_1 a^{n-1} \qquad (n = 1 - n_{\max}) , \\
R_N &= R_1 A^{N-1} \quad (N = 1 - N_{\max}) , \\
\rho_\nu &= \rho_1 \alpha^{\nu-1} \qquad (\nu = 1 - \nu_{\max}) .
\end{aligned} \tag{8}
$$

These basis functions have been verified to be suited for describing both the short-range correlations and the long-range tail behaviors of few-body systems [17–19]. The eigenenergy E in Eq.(2.1) and the coefficients C in Eq.(2.3) are to be determined by the Rayleigh-Ritz variational method.

The angular-momentum space of the wavefunction with $l, L, \lambda \leq 2$ was found to be enough for getting satisfactorily convergence of the binding energies of the states studied below (note that no truncation is taken of the *interactions* in the angular-momentum space). As for the numbers of the Gaussian basis, $n_{\max}$, $N_{\max}$ and $\nu_{\max}$, $4 - 10$ are enough.

III. INTERACTIONS

A. αN interaction

For the interaction $V_{N\alpha}$ between α and a valence nucleon, we employ the effective potential proposed in Ref.[20], which is designed so as to reproduce well the low-lying states and low-energy scattering phase shifts of the αn system.

The Pauli principle between nucleons belonging to α and valence nucleons is taken into account by the orthogonality condition model (OCM) [16]. The OCM projection operator V_{Pauli} is represented by

$$
V_{\text{Pauli}} = \lim_{\lambda\to\infty} \lambda\ |\phi_{0s}(\mathbf{r}_{N\alpha})\rangle\langle\phi(\mathbf{r}'_{N\alpha})| \tag{9}
$$

which excludes the amplitude of the Pauli forbidden state $\phi_{0s}(\mathbf{r})$ from the four-body total wavefunction [21]. The Gaussian range parameter b of the single-particle $0s$ orbit in the α particle is taken to be $b = 1.358$ fm so as to reproduce the size of the α particle. In the actual calculation, the strength λ for V_{Pauli} is taken to be 10^5 MeV, which is large enough to push the unphysical forbidden states into the very high energy region while keeping the physical states unchanged. Usefulness of this Pauli operator method of OCM has been verified in many cluster-model calculations.

B. NN interaction

In order to study the fine structure of our $\alpha + n + p + \Lambda$ system (${}^{7}_{\Lambda}$Li), it is necessary to use an NN interaction which reproduces accurately the energy spectrum of the $\alpha + n + p$ subsystem (^{6}Li). Such an NN interaction is given here as follows: We start from the AV'8 [22] potential, V_{NN} , derived from the AV18 [23] by neglecting the $(L \cdot S)^2)$ term. In our model, this potential gives the calculated values of -3.38 MeV and -0.98 MeV for the 1^+ and 3^+ states of ^{6}Li, respectively, being of less binding compared to the experimental data. Next, we adjust the central and tensor parts of V_{NN} together with the slight modification of $V_{N\alpha}$ so that the experimental energies of ^{6}Li $(1^+, 3^+)$ and deuteron are reproduced.

C. $\alpha\Lambda$ interaction

The interaction between the Λ particle and α cluster is derived by folding the ΛN G-matrix interaction with a three-range Gaussian form into the density of the α cluster in the same manner as our previous work in Ref. [24]. In the present work, we employ the G-matrix interaction for Nijmegen model F(NF) [25], the parameters of which are also listed in Ref. [24]. Even if the versions for the other Nijmegen models are used, the obtained results are almost the same as the present one. This is because our ΛN folding interaction is adjusted so as to reproduce the experimental value of $B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He})$.

