Few-body aspects of hypernuclear physics*

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Abstract

Two spin-doublet, $3/2^+$-$1/2^+$ and $7/2^+$-$5/2^+$, in ${}^{7}_{\Lambda}$Li are studies on the basis of the $\alpha + \Lambda + n + p$ four-body model. The calculated energy splittings of $3/2^+$-$1/2^+$ and $7/2^+$-$5/2^+$ states in ${}^{7}_{\Lambda}$Li are 0.69 MeV and 0.46 MeV, which are in good agreement with the recent observed data.

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I. INTRODUCTION

It is a fundamental problem in hypernuclear physics to explore the features of underlying interactions between hyperons(Y) and nucleons(N) through analysis of many-body phenomena, because YN scattering data in free space are quite limited. Then, quantitative analysis for light Λ hypernuclei, are of a special significance. In this connection, it is very important that accurate measurements for γ-ray spectra have been performed systematically [1–5], which can be used to extract the spin-dependent components of ΛN interactions though the detailed analysis of hypernuclear structures. For light p-shell hypernuclei, the cluster models can represent most excellently. Recently, it is possible to study underlying ΛN interactions in comparison with the hypernuclear data observed in the γ-ray experiments[1, 5].

The purpose in this work is to analyze the ground $1/2^+$-$3/2^+$ and excited $5/2^+$-$7/2^+$ doublets in ${}^{7}_{\Lambda}$Li keeping the consistency with the $5/2^+$-$3/2^+$ doublet in ${}^{9}_{\Lambda}$Be, and to determine the ΛN spin-spin and spin-orbit interactions accurately based on the experimental data for ${}^{9}_{\Lambda}$Be, ${}^{7}_{\Lambda}$Li and ${}^{4}_{\Lambda}$H.

II. METHOD

The hypernucleus ${}^{7}_{\Lambda}$Li is considered to be composed of α cluster, Λ particle and two nucleons. The core nucleus α is considered to be an inert core and to have the $(0s)^4$ configuration, $\Psi(\alpha)$. The Pauli principle between the valence nucleon and the core nucleon is taken into account by the orthogonality condition model (OCM) [6], as the valence nucleon's wavefunction should be orthogonal to that the core nucleon.

We took nine sets of the Jacobian coordinates of the four-body system of ${}^{7}_{\Lambda}$Li. The Schrödinger equation are given by

$$(H-E)\,\Psi_{JM}({}^{7}_{\Lambda}\mathrm{Li})=0\ , \tag{1}$$

$$H=T+\sum_{a,b}V_{ab}+V_{\mathrm{Pauli}}\ , \tag{2}$$

where T is the kinetic-energy operator and V_{ab} is the interaction between the constituent particles a and b. The Pauli principle between the α particle and two nucleons is taken into account by the Pauli projection operator V_{Pauli} which is explained in the terms of $\lim_{\lambda\to\infty}\ \lambda\ |\phi_{0s}(\mathbf{r}_{N\alpha})\rangle\langle\phi(\mathbf{r}'_{N\alpha})|$ which excludes the amplitude of the Pauli forbidden state $\phi_{0s}(\mathbf{r})$ from the four-body total wavefunction [10]. The total wavefunction is described as a sum of amplitudes of the rearrangement channels Fig. 1 in the LS coupling scheme:

$$\begin{aligned}\Psi_{JM}\ (\ {}^{A}_{\Lambda}\mathrm{Li}) &= \sum_{c=1}^{9}\sum_{n,N,\nu}\sum_{l,L,\lambda}\sum_{S,\Sigma,I,K} C^{(c)}_{nlNL\nu\lambda S\Sigma IK}\times\mathcal{A}_N\Big[\Phi(\alpha)[\chi_s(\Lambda)[\chi_{\frac{1}{2}}(N_1)\chi_{\frac{1}{2}}(N_2)]_S]_\Sigma \\ &\times\ [[\phi^{(c)}_{nl}(\mathbf{r}_c)\psi^{(c)}_{NL}(\mathbf{R}_c)]_I\xi^{(c)}_{\nu\lambda}(\boldsymbol{\rho}_c)]_K\Big]_{JM}\ .\end{aligned} \tag{3}$$

Here the operator $\mathcal{A}_N$ stands for antisymmetrization between the two nucleons. $\chi_{\frac{1}{2}}(N_i)$ and $\chi_{\frac{1}{2}}(\Lambda)$ are the spin functions of the i-th nucleon and Λ particle. Following the Gaussian Expansion Method (GEM) [7–9], we take the functional form of $\phi_{nlm}(\mathbf{r})$, $\psi_{NLM}(\mathbf{R})$ and $\xi^{(c)}_{\nu\lambda\mu}(\boldsymbol{\rho}_c)$ as Gaussian form and spherical harmonics. The eigenenergy E and the coefficients C are to be determined by the Rayleigh-Ritz variational method.

III. INTERACTIONS

For the interaction $V_{N\alpha}$ between α and a valence nucleon, we employ the effective potential which is designed so as to reproduce well the low-lying states and low-energy scattering phase shifts of the αn system.

