THEF-NIJM 09.02
arXiv:0905.1407

Pion-Nucleon Scattering in Kadyshevsky Formalism: I Meson Exchange Sector*

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Abstract

In a series of two papers we present the theoretical results of πN/meson-baryon scattering in the Kadyshevsky formalism. In this paper the results are given for meson exchange diagrams. On the formal side we show, by means of an example, how general couplings, i.e. couplings containing multiple derivatives and/or higher spin fields, should be treated. We do this by introducing and applying the Takahashi-Umezawa and the Gross-Jackiw method. For practical purposes we introduce the $\bar{P}$ method. We also show how the Takashashi-Umezawa method can be derived using the theory of Bogoliubov and collaborators and the Gross-Jackiw method is also used to study the n-dependence of the Kadyshevsky integral equation. Last but not least we present the second quantization procedure of the quasi particle in Kadyshevsky formalism.

* Submitted for publication

I. INTRODUCTION

Over the years the Nijmegen group has constructed very successful baryon-baryon models (NN and YN). As for instance in [1] and [2] soft-core One-Boson-Exchange NN and YN models are constructed based on Regge-pole theory. The models are linked via $SU_f(3)$ symmetry in order to have more control on the parameters.

Based on the same ideas, the Nijmegen group recently broadened its horizon by also including meson-baryon models [3]. Here, a simultaneous πN and K^+N model is constructed using one-meson and one-baryon exchange potentials.

This work is presented in two articles, referred to as paper I (this paper) and paper II [4], and can be regarded as an extension of [3], since we also consider meson-baryon scattering or pion-nucleon, more specifically. The reason for considering pion-nucleon scattering is, besides the interest in its own, that there is a large amount of experimental data. Using the aforementioned $SU_f(3)$ symmetry the extension to other meson-baryon systems is easily made. Last but not least we would like to mention the connection to photo/electro-production models.

Compared to [3] our focus is more on the theoretical background. For instance we formally include what is called "pair suppression", whereas this was assumed in [3]. Pair suppression comes down to the suppression of negative energy contributions. For the first time, at least to our knowledge, we incorporate pair suppression in a covariant and frame independent way. This may also be interesting for relativistic many body theories. The details of the formal incorporation of pair suppression are discussed in paper II.

In order to have this covariant and frame independent pair suppression, we use the Kadyshevsky formalism [5–8]. This formalism is equivalent to Feynman formalism, since it can be derived from the same S-matrix formula. It covariantly, though frame dependently [25], separates positive and negative energy contributions. Generally, the number of diagrams increases: $1 \rightarrow n!$ at order n as in old-fashioned perturbation theory. Contrary to the Feynman formalism all particles in the Kadyshevsky formalism remain on their mass shell, at the cost of the introduction of an extra quasi particle, which carries four momentum only. A second quantization formalism of this quasi field is presented in appendix B. An other advantage of the Kadyshevsky formalism is that it brings about a three dimensional Lippmann-Schwinger type of integral equation [8], whereas a three dimensional integral equation was achieved in [3] only after approximations of the Bethe-Salpeter equation [9]. We study the n-dependence of the Kadyshevsky integral equation with tree level amplitudes as input in section II A. As compared to the original Kadyshevsky rules we use a slightly different version, introduced and discussed in appendix A.

Couplings containing derivatives and higher spin fields may cause differences and problems as far as the results in the Kadyshevsky formalism and the Feynman formalism are concerned. This is discussed in section IV B by means of an example of simplified vector meson exchange. After a second glance the results in both formalisms are the same, however, they contain extra frame dependent contact terms. Two methods are introduced and applied, which discuss a second source of extra terms: the Takahashi-Umezawa (TU) [10–12] and the Gross-Jackiw (GJ) [13] method. The extra terms coming from this second source cancel the former ones exactly. Both formalisms, however, yield the same results. With the use of (one of) these methods the final results for the S-matrix or amplitude are covariant and frame independent (n-independent). In section IV B 4 we introduce and discusse the $\bar{P}$-method, which is quite useful for practical purposes. We derive the TU method from the

BMP [14–16] theory in appendix C and in light of this TU method we make some remarks about the Haag theorem [17] in appendix D.

Although we already discussed some content, this paper is organized as follows: we start in section II with some meson-baryon scattering kinematics in Kadyshevsky formalism together with the discussion of the n-dependence of the integral equation. We start the application of the Kadyshevsky formalism to the πN system by first discussing the ingredients of the model in section III. The meson exchange amplitudes are calculated in section IV, which contains the results for equal initial and final states. For the results for general meson-baryon initial and final states we refer to appendix A of paper II. For the results for baryon exchange we refer to paper II as well. As mentioned before section IV also contains the discussion of how general couplings, i.e. couplings containing multiple derivatives and/or higher spin fields, should be treated in the Kadyshevsky formalism.

II. MESON-BARYON SCATTERING KINEMATICS

We consider the pion-nucleon or more general the meson-baryon reactions

$$M_i(q) + B_i(p,s) \rightarrow M_f(q') + B_f(p',s') \ . \tag{1}$$

where M stands for a meson and B is a baryon. For the four momentum of the baryons and mesons we take, respectively

$$\begin{aligned} p_c^\mu &= (E_c, \mathbf{p}_c) \quad , \ where \quad E_c = \sqrt{\mathbf{p}_c^2 + M_c^2} \ , \\ q_c^\mu &= (\mathcal{E}_c, \mathbf{q}_c) \quad , \ where \quad \mathcal{E}_c = \sqrt{\mathbf{q}_c^2 + m_c^2} \ . \end{aligned} \tag{2}$$

Here, c stands for either the initial state i or the final state f. In some cases we find it useful to use the definitions (2) for the intermediate meson-baryon states n.

Using the Kadyshevsky formalism (appendix A) and especially the second quantization procedure (appendix B) external quasi particles may occur with initial and final state momenta $n\kappa$ and $n\kappa'$, respectively. Therefore, the usual overall four-momentum conservation is generally replaced by

$$p + q + \kappa\ n = p' + q' + \kappa'\ n \ . \tag{3}$$

As (3) and (1) make clear a "prime" notation is used to indicate final state momenta; no prime means initial state momenta. We will maintain this notation (also for the energies) throughout these articles, unless indicated otherwise.

Furthermore we find it useful to introduce the Mandelstam variables in the Kadyshevsky formalism

$$\begin{aligned} s_{pq} &= (p+q)^2 \quad , \quad s_{p'q'} = (p'+q')^2 \ , \\ t_{p'p} &= (p'-p)^2 \quad , \quad t_{q'q} = (q'-q)^2 \ , \\ u_{p'q} &= (p'-q)^2 \quad , \quad u_{pq'} = (p-q')^2 \ , \end{aligned} \tag{4}$$

where s_{pq} and $s_{p'q'}$ etc., are only identical for $\kappa' = \kappa = 0$. These Mandelstam variables satisfy the relation

$$2\sqrt{s_{p'q'}s_{pq}} + t_{p'p} + t_{q'q} + u_{pq'} + u_{p'q} = 2\left(M_f^2 + M_i^2 + m_f^2 + m_i^2\right) \ . \tag{5}$$

The total and relative four-momenta of the initial, final, and intermediate channel ($c = i, f, n$) are defined by

$$P_c \;=\; p_c + q_c \;,\; k_c = \mu_{c,2}\; p_c - \mu_{c,1}\; q_c \;, \tag{6}$$

where the weights satisfy $\mu_{c,1} + \mu_{c,2} = 1$. We choose the weights to be

$$\begin{aligned} \mu_{c,1} &= \frac{E_c}{E_c + \mathcal{E}_c} \;, \\ \mu_{c,2} &= \frac{\mathcal{E}_c}{E_c + \mathcal{E}_c} \;. \end{aligned} \tag{7}$$

Since in the Kadyshevsky formalism all particles are on their mass shell, the choice (7) means that always $k_c^0 = 0$.

In the center-of-mass (CM) system $\mathbf{p} = -\mathbf{q}$ and $\mathbf{p}' = -\mathbf{q}'$, therefore

$$\begin{aligned} P_i &= (W, \mathbf{0}) \;, \quad P_f = (W', \mathbf{0}) \;, \\ k_i &= (0, \mathbf{p}) \;, \quad k_f = (0, \mathbf{p}') \;. \end{aligned} \tag{8}$$

where $W = E + \mathcal{E}$ and $W' = E' + \mathcal{E}'$. Furthermore we take $n^\mu = (1, \mathbf{0})$.

Also we take as the scattering plane the xz-plane, where the 3-momentum of the initial baryon is oriented in the positive z-direction.

In the CM system the unpolarized differential cross section is defined to be

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{|\mathbf{p}'|}{2|\mathbf{p}|} \sum \left| \frac{M_{fi}}{8\pi\sqrt{s}} \right|^2 \;, \tag{9}$$

where the amplitude M_{fi} is defined in appendix A and the sum is over the spin components of the final baryon.

To generate amplitudes at all orders we use the Kadyshevsky integral equation in the CM system

$$\begin{aligned} M(W'\,\mathbf{p}'; W\,\mathbf{p}) \;=\; & M^{irr}_{00}(W'\,\mathbf{p}'; W\,\mathbf{p}) + \int d^3k_n\, M^{irr}_{0\kappa}(W'\,\mathbf{p}'; W_n\,\mathbf{k}_n) \\ & \times \frac{1}{(2\pi)^3}\; \frac{1}{4\mathcal{E}_n E_n}\; \frac{1}{\sqrt{s} - \sqrt{s_n} + i\varepsilon}\; M_{\kappa 0}(W_n\,\mathbf{k}_n; W\,\mathbf{p}) \;. \end{aligned} \tag{10}$$

Although there are still κ-labels in (10), they're fixed at $\kappa = P_i^0 - P_n^0$. Also we have included the spinors of the projection operator of the fermion propagator

$$\begin{aligned} S^{(+)}(p_n) \;&=\; \Lambda^{(1/2)}(p_n)\; \theta(p_n^0)\delta(p_n^2 - M^2) \;, \\ &= \sum_{s_n} u(p_n s_n)\bar{u}(p_n s_n)\; \theta(p_n^0)\delta(p_n^2 - M^2) \;, \end{aligned} \tag{11}$$

in the amplitudes $M_{0\kappa}(p'q'; p_n q_n)$ and $M_{\kappa 0}(p_n q_n; pq)$.

We have put the intermediate negative energy states ($\Delta^{(-)}(x-y; m_\pi^2)$ and $S^{(-)}(x-y; M_N^2)$) in $M^{irr}_{\kappa\kappa'}$, but in principle they could also participate in the integral equation. However, using pair suppression in the way we do in paper II, these terms vanish.

A. n-independence of Kadyshevsky Integral Equation

When generating Kadyshevsky diagrams to random order using the Kadyshevsky integral equation, the (full) amplitude is identical to the one obtained in Feynman formalism when the external quasi particle momenta are put to zero. It is therefore n-independent, i.e. frame independent.

Since an approximation is used to solve the Kadyshevsky integral equation, namely tree level diagrams as driving terms, it is not clear whether the full amplitude remains to be n-independent when the external quasi particle momenta are put to zero.