D. ΛN interaction

For ΛN interactions, meson-theoretical models have been proposed on the basis of the SU(3) symmetry of meson-baryon coupling constants. In principle, these realistic interactions can be used directly in our four-body model of ${}^{7}_{\Lambda}$Li. However, the purpose of this work is to extract the information on the spin-dependent parts of the ΛN interaction as quantitatively as possible using the measured splitting energies of spin-doublet states. We employ effective ΛN single-channel interactions simulating the basic features of the Nijmegen meson-theoretical models NSC97f [26], in which some potential parameters are adjusted phenomenologically so as to reproduce the experimental data.

Our ΛN interactions, composed of central, SLS and ALS parts are represented as

$$V_{\Lambda N}^{(\mathrm{C})}(r) = \sum_{\alpha} \sum_{i=1}^{3} v_i^{(\alpha)} \; \exp\left(-(r/\beta_i)^2\right) \quad , \tag{10}$$

$$V_{\Lambda N}^{\mathrm{LS}} = V^{SLS}\mathbf{L}\mathbf{S}_{+} + V^{ALS}\mathbf{L}\mathbf{S}_{-} \tag{11}$$

with $\mathbf{S}_{\pm} = \mathbf{s}_{\mathbf{\Lambda}} \pm \mathbf{s}_{\mathbf{N}}$. Here, the central potential, $V_{\Lambda N}^{\mathrm{C}}$, with three-range Gaussian forms are given separately for spin-parity states of α= ${}^{3}E$ (triplet even), ${}^{1}E$ (singlet even), ${}^{3}O$ (triplet odd), ${}^{1}O$ (singlet odd). The even- and odd-state spin-spin interaction are defined by $(V_{\Lambda N}^{(^3E)} - V_{\Lambda N}^{(^1E)})/4$ and $(V_{\Lambda N}^{(^3O)} - V_{\Lambda N}^{(^1O)})/4$, respectively.

The potential parameters in the central parts are chosen so as to simulate ΛN scattering phase shifts calculated by NSC97f. The determined parameters are given in Table I. It should be noted here that the ΛN-ΣN coupling interactions are included explicitly in NSC97f, and their contributions in many-body systems are different from those in free space. This

TABLE I: Parameters of the ΛN interaction defined in Eq.(3.2). Range parameters are in fm and strength are in MeV. The numbers in parentheses are improved even state strength so as to reproduce observed spin doublet state in ${}^{4}_{\Lambda}$H and odd state strength so as to reproduce observed ground doublet state in ${}^{7}_{\Lambda}$Li .

i	1	2	3
β_i	1.60	0.80	0.35
$v_i(^1E)$	–7.87	–342.5 (–357.4)	6132.
$v_i(^3E)$	–7.89	–242.4 (–217.3)	3139.
$v_i(^1O)$	–1.30	213.7 (513.7)	8119.
$v_i(^3O)$	–3.38	122.9 (22.9)	5952.

means that our obtained phase-shift equivalent potential should be modified appropriately in applications to hypernuclear system: We adjust the second-range strengths $v_2^{(^3E)}$ and $v_2^{(^1E)}$ so that calculated energies of 0^+-1^+ doublet state in our $NNN\Lambda$ four-body calculation reproduce the experimental values obtained by those of ${}^{4}_{\Lambda}$H. In Table I, the adjusted values of $v_2^{(^1E)}$ and $v_2^{(^3E)}$ are shown in parentheses. On the other hand, there was no clear experimental data to determine quantitatively the odd-state parts, which leads to remarkable differences among theoretical interaction models. Our present analysis for the splitting energies of ${}^{7}_{\Lambda}$Li gives some constraint on the odd-state spin-spin part. The second-range values of $v_2^{(^1O)}$ and $v_2^{(^3O)}$ in parentheses are determined on the basis of the ${}^{7}_{\Lambda}$Li data, as shown later.