As for the NN interaction, AV8 realistic force was employed.

The interaction between the Λ particle and α cluster is derived by folding the ΛN G-matrix interaction with a three-range Gaussian form into the density of the α cluster in the same manner as our previous work in Ref. [11]. In the present work, we employ the G-matrix interaction for Nijmegen model F(NF) [12], the parameters of which are also listed in Ref. [11].

For ΛN interactions, meson-theoretical models have been proposed on the basis of the SU(3) symmetry of meson-baryon coupling constants. In principle, these realistic interactions can be used directly in our four-body model of ${}^{7}_{\Lambda}$Li. However, the purpose of this work is to extract the information on the spin-dependent parts of the ΛN interaction as quantitatively as possible using the measured splitting energies of spin-doublet states. We employ effective ΛN single-channel interactions simulating the basic features of the Nijmegen meson-theoretical models NSC97f [13], in which some potential parameters are adjusted phenomenologically so as to reproduce the experimental data.

Our ΛN interaction are composed of the central, symmetric LS(SLS) and antisysmetric LS (ALS) parts. The potential parameters in the central parts are chosen so as to simulate ΛN scattering phase shifts calculated by NSC97f.

The SLS and ALS interactions here are chosen so as to reproduce the ${}^{9}_{\Lambda}$Be data.

IV. RESULTS

In Fig. 1 , we illustrate our result for the $1/2^+$-$3/2^+$ and $5/2^+$-$7/2^+$ doublet states of ${}^{7}_{\Lambda}$Li . The energies of the 1^+-3^+ doublet state of ^{6}Li nucleus calculated in the framework of the $\alpha+n+p$ three-body model are -3.7 MeV and -1.6 MeV, being measured from the $\alpha+n+p$ three-body threshold. As shown in the left side of the figure, the calculated splitting energies for both doublets are about 1 MeV very similar to that for 0^+-1^+ doublet state of ${}^{4}_{\Lambda}$H (${}^{4}_{\Lambda}$He), when only the even-state central interaction is used. Namely, the even-state spin-spin interaction turns out to contribute similarly to the 0^+-1^+ splitting energy of ${}^{4}_{\Lambda}$H (${}^{4}_{\Lambda}$He) and the $1/2^+$-$3/2^+$ and $5/2^+$-$7/2^+$ ones of ${}^{7}_{\Lambda}$Li.

Next, let us switch on the odd-state central interaction. When only the even-state interaction is used, the obtained value of the ground-state energy is -9.79 MeV. When we use the ${}^{1}O$ and ${}^{3}O$ interactions derived from NSC97f, the ground $1/2+$ state is obtained at -9.23 MeV. This energy changes only slightly (-0.06 MeV) with the inclusion of SLS and ALS, because the spin-orbit interactions have essentially no effect on the $1/2+$ state due to its $L = 0$ structure. The final value -9.29 MeV means that the experimental Λ binding energy (5.58 MeV) is reproduced well, because the calculated energy of the $\alpha+p+n$ subunit is -3.7 MeV in our model. Found in the figure, this fact is owing to the peculiar role of our odd-state interaction. The repulsive nature of this part is an important property of NSC97f.

Now, we come to the important stage of looking at the roles of the SLS and ALS interactions for splitting energies. It should be noted here that these interactions work differently for the two doublet states of ${}^{7}_{\Lambda}$Li: The contributions to the ground-state $1/2^+$-$3/2^+$ doublet are very small, where the pn pair part outside the α core is dominated by the $L = 0$ compo-