In examining the n-dependence of the amplitude we write the Kadyshevsky integral equation schematically as

$$M_{00} = M_{00}^{irr} + \int d\kappa\ M_{0\kappa}^{irr}\ G'_\kappa\ M_{\kappa 0}\ , \tag{12}$$

Since $n^2 = 1$, only variations in a space-like direction are unrestricted, i.e. $n \cdot \delta n = 0$ [13]. We therefore introduce the projection operator

$$P^{\alpha\beta} = g^{\alpha\beta} - n^\alpha n^\beta\ , \tag{13}$$

from which it follows that $n_\alpha P^{\alpha\beta} = 0$. The n-dependence of the amplitude can now be studied

$$\begin{aligned} P^{\alpha\beta}\frac{\partial}{\partial n^\beta}M_{00} &= P^{\alpha\beta}\frac{\partial M_{00}^{irr}}{\partial n^\beta} \\ &+P^{\alpha\beta}\int d\kappa \left[\frac{\partial M_{0\kappa}^{irr}}{\partial n^\beta}\ G'_\kappa\ M_{\kappa 0} + M_{0\kappa}^{irr}\ G'_\kappa\ \frac{\partial M_{\kappa 0}}{\partial n^\beta}\right]\ . \end{aligned} \tag{14}$$

If both Kadyshevsky contributions are considered at second order in M_{00}, then it is n-independent, since it yields the Feynman expression. As far as the second term in (14) is concerned we observe the following

$$\frac{\partial M_{0\kappa}^{irr}}{\partial n^\beta} \propto \kappa f(\kappa)\ \ ,\ \ \frac{\partial M_{\kappa 0}}{\partial n^\beta} \propto \kappa g(\kappa)\ , \tag{15}$$

where $f(\kappa)$ and $g(\kappa)$ are functions that do not contain poles or zero's at $\kappa = 0$. Therefore, the integral in (14) is of the form

$$\int d\kappa\ \kappa\ h(\kappa) G'_\kappa\ . \tag{16}$$

When performing the integral we decompose the G'_κ as follows

$$G'_\kappa \propto \frac{1}{\kappa + i\varepsilon} = P\frac{1}{\kappa} - i\pi\delta(\kappa)\ . \tag{17}$$

As far as the $\delta(\kappa)$-part of (17) is concerned we immediately see that it gives zero when used in the integral (16). For the Principle valued integral, indicated in figure 1 by **I**, we close the integral by connecting the end point ($\kappa = \pm\infty$) via a (huge) semi-circle in the upper half, complex κ-plane (line **II** in figure 1) and by connecting the points around zero via a small semi circle also in the upper half plane (line **III** in figure 1). Since every single (tree

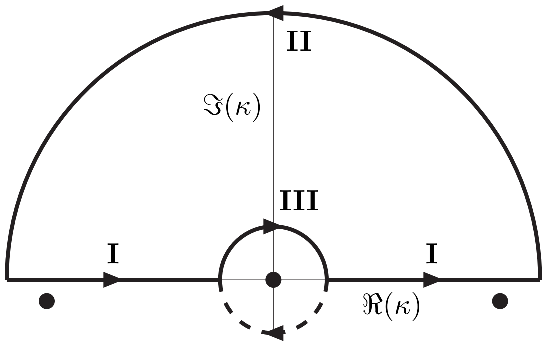

FIG. 1: *Principle value integral*

level) amplitude is proportional to $1/(\kappa + A + i\varepsilon)$, where κ is related to the momentum of the incoming or outgoing quasi particle and A some positive or negative number, the poles will always be in the lower half plane and not within the contour. Therefore, the contour integral is zero.

Since we have added integrals (**II** and **III** in figure 1) we need to know what their contributions are. The easiest part is integral **III**. Its contribution is half the residue at $\kappa = 0$ and since the only remaining integrand part $h(\kappa)$ in (16) doesn't contain a pole at zero it is zero.

If we want the contribution of integral **II** to be zero, than the integrand should at least be of order $O(\frac{1}{\kappa^2})$. Unfortunately, this is not (always) the case as we will see in sections IV and paper II. To this end we introduce a phenomenological "form factor"

$$F(\kappa) = \left(\frac{\Lambda_\kappa^2}{\Lambda_\kappa^2 - \kappa^2 - i\epsilon(\kappa)\varepsilon} \right)^{N_\kappa} , \tag{18}$$

where Λ_κ is large and N_κ is some positive integer. In (18) ε is real, positive, though small and $\epsilon(\kappa) = \theta(\kappa) - \theta(-\kappa)$.

The effect of the function $F(\kappa)$ (18) on the original integrand in (16) is little, since for large Λ_κ it is close to unity. However, including this function in the integrand makes sure that it is at least of order $O(\frac{1}{\kappa^2})$ so that integral **II** gives zero contribution. The $-i\epsilon(\kappa)\varepsilon$ part ensures that there are now poles on or within the closed contour, since they are always in the lower half plane (indicated by the dots in figure 1).

III. APPLICATION: PION-NUCLEON SCATTERING

In the following sections we're going to apply the Kadyshevsky formalism to the pion-nucleon system, although we present it in such a way that it can easily be extended to other meson-baryon systems. The isospin factors are not included in our treatment; we are only concerned about the Lorentz and Dirac structure. For the details about the isospin factors we refer to [3].

The ingredients of the model are tree level, exchange amplitudes as mentioned before. These amplitudes serve as input for the integral equation. Very similar to what is done in [3] we consider for the amplitudes the exchanged particles as in table I. Graphically, this shown in figure 2. Contrary to [3] we do not consider the exchange of the tensor mesons, since their

Channel	Exchanged particle
t	f_0, σ, P, ρ
u	$N, N^*, S_{11}, \Delta_{33}$
s	$N, N^*, S_{11}, \Delta_{33}$

TABLE I: *Exchanged particles in the various channels*

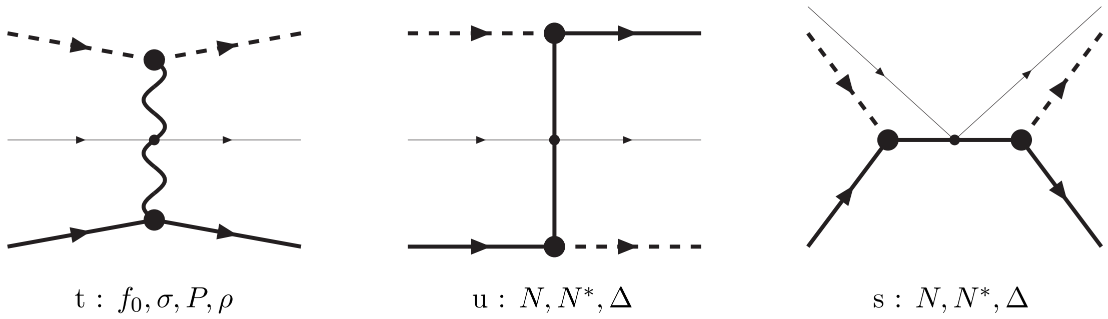

FIG. 2: *Tree level amplitudes as input for integral equation. The inclusion of the quasi particle lines is schematically. Therefore, the diagrams represent either the (a) or the (b) diagram.*

contribution is little. The inclusion of the them can be regarded as an extension of this work.

For the description of the amplitudes we need the interaction Lagrangians, which in our treatment always serve as the starting points

- Triple meson vertices

$$\mathcal{L}_{SPP} = g_{PPS}\, \phi_{P,a}\phi_{P,b} \cdot \phi_S \ , \tag{19a}$$

$$\mathcal{L}_{VPP} = g_{VPP} \left(\phi_a i \overleftrightarrow{\partial}_\mu \phi_b \right) \phi^\mu \ , \tag{19b}$$

  where S, V, P stand for *scalar*, *vector* and *pseudo scalar* to indicate the various mesons. The indices a, b are used to indicate the outgoing and incoming meson, respectively. For the derivative $\overleftrightarrow{\partial_\mu} = \overrightarrow{\partial_\mu} - \overleftarrow{\partial_\mu}$.

- Meson-baryon vertices

$$\mathcal{L}_{SNN} = g_S\, \bar{\psi}\psi \cdot \phi_S \ , \tag{20a}$$

$$\mathcal{L}_{VNN} = g_V\, \bar{\psi}\gamma_\mu\psi\ \phi^\mu - \frac{f_V}{2M_V}\ i\partial^\mu \left(\bar{\psi}\sigma_{\mu\nu}\psi \right) \cdot \phi^\nu \ , \tag{20b}$$

$$\mathcal{L}_{PV} = \frac{f_{PV}}{m_\pi}\, \bar{\psi}\gamma_5\gamma_\mu\psi \cdot \partial^\mu \phi_P \ , \tag{20c}$$

$$\mathcal{L}_{V} = \frac{f_V}{m_\pi}\, \bar{\psi}\gamma_\mu\psi \cdot \partial^\mu \phi_P \ , \tag{20d}$$

  where $\sigma_{\mu\nu} = \frac{1}{2}\left[\gamma_\mu, \gamma_\nu\right]$. The coupling constants f_V of (20b) and (20d) do not necessarily coincide.

We have chosen (20b) in such a way that the vector meson couples to a current, which may contain a derivative. This is a bit different from [3, 18], where the derivative acts on the vector meson. In Feynman theory this does not make a difference, however it does in Kadyshevsky formalism, because of the presence of the quasi particles.

Equation (20c) is used to describe the exchange (u, s-channel) of the nucleon and Roper (N^*) and (20d) is used for the S_{11} exchange. This, because of their intrinsic parities. Note, that we could also have chosen the pseudo scalar and scalar couplings for these exchanges. However, since the interactions (20c) and (20d) are also used in [3] and in chiral symmetry based models, we use these interactions.

- $\underline{\pi N\Delta_{33}\text{ vertex}}$

$$\mathcal{L}_{\pi N\Delta} = g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\left(\partial_\mu\bar{\Psi}_\nu\right)\gamma_5\gamma_\alpha\psi\,(\partial_\beta\phi) + g_{gi}\,\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_5\gamma_\alpha\,(\partial_\mu\Psi_\nu)\,(\partial_\beta\phi)\ , \tag{21}$$

The use of this interaction Lagrangian differs from the one used in [3]. We'll come back to this in paper II.

The meson exchange processes are discussed in section IV. As mentioned before the discussion of the baryon exchange processes (including pair suppression) is postponed to paper II. An other important ingredient of the model is the use of form factors. We also postpone the discussion of them to paper II.

IV. MESON EXCHANGE

Here, we proceed with the discussion of the meson exchange processes. We give the amplitudes for meson-baryon scattering or pion-nucleon scattering, specifically, meaning that we take equal initial and final states ($M_f = M_i = M$ and $m_f = m_i = m$, where M and m are the masses of the nucleon and pion, respectively). The results for general meson-baryon initial and final states are presented in appendix A of paper II.

A. Scalar Meson Exchange

For the description of the scalar meson exchange processes at tree level, graphically shown in figure 3, we use the interaction Lagrangians (19a) and (20a), which lead to the vertices

$$\begin{aligned}\Gamma_{PPS} &= g_{PSS}\ ,\\ \Gamma_S &= g_S\ ,\end{aligned} \tag{22}$$

using $\mathcal{L}_I = -\mathcal{H}_I \to -\Gamma$. For the appropriate propagator we use the first line of (A2).