The SLS and ALS interactions here are chosen so as to reproduce the ${}^{9}_{\Lambda}$Be data. In Ref.[14], the various sets were derived from the Nijmegen models. However, the $5/2^+$-$3/2^+$ splitting energies obtained from these sets are considerably larger than the experimental value. Now, our SLS and ALS interactions are derived as follows: Firstly, the SLS part derived from NSC97f with the G-matrix procedure is represented in the two-range form $V^{SLS} = \sum_{i=1}^{2} v_i^{(+)}\, e^{-(r/\gamma_i)}$ The values of the parameters are $v_1^{(+)} = -110.6$ MeV and $v_2^{(+)} = -1157.$ MeV for $\gamma_1 = 0.70$ fm and $\gamma_2 = 0.40$ fm, respectively as given in Ref.[14]. Next, assuming $V^{ALS} = -\alpha V^{SLS}$, the parameter α is chosen so as to reproduce the measured $5/2^+$-$3/2^+$ splitting energy with the $2\alpha + \Lambda$ cluster model developed in Ref.[14]. Our obtained value is $\alpha = 0.83$. Using these V^{SLS} and V^{ALS}, we also calculated the energy splitting of $1/2^-$-$3/2^-$ doublets in ${}^{13}_{\Lambda}$C based on 3α+Λ four-body model to be 0.2 MeV which is consisten with the observed data within the error [2]. This ALS interaction is fairly stronger than that derived from NSC97f. As discussed later, however, the similar result can be obtained by weakening the SLS part without changing the ALS part, because only the sum of SLS and ALS is fixed by the ${}^{9}_{\Lambda}$Be data. There is a famous quark-model prediction [27] that the ALS is so strong as to substantially cancel the LS one. It should be noted that this prediction is not necessarily proved by our present analysis.

IV. RESULTS

In Fig. 2, we illustrate our result for the $1/2^+$-$3/2^+$ and $5/2^+$-$7/2^+$ doublet states of ${}^{7}_{\Lambda}$Li . The energies of the 1^+-3^+ doublet state of ^{6}Li nucleus calculated in the framework

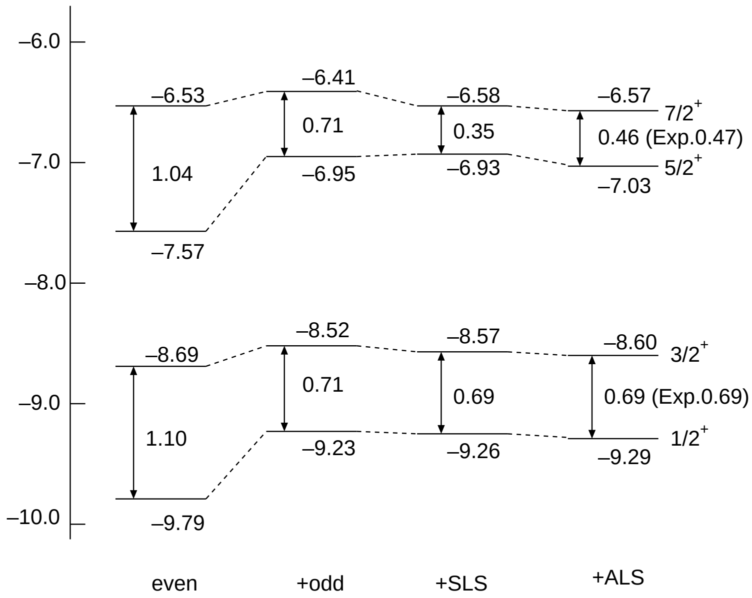

FIG. 2: Calculated energy levels of ${}^{7}_{\Lambda}$Li on the basis of $\alpha+\Lambda+n+p$ model. The energies are measured from the $\alpha+\Lambda+n+p$ threshold. The observed energy splittings of $3/2^+$-$1/2^+$ and $7/2^+$-$5/2^+$ are 0.69 MeV and 0.47 MeV, respectively.

of the $\alpha{+}n{+}p$ three-body model are -3.7 MeV and -1.6 MeV, being measured from the $\alpha{+}n{+}p$ three-body threshold. As shown in the left side of the figure, the calculated splitting energies for both doublets are about 1 MeV very similar to that for 0^+-1^+ doublet state of ${}^{4}_{\Lambda}$H (${}^{4}_{\Lambda}$He), when only the even-state central interaction is used. Namely, the even-state spin-spin interaction turns out to contribute similarly to the 0^+-1^+ splitting energy of ${}^{4}_{\Lambda}$H (${}^{4}_{\Lambda}$He) and the $1/2^+$-$3/2^+$ and $5/2^+$-$7/2^+$ ones of ${}^{7}_{\Lambda}$Li.