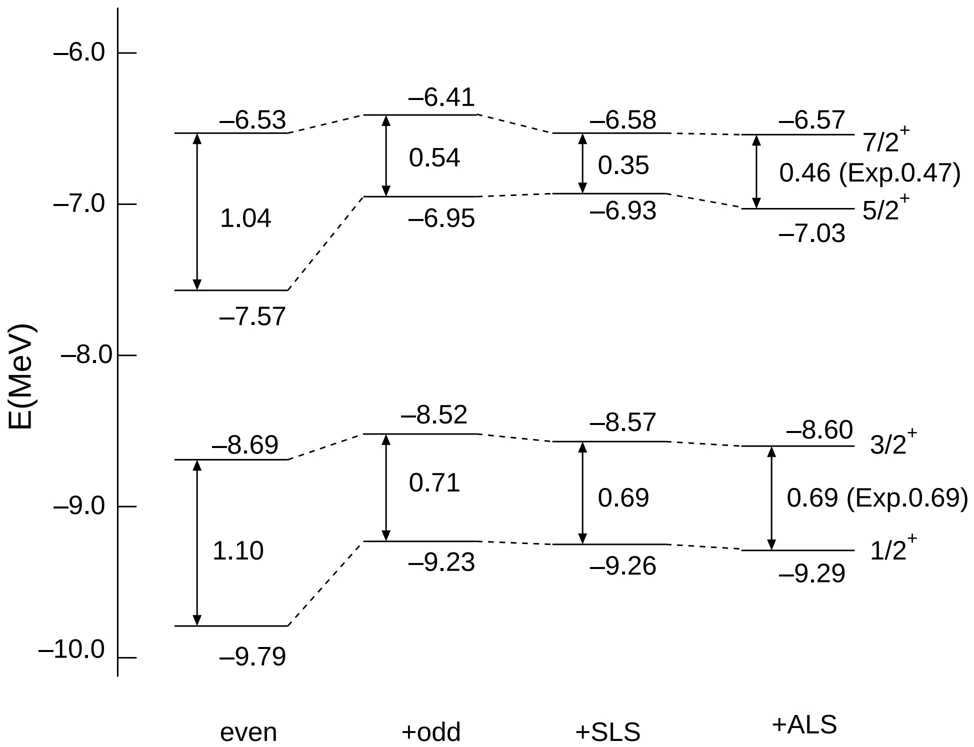

FIG. 1: Calculated energy levels of ${}^{7}_{\Lambda}$Li on the basis of $\alpha + \Lambda + n + p$ model. The energies are measured from the $\alpha + \Lambda + n + p$ threshold. The observed energy splittings of $3/2^{+}$-$1/2^{+}$ and $7/2^{+}$-$5/2^{+}$ are 0.69 MeV and 0.47 MeV, respectively.

nent spatially. On the other hand, in the case of the excited $5/2^{+}$-$7/2^{+}$ doublet composed of the $L = 2$ pn pair, the SLS and ALS interactions play important roles: As seen in Fig. 1, the SLS works attractively (slightly repulsively) for the $7/2^{+}$ ($5/2^{+}$) state, because the $7/2^{+}$ state is dominated by the spin-triplet configuration of the $L = 2$ pn pair and the s-state Λ. On the other hand, the ALS works efficiently in the $5/2^{+}$ state which has both configurations of spin-triplet and spin-singlet. The ALS which acts between $S = 0$ and $S = 1$ ΛN two-body states has essentially no effect on the $7/2^{+}$ state.

Thus, owing to the combined effects of the SLS and ALS, our final result reproduces nicely the observed energies of the spin-doublet states in ${}^{7}_{\Lambda}$Li.

Before going to summary, we would like to comment on the role of the ΛN-ΣN coupling. Our basic assumption in this work is that the ΛN-ΣN coupling interaction can be renormalized into the ΛN-ΛN interaction effectively. In this spirit, the even-state parts of our ΛN-ΛN interaction were adjusted so as to reproduce the 0^{+} and 1^{+} of ${}^{4}_{\Lambda}$H. It is reasonable, however, to consider that the ΛN-ΣN coupling works more repulsively in ${}^{7}_{\Lambda}$Li. It is likely that the role of the odd-state repulsion in our treatment is a substitute for this effect. As shown in Fig. 1, the energy of $5/2^{+}$ state is located above by about 0.2 MeV in comparison with the observed energy of $5/2^{+}$ state. This problem may be solved by taking into account the repulsive effect of the ΛN-ΣN coupling instead of the odd-state repusion, because the SLS/ALS interaction works more efficiently under the attractive odd-state interactions. Some authors [14, 15] pointed out the extra contribution to the ${}^{4}_{\Lambda}$H(0^{+}-1^{+}) splitting energy from the three-body correlated ΛN-ΣN mixing. The present authors also obtained the value of about 0.3 MeV for the three-body contribution of $\Lambda N-\Sigma N$ coupling in the 0^+-1^+ splitting energy in ${}^4_\Lambda$H [16]. In the shell model calculation [17], Millener calculated the spin-doublets states in ${}^7_\Lambda$Li including $\Lambda N-\Sigma N$ coupling and he concluded that this contribution was small in these splitting energies. On the other hand, Fetisov pointed out that the large effect of $\Lambda N-\Sigma N$ coupling was found in both of ${}^4_\Lambda$H and ${}^7_\Lambda$Li [18]. It is an open problem to study ΛN-ΣN coupling effects consistently for ${}^4_\Lambda$H and ${}^7_\Lambda$Li.

V. SUMMARY

We discussed the two spin-doublets of $3/2^+$-$1/2^+$ and $7/2^+$-$5/2^+$ in ${}^7_\Lambda$Li based on $\alpha+\Lambda+n+p$ four-body model. It is found that the even-state ΛN interaction leads to the similar values of the splitting energies of the 0^+-1^+ doublet in ${}^4_\Lambda$H (${}^4_\Lambda$He) and the $1/2^+$-$3/2^+$ and $5/2^+$-$7/2^+$ doublets in ${}^7_\Lambda$Li. Then, the odd-state interactions play important roles to reproduce the difference between the two doublet states in ${}^7_\Lambda$Li. With use of the SLS and ALS interactions adjusted so as to reproduce the $5/2^+$-$3/2^+$ splitting in ${}^9_\Lambda$Be, the two doublet states in ${}^7_\Lambda$Li can be reproduced exactly by tuning the odd-state spin-spin interaction.

Acknowledgments

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