Applying the Kadyshevsky rules as discussed in appendix A, the amplitudes read

$$M^{a,b}_{\kappa'\kappa} = g_{PSS}g_S\int\frac{d\kappa_1}{\kappa_1+i\varepsilon}\left[\bar{u}(p's')u(ps)\right]\theta(P^0_{a,b})\delta(P^2_{a,b}-M^2_S)\ , \tag{23}$$

where $P_{a,b} = \pm\Delta_t + \frac{1}{2}(\kappa'+\kappa) - n\kappa_1$ (here a corresponds to the $+$ sign and b to the $-$ sign)

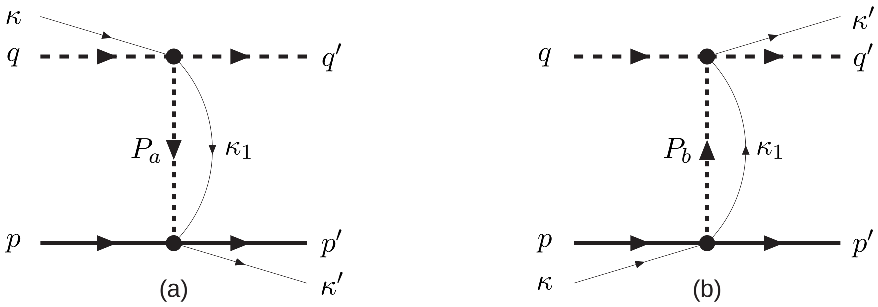

FIG. 3: *Scalar meson exchange*

and $\Delta_t = \frac{1}{2}(p' - p - q' + q)$. For the κ_1 integration we consider the δ-function in (23)

$$\begin{aligned}
(a): \quad \delta(P_a^2 - M_S^2) &= \frac{1}{|\kappa_1^+ - \kappa_1^-|}\left(\delta(\kappa_1 - \kappa_1^+) + \delta(\kappa_1 - \kappa_1^-)\right) \ , \\
\kappa_1^\pm &= \Delta_t \cdot n + \frac{1}{2}(\kappa' + \kappa) \pm A_t \ , \\
(b): \quad \delta(P_b^2 - M_S^2) &= \frac{1}{|\kappa_1^+ - \kappa_1^-|}\left(\delta(\kappa_1 - \kappa_1^+) + \delta(\kappa_1 - \kappa_1^-)\right) \ , \\
\kappa_1^\pm &= -\Delta_t \cdot n + \frac{1}{2}(\kappa' + \kappa) \pm A_t \ ,
\end{aligned} \tag{24}$$

where $A_t = \sqrt{(n \cdot \Delta_t)^2 - \Delta_t^2 + M_S^2}$. In both cases $\theta(P^0_{a,b})$ selects the κ_1^- solution. Therefore,

$$\begin{aligned}
P_a &= \Delta_t - (\Delta_t \cdot n)\, n + A_t n \ , \\
P_b &= -\Delta_t + (\Delta_t \cdot n)\, n + A_t n \ .
\end{aligned} \tag{25}$$

With these expression we find for the amplitudes

$$\begin{aligned}
M^{(a)}_{\kappa'\kappa} &= g_{PSS} g_S \ [\bar{u}(p's')u(ps)] \, \frac{1}{2A_t} \cdot \frac{1}{\Delta_t \cdot n + \bar{\kappa} - A_t + i\varepsilon} \ , \\
M^{(b)}_{\kappa'\kappa} &= g_{PSS} g_S \ [\bar{u}(p's')u(ps)] \, \frac{1}{2A_t} \cdot \frac{1}{-\Delta_t \cdot n + \bar{\kappa} - A_t + i\varepsilon} \ ,
\end{aligned} \tag{26}$$

where $\bar{\kappa} = \frac{1}{2}(\kappa' + \kappa)$.

Adding the two together and putting $\kappa' = \kappa = 0$ we get

$$M_{00} = g_{PSS} g_S \ [\bar{u}(p's')u(ps)] \, \frac{1}{t - M_S^2 + i\varepsilon} \ , \tag{27}$$

which is Feynman result [3].

In subsection II A we discussed the n-dependence of the Kadyshevsky integral equation. In order to do that we need to know the n-dependence of the amplitude (14)

$$\begin{aligned}
M^{(a+b)}_{0\kappa} &= M^{(a)}_{0\kappa} + M^{(b)}_{0\kappa} \ , \\
\frac{\partial M^{(a+b)}_{0\kappa}}{\partial n^\beta} &= \kappa \ g_{PSS} g_S \ [\bar{u}(p's')u(ps)] \\
&\quad \times \frac{n \cdot \Delta_t (\Delta_t)_\beta}{2A_t^3} \, \frac{(n \cdot \Delta_t)^2 - 3A_t^2 - \frac{\kappa^2}{4} + 2\kappa A_t}{\left((n \cdot \Delta_t)^2 - \left(A_t - \frac{\kappa}{2}\right)^2 + i\varepsilon\right)^2} \ .
\end{aligned} \tag{28}$$

If we would only consider scalar meson exchange in the Kadyshevsky integral equation the integrand would be of the form (16), where $h(\kappa)$ would by itself be of order $O(\frac{1}{\kappa^2})$ as can be seen from (28). Therefore, the phenomenological "form factor" (18) would not be needed.

Since there's no propagator as far as Pomeron exchange is concerned, the Kadyshevsky amplitude is the same as the Feynman amplitude for Pomeron exchange [3]

$$M_{\kappa'\kappa} = \frac{g_{PPP}g_P}{M}\left[\bar{u}(p's')u(p)\right] . \tag{29}$$

B. Vector Meson Exchange: Example

Before we go on with real vector meson exchange, we consider simplified vector meson exchange. We use this as an example to illustrate seaming problems that might occur in the results in the Kadyshevsky formalism, especially when compared to those in the Feynman formalism. We stress that although we consider the example of simplified vector meson exchange, these peculiarities are generally present when interaction Lagrangians containing derivatives and/or higher spin fields ($s \geq 1$) are considered.

In order to study simplified vector meson exchange we take interaction Lagrangian (19b) and (20b), without the $\sigma_{\mu\nu}$-term

$$\mathcal{L}_I = g\,\phi_a i\overleftrightarrow{\partial_\mu}\phi_b \cdot \phi^\mu + g\,\bar{\psi}\gamma_\mu\psi \cdot \phi^\mu , \tag{30}$$

1. Naive Kadyshevsky Approach

The Kadyshevsky diagrams for the (simplified) vector meson exchange are shown in figure 4. For the various components of the diagrams we take the following vertex functions

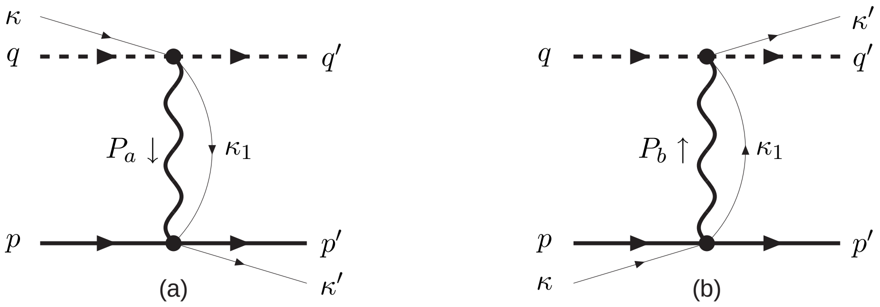

FIG. 4: *Vector meson exchange in Kadyshevsky formalism.*

$$\begin{aligned}\Gamma_\mu^{\bar{\psi}\psi} &= g\,\gamma_\mu ,\\ \Gamma_\mu^{\phi\phi} &= g\,\left(q' + q\right)_\mu ,\end{aligned} \tag{31}$$

following from (30), and the third line of (A2) for the propagator.

Applying the Kadyshevsky rules as given in appendix A straightforwardly we get the following amplitudes

$$
\begin{aligned}
M^{(a,b)}_{\kappa'\kappa} &= -g^2 \int \frac{d\kappa_1}{\kappa_1 + i\varepsilon} \left[\bar{u}(p's')\gamma_\mu u(ps)\right] \left(g^{\mu\nu} - \frac{P^\mu_{a,b} P^\nu_{a,b}}{M_V^2}\right) \\
&\qquad \times \theta(P^0_{a,b})\delta(P^2_{a,b} - M_V^2)\,(q' + q)_\nu \ ,
\end{aligned} \tag{32}
$$

The κ_1-integral is discussed in (24) and (25). We, therefore, give the results immediately

$$
\begin{aligned}
M^{(a)}_{\kappa'\kappa} &= -g^2\, \bar{u}(p's') \Bigg[2\not{Q} - \frac{1}{M_V^2} \left((M_f - M_i) + \frac{1}{2}\not{n}(\kappa' - \kappa) - (\Delta_t \cdot n - A_t)\not{n} \right) \\
&\qquad \times \left(\frac{1}{4}\left(s_{p'q'} - s_{pq}\right) + \frac{1}{4}\left(u_{pq'} - u_{p'q}\right) - (m_f^2 - m_i^2) \right. \\
&\qquad \left. - 2(\Delta_t \cdot n - A_t) n \cdot Q \right) \Bigg] u(ps) \\
&\times \frac{1}{2A_t}\, \frac{1}{\Delta_t \cdot n + \frac{1}{2}(\kappa' + \kappa) - A_t + i\varepsilon} \ , \\
M^{(b)}_{\kappa'\kappa} &= -g^2\, \bar{u}(p's') \Bigg[2\not{Q} - \frac{1}{M_V^2} \left((M_f - M_i) + \frac{1}{2}\not{n}(\kappa' - \kappa) - (\Delta_t \cdot n + A_t)\not{n} \right) \\
&\qquad \times \left(\frac{1}{4}\left(s_{p'q'} - s_{pq}\right) + \frac{1}{4}\left(u_{pq'} - u_{p'q}\right) - (m_f^2 - m_i^2) \right. \\
&\qquad \left. - 2(\Delta_t \cdot n + A_t) n \cdot Q \right) \Bigg] u(ps) \\
&\times \frac{1}{2A_t}\, \frac{1}{-\Delta_t \cdot n + \frac{1}{2}(\kappa' + \kappa) - A_t + i\varepsilon} \ .
\end{aligned} \tag{33}
$$

Adding the two together and putting $\kappa' = \kappa = 0$ we should get back the Feynman expression

$$
\begin{aligned}
M_{00} &= M^{(a)}_{00} + M^{(b)}_{00} \\
&= -g^2 \bar{u}(p's') \left[2\not{Q} + \frac{(M_f - M_i)}{M_V^2}\,(m_f^2 - m_i^2) \right] u(ps)\, \frac{1}{t - M_V^2 + i\varepsilon} \\
&\quad -g^2 \bar{u}(p's') \left[\not{n}\right] u(ps)\, \frac{2Q \cdot n}{M_V^2} \ .
\end{aligned} \tag{34}
$$

From (34) we see that the first term on the rhs is indeed the Feynman result. However, the second term on the rhs is an unwanted, n-dependent, contact term.

As mentioned before, similar discrepancies are obtained when couplings containing higher spin fields ($s \geq 1$) are used. Therefore, it seems that the Kadyshevsky formalism doesn't yield the same results in these cases as the Feynman formalism when κ' and κ are put to zero. Since the real difference between Feynman formalism and Kadyshevsky formalism lies in the treatment of the Time Ordered Product (TOP) or θ-function also the difference in results should find its origin in this treatment.

In Feynman formalism derivatives are taken out of the TOP in order to get Feynman functions, which may yield extra terms. This is also the case in the above example

$$\begin{aligned}
T[\phi^\mu(x)\phi^\nu(y)] &= -\left[g^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{M_V^2}\right] i\Delta_F(x-y) - i\delta_0^\mu\delta_0^\nu\,\delta^4(x-y)\ , \\
S_{fi} &= (-i)^2 g^2 \int d^4x d^4y\,\left[\bar{\psi}\gamma_\mu\psi\right]_x T[\phi^\mu(x)\phi^\nu(y)]\left[\phi_a i\overleftrightarrow{\partial_\nu}\phi_b\right]_y\ , \\
\Rightarrow M_{extra} &= -g^2\bar{u}(p's')\,[\not{n}]\,u(ps)\,\frac{2Q\cdot n}{M_V^2}\ .
\end{aligned} \tag{35}$$

[26] If we include the extra term of (35) on the Feynman side we see that both formalisms yield the same result.