Next, let us switch on the odd-state central interaction. When only the even-state interaction is used, the obtained value of the ground-state energy is -9.79 MeV. When we use the ${}^{1}O$ and ${}^{3}O$ interactions derived from NSC97f, the ground 1/2+ state is obtained at -9.23 MeV. This energy changes only slightly (-0.06 MeV) with the inclusion of SLS and ALS, because the spin-orbit interactions have essentially no effect on the 1/2+ state due to its $L=0$ structure. The final value -9.29 MeV means that the experimental Λ binding energy (5.58 MeV) is reproduced well, because the calculated energy of the $\alpha+p+n$ subunit is -3.7 MeV in our model. Found in the figure, this fact is owing to the peculiar role of our odd-state interaction. Therefore, the repulsive contribution from the spin-independent part $(3V_{\Lambda N}^{({}^3O)}+V_{\Lambda N}^{({}^1O)})/4$ turns out to be decisive to reproduce the experimental value. The repulsive nature of this part is an important property of NSC97f. On the other hand, the corresponding part in the recent model ESC04 [28] is attractive in contrast to NSC97f. The important role of the repulsive odd-state interaction in our analysis does not necessarily mean that the odd-state part in NSC97f is more realistic than the one in ESC04, as discussed later.

As for the $1/2^+$-$3/2^+$ splitting, the addition of the NSC97f odd-state central interaction leads to 0.97 MeV. This splitting is too large in view of the experimental value (0.69 MeV), because the contribution of the SLS/ALS interactions to this $1/2^+$-$3/2^+$ splitting is quite small. We add here the attractive (repulsive) correction on the ${}^{3}O$ (${}^{1}O$) state interaction, which works efficiently in high-spin (low-spin) partners of doublets, by making the spin-spin interaction more attractive: We introduce the attractive spin-spin interaction in second range (0.8 fm), $\Delta v_{ss} = -100.0$ MeV. The modified values of $v_2^{(^1O)}$ and $v_2^{(^3O)}$ are shown in parentheses of Table.I, which leads to the calculated values of 0.71 MeV and 0.54 MeV for the lower and higher doublets, respectively (cf. Fig. 2), as seen in the figure.

Now, we come to the important stage of looking at the roles of the SLS and ALS interactions for splitting energies. It should be noted here that these interactions work differently for the two doublet states of $^7_\Lambda$Li: The contributions to the ground-state $1/2^+$-$3/2^+$ doublet are very small, where the pn pair part outside the α core is dominated by the $L = 0$ component spatially. On the other hand, in the case of the excited $5/2^+$-$7/2^+$ doublet composed of the $L = 2$ pn pair, the SLS and ALS interactions play important roles: As seen in Fig. 2, the SLS works attractively (slightly repulsively) for the $7/2^+$ ($5/2^+$) state, because the $7/2^+$ state is dominated by the spin-triplet configuration of the $L = 2$ pn pair and the s-state Λ. On the other hand, the ALS works efficiently in the $5/2^+$ state which has both configurations of spin-triplet and spin-singlet. The ALS which acts between $S = 0$ and $S = 1$ ΛN two-body states has essentially no effect on the $7/2^+$ state.

Thus, owing to the combined effects of the SLS and ALS, our final result reproduces nicely the observed energies of the spin-doublet states in $^7_\Lambda$Li. Recently, Millner calculate $3/2^+$-$1/2^+$ and $7/2^+$-$5/2^+$ spin-doublets states using a shell model with the phenomenological interaction matrix element [5, 32]. His calculation is also in good agreement with the recent data. It should be noted here that the strength of the ALS part is not necessarily determined by our present analysis. The above result is obtained by making the ALS part stronger than that given by NSC97f so as to reproduce the $^9_\Lambda$Be data. The very similar result, however, can be obtained by weakening the SLS part without changing the ALS part.

Before going to summary, we would like to comment on the role of the ΛN-ΣN coupling. Our basic assumption in this work is that the ΛN-ΣN coupling interaction can be renormalized into the ΛN-ΛN interaction effectively. In this spirit, the even-state parts of our ΛN-ΛN interaction were adjusted so as to reproduce the 0^+ and 1^+ of $^4_\Lambda$H. It is reasonable, however, to consider that the ΛN-ΣN coupling works more repulsively in $^7_\Lambda$Li. It is likely that the role of the odd-state repulsion in our treatment is a substitute for this effect. This is the reason why the attractive odd-state interaction in ESC04 models cannot be denied. As shown in Fig. 2, the energy of $5/2^+$ state is located above by about 0.2 MeV in comparison with the observed energy of $5/2^+$ state. This problem may be solved by taking into account the repulsive effect of the ΛN-ΣN coupling instead of the odd-state repusion, because the SLS/ALS interaction works more efficiently under the attractive odd-state interactions. Some authors [30, 31] pointed out the extra contribution to the $^4_\Lambda$H(0^+-1^+) splitting energy from the three-body correlated ΛN-ΣN mixing. The present authors also obtained the value of about 0.3 MeV for the three-body contribution of $\Lambda N - \Sigma N$ coupling in the 0^+-1^+ splitting energy in $^4_\Lambda$H [29]. In the shell model calculation [32], Millener calculated the spin-doublets states in $^7_\Lambda$Li including $\Lambda N - \Sigma N$ coupling and he concluded that this contribution was small in these splitting energies. On the other hand, Fetisov pointed out that the large effect of $\Lambda N - \Sigma N$ coupling was found in both of $^4_\Lambda$H and $^7_\Lambda$Li [33]. It is an open problem to study ΛN-ΣN coupling effects consistently for $^4_\Lambda$H and $^7_\Lambda$Li.

In summary, we discussed the two spin-doublets of $3/2^+$-$1/2^+$ and $7/2^+$-$5/2^+$ in $^7_\Lambda$Li based on $\alpha + \Lambda + n + p$ four-body model. Here, it is important that all the two-body interactions are chosen so as to reproduce both the binding energy of any subsystem composed of two-and three-constituent particles. Our ΛN interactions, simulating ΛN scattering phase shifts calculated by NSC97f, are adjusted so as to reproduce the observed data of spin-doublet states. It is found that the even-state ΛN interaction leads to the similar values of the splitting energies of the 0^+-1^+ doublet in ${}^4_\Lambda$H (${}^4_\Lambda$He) and the $1/2^+$-$3/2^+$ and $5/2^+$-$7/2^+$ doublets in ${}^7_\Lambda$Li. Then, the odd-state interactions play important roles to reproduce the difference between the two doublet states in ${}^7_\Lambda$Li. With use of the SLS and ALS interactions adjusted so as to reproduce the $5/2^+$-$3/2^+$ splitting in ${}^9_\Lambda$Be, the two doublet states in ${}^7_\Lambda$Li can be reproduced exactly by tuning the odd-state spin-spin interaction.

The basic assumption in our present approach is that the ΛN-ΣN coupling interactions are renormalized reasonably into our ΛN interactions. The validity of this assumption will be investigated in our future studies. The coupled four-body calculation of $\alpha + \Lambda + N + N$ and $\alpha + \Sigma + N + N$ is in progress.

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