Although we have exact equivalence between the two formalisms, the result, though covariant, is still n-dependent, i.e. frame-dependent. Of course this is not what we want. As it will turn out there is another source of extra terms exactly cancelling for instance the one that pops-up in our example ((34), (35)). As mentioned in the introduction we present two methods for getting these extra terms cancelling the one in (34) and (35): the TU method is more fundamental and the GJ method is more systematic and pragmatic. Both methods we will shortly introduce and apply to the problem in sections IV B 2 and IV B 3, respectively.

2. *Takahahsi & Umezawa Solution*

In order to demonstrate the TU method [10–12] we start with a rewritten version of the Yang-Feldman (YF) equations [19] for a general interaction

$$\mathbf{\Phi}_\alpha(x) = \Phi_\alpha(x) - \int d^4y\ R_{\alpha\beta}(\partial)\ D_a(y)\ \Delta_{ret}(x-y)\cdot\mathbf{j}_{\beta;a}(y)\ , \tag{36}$$

where $\mathbf{\Phi}_\alpha(x)$ and $\Phi_\alpha(x)$ are fields in the *Heisenberg Representation* (H.R.) and *Interaction Representation* (I.R.), respectively. Furthermore, the vectors $D_a(x)$ and $\mathbf{j}_{\alpha;a}(x)$ are defined to be

$$\begin{aligned}
D_a(x) &\equiv (1, \partial_{\mu_1}, \partial_{\mu_1}\partial_{\mu_2}, \ldots)\ , \\
\mathbf{j}_{\alpha;a}(x) &\equiv \left(-\frac{\partial\mathcal{L}_I}{\partial\mathbf{\Phi}_\alpha(x)}\ ,\ -\frac{\partial\mathcal{L}_I}{\partial\left(\partial_{\mu_1}\mathbf{\Phi}_\alpha(x)\right)}\ ,\ -\frac{\partial\mathcal{L}_I}{\partial\left(\partial_{\mu_1}\partial_{\mu_2}\mathbf{\Phi}_\alpha(x)\right)}\ ,\ \ldots\right)\ ,
\end{aligned} \tag{37}$$

Next, a free auxiliary field $\Phi_\alpha(x,\sigma)$ is introduced, where σ is a space-like surface and x does not necessarily lie on σ. We pose that it has the following form

$$\Phi_\alpha(x,\sigma) \equiv \Phi_\alpha(x) + \int_{-\infty}^{\sigma} d^4y\ R_{\alpha\beta}(\partial) D_a(y)\ \Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y)\ , \tag{38}$$

Combining (38) with (36) leads to

$$\mathbf{\Phi}_\alpha(x) = \Phi_\alpha(x/\sigma) + \frac{1}{2}\int d^4y\left[R_{\alpha\beta}(\partial)D_a(y), \epsilon(x-y)\right]\Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y)\ . \tag{39}$$

This equation will be used to express the fields in the H.R. in terms of fields in the I.R.

In appendix C it is explained that the auxiliary fields and the fields in the I.R. are related by a unitary operator using the BMP theory. Also it is shown how the interaction Hamiltonian should be deduced.

Applying these concepts to our example we determine the "currents" via (37)

$$\begin{aligned} \boldsymbol{j}_{\phi_a,a} &= (-g\ i\partial_\mu \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu, ig\ \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu)\ , \\ \boldsymbol{j}_{\phi_b,a} &= (g\ i\partial_\mu \boldsymbol{\phi}_a \cdot \boldsymbol{\phi}^\mu, -ig\ \boldsymbol{\phi}_a \cdot \boldsymbol{\phi}^\mu)\ , \\ \boldsymbol{j}_{\psi,a} &= (-g\ \gamma_\mu \boldsymbol{\psi} \cdot \boldsymbol{\phi}^\mu, 0)\ , \\ \boldsymbol{j}_{\phi^\mu,a} &= \left(-g\ \boldsymbol{\phi}_a i\overleftrightarrow{\partial_\mu} \boldsymbol{\phi}_b - g\ \bar{\boldsymbol{\psi}}\gamma_\mu \boldsymbol{\psi}, 0\right)\ . \end{aligned} \tag{40}$$

Using (39) we can express the fields in the H.R. in terms of fields in the I.R., i.e. free fields

$$\begin{aligned} \boldsymbol{\phi}_a(x) &= \phi_a(x/\sigma)\ , \\ \boldsymbol{\phi}_b(x) &= \phi_b(x/\sigma)\ , \\ \partial_\mu \boldsymbol{\phi}_a(x) &= \left[\partial_\mu \phi_a(x,\sigma)\right]_{x/\sigma} + \frac{1}{2}\int d^4y \left[\partial^x_\mu \partial^y_\nu, \epsilon(x-y)\right] \Delta(x-y) \left(ig\phi_b \cdot \phi^\nu\right)_y \\ &= \left[\partial_\mu \phi_a(x,\sigma)\right]_{x/\sigma} + ign_\mu \phi_b\ n\cdot\phi\ , \\ \partial_\mu \boldsymbol{\phi}_b(x) &= \left[\partial_\mu \phi_b(x,\sigma)\right]_{x/\sigma} + \frac{1}{2}\int d^4y \left[\partial^x_\mu \partial^y_\nu, \epsilon(x-y)\right] \Delta(x-y) \left(-ig\phi_a \cdot \phi^\nu\right)_y \\ &= \left[\partial_\mu \phi_b(x,\sigma)\right]_{x/\sigma} - ign_\mu \phi_a\ n\cdot\phi\ , \\ \boldsymbol{\psi}(x) &= \psi(x/\sigma)\ , \\ \boldsymbol{\phi}^\mu(x) &= \phi^\mu(x/\sigma) + \frac{1}{2}\int d^4y \left[\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right), \epsilon(x-y)\right] \Delta(x-y) \\ &\qquad \times \left(-g\phi_a i\overleftrightarrow{\partial_\nu}\phi_b - g\bar{\psi}\gamma_\nu\psi\right)_y \\ &= \phi^\mu(x/\sigma) - \frac{g\, n^\mu}{M_V^2}\left(\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b + \bar{\psi}\not{n}\psi\right)\ . \end{aligned} \tag{41}$$

As can be seen from (39) the first term on the rhs is a free field and the second term contains the current expressed in terms of fields in the H.R., which on their turn are expanded similarly. Therefore, one gets coupled equations. In solving these equations we assumed that the coupling constant is small and therefore considered only terms up to first order in the coupling constant in the expansion of the fields in the H.R. Practically speaking, the currents on the rhs of (41) are expressed in terms of free fields.

These expansions (41) are used in the commutation relations of the fields with the interaction Hamiltonian ((C17) of appendix C)

$$
\begin{aligned}
[\phi_a(x), \mathcal{H}_I(y)] &= iU^{-1}(\sigma)\Delta(x-y)\left[-g\ i\partial_\mu \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu + g\ \overleftarrow{i\partial_\mu} \boldsymbol{\phi}_b \cdot \boldsymbol{\phi}^\mu\right]_y U(\sigma) \\
&= i\Delta(x-y)\left[-g\ \overleftrightarrow{i\partial_\mu}\phi_b \cdot \phi^\mu \right. \\
&\quad \left. + \frac{g^2}{M_V^2}\ n\cdot \overleftrightarrow{i\partial}\phi_b \left(\phi_a n \cdot \overleftrightarrow{i\partial}\phi_b + \bar{\psi}\not{n}\psi\right) - g^2\ \phi_a (n\cdot\phi)^2\right]_y \\
[\psi(x), \mathcal{H}_I(y)] &= iU^{-1}(\sigma)(i\not{\partial} + M)\Delta(x-y)\left[-g\ \gamma_\mu \boldsymbol{\psi}\cdot\boldsymbol{\phi}^\mu\right]_y U(\sigma) \\
&= i(i\not{\partial}+M)\Delta(x-y) \\
&\quad \times \left[-g\ \gamma_\mu \psi\cdot\phi^\mu + \frac{g^2}{M_V^2}\ \not{n}\psi\left(\phi_a n\cdot\overleftrightarrow{i\partial}\phi_b + \bar{\psi}\not{n}\psi\right)\right]_y , \\
[\phi^\mu(x), \mathcal{H}_I(y)] &= iU^{-1}(\sigma)\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right)\Delta(x-y) \\
&\quad \times \left[-g\ \boldsymbol{\phi}_a \overleftrightarrow{i\partial_\nu}\boldsymbol{\phi}_b - g\ \bar{\boldsymbol{\psi}}\gamma_\nu\boldsymbol{\psi}\right]_y U(\sigma) \\
&= i\left(-g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_V^2}\right)\Delta(x-y)\left[-g\ \phi_a\overleftrightarrow{i\partial_\nu}\phi_b - g\ \bar{\psi}\gamma_\nu\psi \right. \\
&\quad \left. - g^2\, n_\nu\, \phi_a^2 n\cdot\phi - g^2\, n_\nu\, \phi_b^2 n\cdot\phi\right]_y .
\end{aligned}
\tag{42}
$$

As stated below (C17) these are the fundamental equations from which the interaction Hamiltonian can be determined

$$
\begin{aligned}
\mathcal{H}_I &= -g\ \phi_a\overleftrightarrow{i\partial_\mu}\phi_b\cdot\phi^\mu - g\ \bar{\psi}\gamma_\mu\psi\cdot\phi^\mu - \frac{g^2}{2}\ \phi_a^2(n\cdot\phi)^2 - \frac{g^2}{2}\ \phi_b^2(n\cdot\phi)^2 \\
&\quad + \frac{g^2}{2M_V^2}\ \left[\bar{\psi}\not{n}\psi\right]^2 + \frac{g^2}{M_V^2}\ \left[\bar{\psi}\not{n}\psi\right]\left[\phi_a n\cdot\overleftrightarrow{i\partial}\phi_b\right] + \frac{g^2}{2M_V^2}\ \left[\phi_a n\cdot\overleftrightarrow{i\partial}\phi_b\right]^2 \\
&\quad + O(g^3)\ldots .
\end{aligned}
\tag{43}
$$

If equation (41) was solved completely, then the rhs of (41) would contain higher orders in the coupling constant and therefore also the interaction Hamiltonian (43). These terms are indicated by the ellipsis.

If we want to include the external quasi fields as in appendix B, then the easy way to do this is to apply (B7) straightforwardly. However, since we want to derive the interaction Hamiltonian from the interaction Lagrangian we would have to include a $\bar{\chi}(x)\chi(x)$ pair in (30) similar to (B7). This would mean that the terms of order g^2 in (43) are quartic in the quasi field, where two of them can be contracted

$$
\bar{\chi}(x)\underbrace{\chi(x)\bar{\chi}}(x)\chi(x) = \bar{\chi}(x)\theta[n(x-x)]\chi(x) . \tag{44}
$$

Defining the θ-function to be 1 in its origin we assure that all terms in the interaction Hamiltonian (43) relevant to πN-scattering are quadratic in the external quasi fields, even higher order terms in the coupling constant.

The only term of order g^2 in (43) that gives a contribution to the first order in the S-matrix describing πN-scattering is the second term on the second line in the rhs of (43). Its contribution to the first order in the S-matrix is

$$
\begin{aligned}
S_{fi}^{(1)} &= -i\int d^4x\mathcal{H}_I(x) = \frac{-ig^2}{M_V^2}\int d^4x\,\left[\bar{\psi}\not{n}\psi\right]\left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b\right]_x \\
&= \frac{-ig^2}{M_V^2}\,\bar{u}(p's')\not{n}u(ps)n\cdot(q'+q)\ , \\
\Rightarrow M_{canc} &= g^2\,\bar{u}(p's')\not{n}u(ps)\frac{2n\cdot Q}{M_V^2}\ .
\end{aligned}
\tag{45}
$$

Indeed we see that this term (45) cancels the extra term in (34).

From (43) one can see that the interaction Hamiltonian contains not only terms of order g, but also higher order terms. In our example we see that the g^2 terms in the interaction Hamiltonian is responsible for the cancellation. In this light we would also like to mention the specific example of scalar electrodynamics as described in [20], section 6-1-4. There the interaction Hamiltonian also contains a term of order g^2, which has the same purpose as in our case. The method described in [20] is not generally applicable, whereas the above described method, although applied to a specific example, is.

3. Gross & Jackiw Solution

The essence of the Gross and Jackiw method [13] is to define a different TOP: the T^* product, which is by definition n-independent

$$
T^*(x,y)\ =\ T(x,y;n)+\tau(x,y;n)\ , \tag{46}
$$

Studying the n-dependence is done in the same way as described in subsection II A

$$
P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T^*(x,y)\ =\ P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T(x,y;n)+P^{\alpha\beta}\frac{\delta}{\delta n^\beta}\tau(x,y;n)\equiv 0\ . \tag{47}
$$

In our applications we are interested in second order contributions to πN-scattering. Therefore, we analyze the n-dependence of the TOP of two interaction Hamiltonians, where we take it to be just $\mathcal{H}_I=-\mathcal{L}_I$

$$
P^{\alpha\beta}\frac{\delta}{\delta n^\beta}T(x,y;n)\ =\ P^{\alpha\beta}(x-y)_\beta\delta\left[n\cdot(x-y)\right]\ \left[\mathcal{H}_I(x),\mathcal{H}_I(y)\right]\ . \tag{48}
$$

In general one has for equal time commutation relations

$$
\begin{aligned}
\delta[n(x-y)]\left[\mathcal{H}_I(x),\mathcal{H}_I(y)\right] \ =\ & \big[C(n)+P^{\alpha\beta}S_\alpha(n)\partial_\beta \\
&+P^{\alpha\beta}P^{\mu\nu}Q_{\alpha\mu}(n)\partial_\beta\partial_\nu+\ldots\big]\,\delta^4(x-y)\ .
\end{aligned}
\tag{49}
$$

where the ellipsis stand for higher order derivatives. We will only consider (and encounter) up to quadratic derivatives. The S^α and $Q^{\alpha\beta}$ terms in (49) are known in the literature as *Schwinger terms.*

It should be mentioned that in [13] only the first two terms on the rhs of (49) are considered.

Using the fact that the TOP and therefore also the T^* product appears in the S-matrix as an integrand, we are allowed to use partial integration for the $S_\alpha(n)$ and $Q_{\alpha\beta}(n)$ terms.

The $C(n)$ always vanishes. Furthermore, we use the fact that $P^{\alpha\beta}$ is a projection operator. With these considerations we find from (47)-(49) the extra terms

$$\tau(x-y;n) = \int^n dn'^\beta \left[S_\beta(n') + P^{\mu\nu}\left(Q_{\beta\mu}(n') + Q_{\mu\beta}(n')\right)\partial_\nu\right]\delta^4(x-y) \ . \tag{50}$$

In principle the rhs of (50) can also contain a constant term, i.e. independent of n^μ. But since we are looking for n^μ-dependent terms only, this term is irrelevant.

Now, we're going to apply the method of Gross and Jackiw. The "covariantized" equal time commutator of interaction Hamiltonians is

$$\begin{aligned}
&\delta[n(x-y)]\left[\mathcal{H}_I(x), \mathcal{H}_I(y)\right] = \\
&= g^2\left\{\frac{1}{M_V^2}\left([\psi\not{n}\psi]_x\left[\phi_a i\overleftrightarrow{\partial_\mu}\phi_b\right]_y + [\psi n_\mu\psi]_x\left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b\right]_y\right.\right. \\
&\qquad + \left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b\right]_x[\psi n_\mu\psi]_y + \left[\phi_a i\overleftrightarrow{\partial_\mu}\phi_b\right]_x[\psi\not{n}\psi]_y \\
&\qquad + [\psi\not{n}\psi]_y[\psi\gamma_\mu\psi]_x + [\psi\gamma_\mu\psi]_y[\psi\not{n}\psi]_x \\
&\qquad \left. + \left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b\right]_y\left[\phi_a i\overleftrightarrow{\partial_\mu}\phi_b\right]_x + \left[\phi_a i\overleftrightarrow{\partial_\mu}\phi_b\right]_y\left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b\right]_x\right) \\
&\quad + \phi_a(y)n\cdot\phi(x)\phi_a(x)\phi_\mu(y) + \phi_a(y)\phi_\mu(x)\phi_a(x)n\cdot\phi(y) \\
&\quad \left. + [\phi_b n\cdot\phi]_x[\phi_b\phi_\mu]_y + [\phi_b\phi_\mu]_x[\phi_b n\cdot\phi]_y\right\} P^{\mu\rho}i\partial_\rho\delta^4(x-y) \ .
\end{aligned} \tag{51}$$

Comparing this with (49) we see that the terms between curly brackets coincide with $-iS_\alpha(n)$; the $Q_{\alpha\beta}(n)$ terms are absent. Therefore, the τ-function, representing the compensating terms, becomes by means of (50) and (51)

$$\begin{aligned}
\tau(x-y;n) \ &= \ ig^2\left[\frac{1}{M_V^2}\left(2[\psi\not{n}\psi]\left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b\right] + [\psi\not{n}\psi]^2 + \left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b\right]^2\right)\right. \\
&\qquad \left. + \phi_a^2(n\cdot\phi)^2 + \phi_b^2(n\cdot\phi)^2\right]\delta^4(x-y) \ .
\end{aligned} \tag{52}$$

Its contribution to πN-scattering S-matrix and amplitude is

$$\begin{aligned}
S^{(2)}_{canc} \ &= \ \frac{(-i)^2}{2!}\int d^4x d^4y \ \frac{2ig^2}{M_V^2} \ [\psi\not{n}\psi]\left[\phi_a n\cdot i\overleftrightarrow{\partial}\phi_b)\right]\delta^4(x-y) \ , \\
M_{canc} \ &= \ g^2 \ \bar{u}(p's')\not{n}u(ps)\frac{2n\cdot Q}{M_V^2} \ ,
\end{aligned} \tag{53}$$

which is the same expression as the cancelling amplitude derived from the TU scheme in (45).

4. $\bar{P}$ Approach

From the forgoing subsections (sections IV B 3 and IV B 2) we have seen that if we add all contributions, results in the Feynman formalism and in the Kadyshevsky formalism are the same (of course we need to put $\kappa' = \kappa = 0$). Also, we have seen from (35) and the forgoing subsections that if we bring out the derivatives out of the TOP in Feynman formalism not only do we get Feynman functions, but also the n-dependent contact terms cancel out.

Unfortunately, this is not the case in Kadyshevsky formalism. There, all n-dependent contact terms cancel out after adding up the amplitudes. So, when calculating an amplitude according to the Kadyshevsky rules in appendix A one always has to keep in mind the contributions as described in sections IV B 2 and IV B 3. For practical purposes this is not very convenient.

Inspired by the Feynman procedure we could also do the same in Kadyshevsky formalism, namely let the derivatives not only act on the vector meson propagator [27] but also on the quasi particle propagator (θ-function). In doing so, we know that all contact terms cancel out; just as in Feynman formalism.

We show the above in formula form.

$$\begin{aligned}
& \theta[n(x-y)]\partial^\mu_x\partial^\nu_x\Delta^{(+)}(x-y) + \theta[n(y-x)]\partial^\mu_x\partial^\nu_x\Delta^{(+)}(y-x) \\
= \; & \partial^\mu_x\partial^\nu_x\theta[n(x-y)]\Delta^{(+)}(x-y) + \partial^\mu_x\partial^\nu_x\theta[n(y-x)]\Delta^{(+)}(y-x) \\
& + in^\mu n^\nu \delta^4(x-y) \\
= \; & \frac{i}{2\pi}\int \frac{d\kappa_1}{\kappa_1 + i\varepsilon}\int \frac{d^4P}{(2\pi)^3}\,\theta(P^0)\delta(P^2 - M_V^2)\left(-\bar{P}_\mu\bar{P}_\nu\right) \\
& \times\left(e^{-i\kappa_1 n(x-y)}e^{-iP(x-y)} + e^{i\kappa_1 n(x-y)}e^{iP(x-y)}\right) \\
& + in^\mu n^\nu\delta^4(x-y)\ ,
\end{aligned} \tag{54}$$

where $\bar{P} = P + n\kappa_1$. In this way the second order in the S-matrix becomes

$$\begin{aligned}
S^{(2)}_{fi} = & -g^2\int d^4x d^4y\, [\bar{u}(p's')\gamma_\mu u(ps)]\, (q'+q)_\nu\, e^{-ix(q-q')}e^{iy(p'-p)} \\
& \times\frac{i}{2\pi}\int \frac{d\kappa_1}{\kappa_1 + i\varepsilon}\int \frac{d^4P}{(2\pi)^3}\theta(P^0)\delta(P^2 - M_V^2)\left(-g^{\mu\nu} + \frac{\bar{P}^\mu\bar{P}^\nu}{M_V^2}\right) \\
& \times\left(e^{-i\kappa_1 n(x-y)}e^{-iP(x-y)}e^{in\kappa' x - in\kappa y} + e^{i\kappa_1 n(x-y)}e^{iP(x-y)}e^{-in\kappa' x + in\kappa y}\right) \\
& + ig^2\int d^4x\, [\bar{u}(p's')\not{n} u(ps)]\, n\cdot(q'+q)\, e^{-ix(q-q'-p'+p-n\kappa'+n\kappa)}\ .
\end{aligned} \tag{55}$$

We see that the second term on the rhs of (55) brings about an amplitude, which is exactly the same as in (34) and (35) and is to be cancelled by (45) and (53).

Performing the various integrals correctly we get

$$\begin{aligned}
(a) \;\Rightarrow & \left\{\begin{array}{lcl} \kappa_1 & = & \Delta_t\cdot n - A_t + \frac{1}{2}(\kappa'+\kappa)\,n \\ \bar{P} & = & \Delta_t + \frac{1}{2}(\kappa'+\kappa)\,n \end{array}\right. \\
(b) \;\Rightarrow & \left\{\begin{array}{lcl} \kappa_1 & = & -\Delta_t\cdot n - A_t + \frac{1}{2}(\kappa'+\kappa)\,n \\ \bar{P} & = & -\Delta_t + \frac{1}{2}(\kappa'+\kappa)\,n \end{array}\right. .
\end{aligned} \tag{56}$$

This yields for the invariant amplitudes

$$
\begin{aligned}
M^{(a)}_{\kappa'\kappa} &= -g^2\ \bar{u}(p's')\left[2\not{Q}+\frac{1}{M_V^2}\left((M_f-M_i)+\frac{1}{2}(\kappa'-\kappa)\not{n}+\not{n}\bar{\kappa}\right)\right.\\
&\quad\times\left.\left(\left(m_f^2-m_i^2\right)+\frac{1}{4}\left(s_{pq}-s_{p'q'}+u_{p'q}-u_{pq'}\right)+2\bar{\kappa}Q\cdot n\right)\right]u(ps)\\
&\quad\times\frac{1}{2A_t}\ \frac{1}{\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}\ ,\\
M^{(b)}_{\kappa'\kappa} &= -g^2\ \bar{u}(p's')\left[2\not{Q}+\frac{1}{M_V^2}\left((M_f-M_i)+\frac{1}{2}(\kappa'-\kappa)\not{n}-\not{n}\bar{\kappa}\right)\right.\\
&\quad\times\left.\left(\left(m_f^2-m_i^2\right)+\frac{1}{4}\left(s_{pq}-s_{p'q'}+u_{p'q}-u_{pq'}\right)-2\bar{\kappa}Q\cdot n\right)\right]u(ps)\\
&\quad\times\frac{1}{2A_t}\ \frac{1}{-\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}\\
M &= M^{(a)}_{00}+M^{(b)}_{00}\\
&= -g^2\bar{u}(p's')\left[2\not{Q}+\frac{(M_f-M_i)}{M_V^2}\left(m_f^2-m_i^2\right)\right]u(ps)\frac{1}{t-M_V^2+i\varepsilon}\ , \qquad (57)
\end{aligned}
$$

where $\bar{\kappa}=\frac{1}{2}(\kappa'+\kappa)$. As before we get back the Feynman expression for the amplitude if we add both amplitudes obtained in Kadyshevsky formalism and put $\kappa'=\kappa=0$. The big advantage of this procedure is that we do not need to worry about the contribution n-dependent contact terms because they cancelled out when introducing $\bar{P}$.

It should be noticed however that the $\bar{P}$-method is only possible when both Kadyshevsky contributions at second order are added. This becomes clear when looking at the first two lines of (54): Letting the derivatives also act on the θ-function gives compensating terms for the $\Delta^{(+)}(x-y)$-part and for the $\Delta^{(-)}(x-y)$-part. Only when added together they combine to the $\delta^4(x-y)$-part.

Also it becomes clear from (54) that at least two derivatives are needed to generate the $\delta^4(x-y)$-part. Therefore, when there's only one derivative, for instance in the case of baryon exchange (so, no derivatives in coupling only in the propagator) at second order, the $\delta^4(x-y)$-part is not present and it is not necessary to use the $\bar{P}$-method. In these cases it doesn't matter for the summed diagrams whether or not the $\bar{P}$-method is used, however for the individual diagrams it does make a difference. This ambiguity is absent in Feynman theory, there derivatives are always taken out of the TOP (which is similar to the $\bar{P}$-method, as discussed above) in order to come to Feynman propagators.

In the forgoing we have demonstrated the $\bar{P}$-method for simplified vector meson exchange and strictly speaking for $\kappa'=\kappa=0$. We stress, however, that this method is generally applicable, i.e. for $\kappa',\kappa\neq 0$ and for general couplings containing multiple derivatives and/or higher spin fields.

C. Real Vector Meson Exchange

Now that we have discussed how to deal with multiple derivatives and/or higher spin fields in the Kadyshevsky formalism by means of the simplified vector meson exchange example, we're prepared to deal with real vector meson exchange. In order to do so we use the interaction Lagrangians as in (19b) and (20b). From these interaction Lagrangians we distillate the already exposed vertex function in (31) (second line) and

$$\Gamma^{\mu}_{VNN} = g_V\,\gamma^{\mu} + \frac{f_V}{2M_V}\,(p'-p)_{\alpha}\,\sigma^{\alpha\mu}\ . \tag{58}$$

The Kadyshevsky diagrams representing vector meson exchange are already exposed in figure 4. Applying the Kadyshevsky rules of appendix A and the $\bar{P}$ method described in section IV B 4 we obtain the following amplitudes

$$\begin{aligned}
M^{(a)}_{\kappa'\kappa} &= -g_{VPP}\,\bar{u}(p's')\Bigg[2g_V\not{Q} \\
&\quad -\frac{g_V}{M_V^2}\,\kappa'\not{n}\left(\frac{1}{4}\,(s_{p'q'}-s_{pq}+u_{pq'}-u_{p'q})+2\bar{\kappa}Q\cdot n\right) \\
&\quad +\frac{f_V}{2M_V}\Bigg(4M\not{Q}+\frac{1}{2}\,(u_{pq'}+u_{p'q})-\frac{1}{2}\,(s_{p'q'}+s_{pq}) \\
&\qquad -\frac{1}{M_V^2}\Bigg(M^2+m^2-\frac{1}{2}\left(\frac{1}{2}(t_{p'p}+t_{q'q})+u_{pq'}+s_{pq}\right. \\
&\qquad\qquad \left.+2M\not{n}\kappa'+\frac{1}{4}\,(\kappa'-\kappa)^2-(p'+p)\cdot n\bar{\kappa}\right) \\
&\qquad\quad \times\left(\frac{1}{4}(s_{p'q'}-s_{pq})+\frac{1}{4}(u_{pq'}-u_{p'q})+2\bar{\kappa}n\cdot Q\right)\Bigg)\Bigg)\Bigg]\,u(ps) \\
&\quad \times\frac{1}{2A_t}\ \frac{1}{\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}\ , \\
M^{(b)}_{\kappa'\kappa} &= -g_{VPP}\,\bar{u}(p's')\Bigg[2g_V\not{Q} \\
&\quad +\frac{g_V}{M_V^2}\,\kappa\not{n}\left(\frac{1}{4}\,(s_{p'q'}-s_{pq}+u_{pq'}-u_{p'q})-2\bar{\kappa}Q\cdot n\right) \\
&\quad +\frac{f_V}{2M_V}\Bigg(4M\not{Q}+\frac{1}{2}\,(u_{pq'}+u_{p'q})-\frac{1}{2}\,(s_{p'q'}+s_{pq}) \\
&\qquad -\frac{1}{M_V^2}\Bigg(M^2+m^2-\frac{1}{2}\left(\frac{1}{2}(t_{p'p}+t_{q'q})+u_{pq'}+s_{pq}\right. \\
&\qquad\qquad \left.-2M\not{n}\kappa+\frac{1}{4}\,(\kappa'-\kappa)^2+(p'+p)\cdot n\bar{\kappa}\right) \\
&\qquad\quad \times\left(\frac{1}{4}(s_{p'q'}-s_{pq})+\frac{1}{4}(u_{pq'}-u_{p'q})-2\bar{\kappa}n\cdot Q\right)\Bigg)\Bigg)\Bigg]\,u(ps) \\
&\quad \times\frac{1}{2A_t}\ \frac{1}{-\Delta_t\cdot n+\bar{\kappa}-A_t+i\varepsilon}\ .
\end{aligned} \tag{59}$$

The sum of the two in the limit of $\kappa'=\kappa=0$ yields

$$\begin{aligned}
M_{00} &= -g_{VPP}\ \bar{u}(p's')\left[2g_V\not{Q}+\frac{f_V}{2M_V}\left((u-s)+4M\not{Q}\right)\right]u(ps) \\
&\qquad \times\frac{1}{t-M_V^2+i\varepsilon}\ ,
\end{aligned} \tag{60}$$

which is, again, the Feynman result [3].

Just as in section IV A we study the n-dependence of the amplitude. This, in light of the n-dependence of the Kadyshevsky integral equation (see section II A).

$$
\begin{aligned}
M_{0\kappa}^{(a+b)} &= M_{0\kappa}^{(a)} + M_{0\kappa}^{(b)}\,, \\
&= -g_{VPP}\;\bar{u}(ps)\left[2g_V\not{Q} + \frac{f_V}{2M_V}\left(4M\not{Q} + \frac{1}{2}\left(u_{pq'} + u_{p'q}\right)\right.\right. \\
&\qquad \left.\left. -\frac{1}{2}\left(s_{p'q'} + s_{pq}\right)\right)\right] u(ps)\frac{A_t - \frac{\kappa}{2}}{A_t}\frac{1}{(\Delta_t\cdot n)^2 - \left(A_t - \frac{\kappa}{2}\right)^2 + i\varepsilon} \\
&\quad -\frac{g_V f_V \kappa}{2M_V^3}\;\bar{u}(ps)\left[\frac{1}{2}(p'+p)\cdot n(Q\cdot n)\kappa\left(A_t - \frac{\kappa}{2}\right)\right. \\
&\qquad +\frac{1}{8}(p'+p)\cdot n\left(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q}\right)\Delta_t\cdot n \\
&\qquad -n\cdot Q\left(M^2 + m^2 - \frac{1}{2}\left(\frac{1}{2}(t_{p'p} + t_{q'q}) + u_{pq'} + s_{pq}\right) + \frac{\kappa^2}{4}\right) \\
&\qquad \left.\times\Delta_t\cdot n\right] u(ps)\,\frac{1}{A_t}\,\frac{1}{(\Delta_t\cdot n)^2 - (\frac{\kappa}{2} - A_t)^2 + i\varepsilon} \\
&\quad +\frac{g_{VPP}\kappa}{M_V^2}\;\bar{u}(p's')\left[\not{n}\left(g_V + \frac{f_V M}{M_V}\right)\left(\frac{1}{4}(s_{p'q'} - s_{pq} + u_{pq'} - u_{p'q})\right.\right. \\
&\qquad \left.\left. + \kappa n\cdot Q\right)\right] u(ps)\,\frac{1}{2A_t}\;\frac{1}{\Delta_t\cdot n + \frac{\kappa}{2} - A_t + i\varepsilon}\;. \qquad (61)
\end{aligned}
$$

Differentiating this with respect to n^α in the same way as in (28) we know that the result will contain an overall factor of κ. This can be seen as follows: The first term in (61) is very similar to $M_{0\kappa}^{(a+b)}$ in (28). Therefore, the overall factor of κ when differentiating with respect to n^α is obvious. All other terms in (61) contain already an overall factor of κ, which doesn't change when differentiating.

As can be seen from (61) the numerator is of higher degree in κ then the denominator. Therefore, the function $h(\kappa)$ in (16) will not be of order $O(\frac{1}{\kappa^2})$ and the "form factor" (18) is necessary.

In (59) as well as in (26) we have taken u and $\bar{u}$ spinors. The reason behind this is pair suppression which we will discuss in paper II.

APPENDIX A: KADYSHEVSKY RULES

Just as in Feynman theory Kadyshevsky amplitudes can be represented by Kadyshevsky diagrams. Since the basic starting points are the same as in Feynman theory we take a general Feynman diagram and give the Kadyshevsky rules from there on to construct the amplitude M_{fi}. Here, we define the amplitude as

$$
S_{fi} = \delta_{fi} - i(2\pi)^2\delta^4\left(P_f - P_i\right)\,M_{fi}\;, \qquad \text{(A1)}
$$

where $P_{f/i}$ is the sum of the final/initial momenta.

Kadyshevsky Rules:

1) Arbitrarily number the vertices of the diagram.

2) Connect the vertices with a quasi particle line, assigned to it a momentum $n\kappa_s$ ($s = 1 \ldots n-1$). Attach to vertex 1 an incoming initial quasi particle with momentum $n\kappa$ and attach to vertex n an outgoing final quasi particle with momentum $n\kappa'$ [28].

3) Orient each internal momentum such that it leaves a vertex with a lower number than the vertex it enters. If 2 fermion lines with opposite momentum direction come together in one vertex assign a $+$ symbol to one line and a $-$ to the other. Each possibility to do this yields a different Kadyshevsky diagram.

4) Assign to each internal quasi particle line a propagator $\frac{1}{\kappa_s + i\varepsilon}$.

5) Assign to all other internal lines the appropriate Wightman function of (A2). Assign to a fermion line with a $\pm$ symbol: $S^{(\pm)}(P)$ (see **3)**)

$$
\begin{aligned}
\Delta^{(+)}(P) &= \theta(P^0)\delta(P^2 - M^2) \ , \\
S^{(\pm)}(P) &= \Lambda^{(1/2)}(\pm P)\ \theta(P^0)\delta(P^2 - M^2) \ , \\
\Delta^{(+)}_{\mu\nu}(P) &= \Lambda^{(1)}_{\mu\nu}(P)\ \theta(P^0)\delta(P^2 - M^2) \ , \\
S^{(\pm)}_{\mu\nu}(P) &= \Lambda^{(3/2)}_{\mu\nu}(\pm P)\ \theta(P^0)\delta(P^2 - M^2) \ ,
\end{aligned} \tag{A2}
$$

where

$$
\begin{aligned}
\Lambda^{(1/2)}_{\mu\nu}(P) &= (\not{P} + M) \ , \\
\Lambda^{(1)}_{\mu\nu}(P) &= \left(-g_{\mu\nu} + \frac{P_\mu P_\nu}{M^2}\right) \ , \\
\Lambda^{(3/2)}_{\mu\nu}(P) &= -(\not{P} + M)\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2P_\mu P_\nu}{3M^2}\right. \\
&\qquad \left. + \frac{1}{3M}\left(P_\mu\gamma_\nu - \gamma_\mu P_\nu\right)\right) \ .
\end{aligned} \tag{A3}
$$

6) There's momentum conservation at the vertices, including the quasi particle momenta.

7) Integrate over the internal quasi momenta: $\int_{-\infty}^{\infty} d\kappa_s$.

8) Integrate over those internal momenta not fixed by momentum conservation at the vertices: $\int_{-\infty}^{\infty} \frac{d^4P}{(2\pi)^3}$.

9) Include a $-$ sign for every fermion loop.

10) Include a $-$ sign for identical initial or final fermions.

11) Repeat the various steps for all different numberings in **1**.

It is clear from **3)** and **11)** that one Feynman diagram leads to several Kadyshevsky diagrams. Generally, one Feynman diagram leads to $n!$ Kadyshevsky diagrams, where n is the number of vertices (or; the order). Especially for higher order diagrams this leads to a dramatic increase of labour. Fortunately, we will only consider second order diagrams.

A few remarks need to be made about these rules as far as the choice of definition is concerned. In **3)** we have followed [5] to orient the internal momenta. Furthermore we have chosen to use the integral representation of the θ-function

$$\theta[n\cdot(x-y)] \;=\; \frac{i}{2\pi}\int d\kappa_1 \frac{e^{-i\kappa_1 n\cdot(x-y)}}{\kappa_1+i\varepsilon}\;, \tag{A4}$$

instead of its complex conjugate. Since the θ-function is real, this is also a proper representation, originally used in the papers of Kadyshevsky. To understand why we have chosen to deviate from the original approach, consider the S-matrix

$$S \;=\; 1+\sum_{n=1}^{\infty}(-i)^n\int_{-\infty}^{\infty} d^4x_1\ldots d^4x_n\;\theta[n(x_1-x_2)]\ldots\theta[n(x_{n-1}-x_n)] \\ \times\,\mathcal{H}_I(x_1)\ldots\mathcal{H}_I(x_n)\;. \tag{A5}$$

In each order S_n there is a factor $(-i)^n$ already in the definition. In that specific order there are $(n-1)$ θ-functions, each containing a factor i from the integral representation (A4). Therefore, every S_n will, regardless the order, contain a factor $(-i)$. Hence, the amplitude M_{fi}, defined in (A1), will not contain overall factors of i, anymore.

The momentum space $S^{(-)}(P)$-functions differ an overall minus sign by their coordinate space analogs $\langle 0|\bar\psi(x)\psi(y)|0\rangle = S^{(-)}(x-y)$. The reason for that is twofold. In many cases the Wightman functions $S^{(-)}(x-y)$, including the overall minus sign, appear in combination with the Normal Ordered Product (NOP): $N(\psi\bar\psi) = -N(\bar\psi\psi)$. Therefore, the minus signs cancel. In all other cases the Wightman functions $S^{(-)}(x-y)$ appear in fermion loops and are therefore responsible for the fermion loop minus sign in **9)**, since every fermion loop will contain an odd number of $S^{(-)}(x-y)$ functions. We stress that this method of defining the Kadyshevsky rules for fermions differs from the original one in [7].

APPENDIX B: SECOND QUANTIZATION

When discussing the Kadyshevsky rules in subsection A and the Kadyshevsky integral equation in (10) we allowed for quasi particles to occur in the initial and final state. In order to do this properly a new theory needs to be set up containing quasi particle creation and annihilation operators. It is set up in such a way that external quasi particles occur in the S-matrix as trivial exponentials so that when the external quasi momenta are taken to be zero the Feynman expression is obtained. We, therefore, require that the vacuum expectation value of the quasi particles is the θ-function

$$<0|\chi(nx)\bar\chi(nx')|0> = \theta[n(x-x')]\;, \tag{B1}$$

and that a quasi field operator acting on a state with quasi momentum $(n)\kappa$ only yields a trivial exponential

$$\begin{aligned}\chi(nx)|\kappa> \;&=\; e^{-i\kappa nx}\;,\\ <\kappa|\bar\chi(nx) \;&=\; e^{i\kappa nx}\;.\end{aligned} \tag{B2}$$

Assuming that a state with quasi momentum $(n)\kappa$ is created in the usual way

$$
\begin{aligned}
a^\dagger(\kappa)|0> &= |\kappa> \ , \\
<0|a(\kappa) &= <\kappa| \ ,
\end{aligned}
\tag{B3}
$$

we have from the requirements (B1) and (B2) the following momentum expansion of the fields

$$
\begin{aligned}
\chi(nx) &= \frac{i}{2\pi}\int \frac{d\kappa}{\kappa+i\varepsilon}\ e^{-i\kappa nx} a(\kappa) \ , \\
\bar{\chi}(nx') &= \frac{i}{2\pi}\int \frac{d\kappa}{\kappa+i\varepsilon}\ e^{i\kappa nx'} a^\dagger(\kappa) \ ,
\end{aligned}
\tag{B4}
$$

and the fundamental commutation relation of the creation and annihilation operators

$$
\left[a(\kappa), a^\dagger(\kappa')\right] = -i2\pi\kappa\delta(\kappa-\kappa') \ . \tag{B5}
$$

From this commutator (B5) it is clear that the quasi particle is not a physical particle nor a ghost.

Now that we have set up the second quantization for the quasi particles we need to include them in the S-matrix. This is done by redefining it

$$
S \ = \ 1 + \sum_{n=1}(-i)^n \int d^4x_1 \ldots d^4x_n \tilde{\mathcal{H}}_I(x_1)\ldots\tilde{\mathcal{H}}_I(x_n) \ , \tag{B6}
$$

where

$$
\tilde{\mathcal{H}}_I(x) \ \equiv \ \mathcal{H}_I(x)\bar{\chi}(nx)\chi(nx) \ . \tag{B7}
$$

In this sense contraction of the quasi fields causes propagation of this field between vertices, just as in the Feynman formalism. Those quasi particles that are not contracted are used to annihilate external quasi particles form the vacuum.

$$
\begin{aligned}
& S^{(2)}(p's'q'n\kappa'; psqn\kappa) = \\
&= (-i)^2 \int d^4x_1 d^4x_2 <\pi N\chi|\tilde{\mathcal{H}}_I(x_1)\tilde{\mathcal{H}}_I(x_2)|\pi N\chi> \\
&= (-i)^2 \int d^4x_1 d^4x_2 <0|b(p's')a(q')a(\kappa') \\
&\quad \times \left[\bar{\chi}(nx_1)\mathcal{H}_I(x_1)\chi(nx_1)\bar{\chi}(nx_2)\mathcal{H}_I(x_2)\chi(nx_2)\right] a^\dagger(\kappa)a^\dagger(q)b^\dagger(ps)|0> \\
&= (-i)^2 \int d^4x_1 d^4x_2 \ e^{in\kappa' x_1} e^{-in\kappa x_2} \\
&\quad \times <0|b(p's')a(q')\mathcal{H}_I(x_1)\theta[n(x_1-x_2)]\mathcal{H}_I(x_2)a^\dagger(q)b^\dagger(ps)|0> \ .
\end{aligned}
\tag{B8}
$$

For the π and N fields we use the well-known momentum expansion

$$
\begin{aligned}
\phi(x) &= \int \frac{d^3l}{(2\pi)^3 2E_l}\left[a(l)e^{-ilx} + a^\dagger(l)e^{ilx}\right] \ , \\
\psi(x) &= \sum_r \int \frac{d^3k}{(2\pi)^3 2E_k}\left[b(k,r)u(k,r)e^{-ikx} + d^\dagger(k,r)v(k,r)e^{ikx}\right] \ ,
\end{aligned}
\tag{B9}
$$

where the creation and annihilation operators satisfy the following (anti-) commutation relations

$$
\begin{aligned}
[a(k), a^\dagger(l)] &= (2\pi)^3\, 2E_k\, \delta^3(k-l) \;, \\
\{b(k,s), b^\dagger(l,r)\} &= (2\pi)^3\, 2E_k\, \delta_{sr}\delta^3(k-l) = \{d(k,s), d^\dagger(l,r)\} \;.
\end{aligned} \tag{B10}
$$

Putting $\kappa' = \kappa = 0$ in (B8) we see that we get the second order in the S-matrix expansion for πN-scattering as in Feynman formalism. Of course this is what we required from the beginning: external quasi particle momenta only occur in the S-matrix as exponentials.

So, we know now how to include the external quasi particles in the S-matrix and therefore we also know what their effect is on amplitudes. For practical purposes we will not use the S-matrix as in (B6), but keep the above in mind. In those cases where the (possible) inclusion of external quasi fields is less trivial we will make some comments.

APPENDIX C: BMP THEORY

According to Haag's theorem [17] in general there does not exist a unitary transformation which relates the fields in the I.R. and the fields in the H.R. On the other hand there is no objection against the existence of an unitary $U[\sigma]$ relating the TU-auxiliary fields and the fields in the I.R.

$$
\Phi_\alpha(x,\sigma) = U^{-1}[\sigma]\; \Phi_\alpha(x)\; U[\sigma] \;. \tag{C1}
$$

Here, we follow the framework of Bogoliubov and collaborators [14–16], to which we refer to as the BMP theory, to prove (C1) in a straightforward way (see appendix C 2).

The BMP theory was originally constructed to bypass the use of an unitary operator U as a mediator between the fields in the H.R. and in the I.R.

1. Set-up

In the description of the BMP theory we will only consider scalar fields. By the assumption of asymptotic completeness the S-matrix is taken to be a functional of the asymptotic fields $\phi_{as,\rho}(x)$, where $as = in, out$. In the following we use *in*-fields, i.e. $\phi_\rho(x) = \phi_{in,\rho}(x)$

$$
\begin{aligned}
S \;=\; 1 + \sum_{n=1}^{\infty} \int d^4x_1 \ldots d^4x_n\; S_n(x_1\alpha_1, \ldots, x_n\alpha_n) \cdot \\
\times :\phi_{\alpha_1}(x_1)\ldots\phi_{\alpha_n}(x_n): \;.
\end{aligned} \tag{C2}
$$

Here, concepts like unitarity and the stability of the vacuum, i.e. $\langle 0|S|0\rangle = 1$, and the 1-particle states, i.e. $\langle 0|S|1\rangle = 0$ are assumed. The *Heisenberg current*, i.e. the current in the H.R., is defined as [29]

$$
\mathbf{J}_\rho(x) = -iS^\dagger\; \frac{\delta S}{\delta\phi_\rho(x)} \;. \tag{C3}
$$

We note that for a hermitean field $\phi_\rho(x)$ the current is also hermitean, due to unitarity. *Microcausality* takes the form, see [15], section 17 [30],

$$
\frac{\delta \mathbf{J}_\rho(x)}{\delta\phi_\lambda(y)} = 0 \quad , \quad \text{for} \;\; x \le y \;. \tag{C4}
$$

It can be shown that the notion of microcausality is reflected in the expression of the S-matrix as the Time-Ordered exponential. See [15] for the details on this point of view. It can also be shown that with the current (C3) the asymptotic fields $\phi_{in/out,\rho}(x)$ satisfy a YF type of equation (as in (C11))

$$\boldsymbol{\phi}_\rho(x) \;=\; \phi_{in/put,\rho}(x) + \int d^4y\, \Delta_{ret/adv}(x-y)\, \mathbf{J}_\rho(y)\ , \tag{C5}$$

giving the Heisenberg fields $\boldsymbol{\phi}_\rho(x)$ in terms of the $\phi_{in/out}(x)$fields.

Lehmann, Symanzik, and Zimmermann (LSZ) [21] formulated an asymptotic condition utilizing the notion of weak convergence in the Hilbert space of state vectors. See e.g. [22] for an detailed exposition of the LSZ-formalism. The correspondence of BMP theory with LSZ is obtained by the identification

$$\mathbf{J}_\rho(x) = -iS^\dagger\ \frac{\delta S}{\delta\phi_\rho(x)} \equiv \left(\Box + m^2\right)\ \boldsymbol{\phi}_\rho(x)\ . \tag{C6}$$

As is explained in for instance [16], the local commutivity of the currents follows from microcausality (C4). Using the YF equations one can show that for space-like separations the fields in the H.R. commute with the currents and among themselves, as was assumed in the LSZ-formalism. For more details and results of BMP see [14–16].

2. Application to Takahashi-Umezawa scheme

In this subsection we introduce the auxiliary field similar to (38)

$$\phi(x,\sigma) \equiv \phi(x) - \int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ \mathbf{J}(x')\ , \tag{C7}$$

and prove that $\phi(x)$ and $\phi(x,\sigma)$ satisfy the same (usual) commutation relations. Such a relation justifies the existence of an unitary operator connecting the two as in (C1).

The difference of the commutation relations is, using (C7),

$$\begin{aligned}
&\Big[\phi(x,\sigma),\phi(y,\sigma)\Big] - \Big[\phi(x),\phi(y)\Big] \\
&= -\int_{-\infty}^{\sigma} d^4y'\ \Delta(y-y')\ \Big[\phi(x),\mathbf{J}(y')\Big] + \int_{-\infty}^{\sigma} d^4x'\ \Delta(x-x')\ \Big[\phi(y),\mathbf{J}(x')\Big] \\
&\quad + \int_{-\infty}^{\sigma}\int_{-\infty}^{\sigma} d^4x' d^4y'\ \Delta(x-x')\Delta(y-y')\ \Big[\mathbf{J}(x'),\mathbf{J}(y')\Big]\ .
\end{aligned} \tag{C8}$$

Since the S-operator is an expansion in asymptotic fields, so is $\mathbf{J}(x)$ by means of its definition in terms of this S-operator (C3). Now, from the commutation relations of the asymptotic fields one has

$$\Big[\phi_\rho(x),\mathbf{J}_\sigma(y)\Big] = i\int d^4x'\ \Delta(x-x')\frac{\delta\mathbf{J}_\sigma(y)}{\delta\phi_\rho(x')}\ . \tag{C9}$$

Using this in (C8) we have

$$\begin{aligned}
&\Big[\phi(x,\sigma),\phi(y,\sigma)\Big] - \Big[\phi(x),\phi(y)\Big] \\
=& -i\int_{-\infty}^{\sigma} d^4y' \int_{-\infty}^{\infty} d^4x' \Delta(x-x')\Delta(y-y')\frac{\delta \mathbf{J}(y')}{\delta\phi(x')} \\
&+i\int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\infty} d^4y'\ \Delta(x-x')\Delta(y-y')\frac{\delta \mathbf{J}(x')}{\delta\phi(y')} \\
&-i\int_{-\infty}^{\sigma} d^4x' \int_{-\infty}^{\sigma} d^4y'\ \Delta(x-x')\Delta(y-y')\left(\frac{\delta \mathbf{J}(x')}{\delta\phi(y')}-\frac{\delta \mathbf{J}(y')}{\delta\phi(x')}\right) \\
=&\ 0\ . \qquad\qquad (C10)
\end{aligned}$$

Cancellation takes place in (C10) when the second integral of the first two term on the rhs in (C10) is split up: $\int_{-\infty}^{\infty} = \int_{-\infty}^{\sigma} + \int_{\sigma}^{\infty}$. The remaining terms are zero because of the microcausility condition (C4). Although we shown the proof for scalar fields only, the generalization to other types of fields is easily made.

Complementary to what is in [10–12] we explicitly show that the unitary operator in (C1) is not any operator but the one connected to the S-matrix. We, therefore, consider (general) *in*- and *out*-fields. Their relation to the fields in the H.R. is

$$\begin{aligned}
\mathbf{\Phi}_\alpha(x) &= \Phi_{in,\alpha}(x) + \int d^4y\ R_{\alpha\beta}(\partial)\ \Delta_{ret}(x-y)\ \mathbf{J}_\beta(y) \\
&= \Phi_{out,\alpha}(x) + \int d^4y\ R_{\alpha\beta}(\partial)\ \Delta_{adv}(x-y)\ \mathbf{J}_\beta(y)\ , \qquad (C11)
\end{aligned}$$

where $\Delta_{ret}(x-y) = -\theta(x^0-y^0)\Delta(x-y)$ and $\Delta_{adv}(x-y) = \theta(y^0-x^0)\Delta(x-y)$.

Equation (C11) makes clear that the choice of the Green function determines the choice of the free field (*in*- or *out*-field) to be used. In this light we make the following identification: $\Phi_\alpha(x,-\infty) \equiv \Phi_{in,\alpha}(x)$, since we have used the retarded Green function in section IV B 2 (text below (36)). With (C11) we can also relate the *out*-field to the auxiliary field $\Phi_\alpha(x,\infty) = \Phi_{out,\alpha}(x)$.

Using these identifications in (C1) we obtain the relation between $\Phi_{\alpha,in}(x)$ and $\Phi_{\alpha,out}(x)$

$$\begin{aligned}
\Phi_{\alpha,in}(x) &= U^{-1}[-\infty]U[\infty]\ \Phi_{\alpha,out}(x)\ U^{-1}[\infty]U[-\infty] \\
\Phi_{in,\alpha}(x) &= S\Phi_{out,\alpha}S^{-1}\ . \qquad (C12)
\end{aligned}$$

Obviously, the operator connecting the *in*- and *out*-fields is the S-matrix, where the relation between $U[\sigma]$ and the S-matrix is

$$\begin{aligned}
U[\sigma] &= T\left[exp\left(-i\int_{-\infty}^{\sigma} d^4x \mathcal{H}_I(x)\right)\right]\ , \\
U[\infty] &= S\ , \qquad U[-\infty] = 1\ . \qquad (C13)
\end{aligned}$$

To make contact with the interaction Hamiltonian we follow [10–12] for completion by realizing that the unitary operator satisfies the Tomonaga-Schwinger equation

$$i\frac{\delta U[\sigma]}{\delta\sigma(x)} = \mathcal{H}_I(x;n)U[\sigma]\Big|_{x/\sigma} = U[\sigma]\ \mathcal{H}_I(x/\sigma;n)\ . \qquad (C14)$$

Here, the interaction Hamiltonian will in general depend on the vector $n_\mu(x)$ locally normal to the surface $\sigma(x)$, i.e. $n^\mu(x)d\sigma_\mu = 0$. It is hermitean because of the unitarity of $U[\sigma]$. Then, from (C1) and (C14) one gets that

$$i\frac{\delta\Phi_\alpha(x,\sigma)}{\delta\sigma(y)} = U^{-1}[\sigma]\left[\Phi_\alpha(x), \mathcal{H}_I(y;n)\right]\ U[\sigma]\ . \tag{C15}$$

On the other hand, varying (38) with respect to $\sigma(y)$ gives

$$i\frac{\delta\Phi_\alpha(x,\sigma)}{\delta\sigma(y)} = i\ D_a(y)\ R_{\alpha\beta}(\partial)\ \Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y)\ . \tag{C16}$$

Comparing (C15) and (C16) gives the relation

$$\left[\Phi_\alpha(x), \mathcal{H}_I(y;n)\right]\ =\ i\ U[\sigma]\left[D_a(y)\ R_{\alpha\beta}(\partial)\ \Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y)\right]U^{-1}[\sigma]\ . \tag{C17}$$

This is the fundamental equation by which the interaction Hamiltonian must be determined.

APPENDIX D: REMARKS ON THE HAAG THEOREM

Here, we take a closer look at the connection between the fields in the H.R. and in the I.R. in the covariant formulation of Tomonaga and Schwinger [23, 24]

$$\mathbf{\Phi}_\alpha(x)\ =\ U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]\ , \tag{D1}$$

This in light of the Haag theorem [17], which states that if there is an unitary operator connecting fields in two representations at some time (as in (D1)), where the field in one representation is free, both fields are free. This would lead to a triviality.

The question is whether this situation (D1) is applicable to our case. In order to answer that question we look at the results of the previous subsection (appendix C). By introducing the auxiliary field in the scalar case as in (38) (or for general fields as in (C7)) we proved (C1) using BMP theory.

Now, we start with (36) and use similar arguments to come to

$$\begin{aligned}
\mathbf{\Phi}_\alpha(x)\ &=\ \Phi_\alpha(x) + \int_{-\infty}^{\infty} d^4y\ D_a(y)\ R_{\alpha\beta}(\partial)\ \theta[n(x-y)]\Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y)\\
&=\ \Phi_\alpha(x) + \int_{-\infty}^{\infty} d^4y\ \theta[n(x-y)]\ D_a(y)\ R_{\alpha\beta}(\partial)\ \Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y)\\
&\quad + \int_{-\infty}^{\infty} d^4y\left[D_a(y)\ R_{\alpha\beta}(\partial), \theta[n(x-y)]\right]\Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y)\ ,\\
\Rightarrow \mathbf{\Phi}_\alpha(x)\ &=\ U^{-1}[\sigma]\ \Phi_\alpha(x)\ U[\sigma]|_{x/\sigma}\\
&\quad + \frac{1}{2}\int_{-\infty}^{\infty} d^4y\left[D_a(y)\ R_{\alpha\beta}(\partial), \epsilon(x-y)\right]\Delta(x-y)\cdot\mathbf{j}_{\beta;a}(y)\ .
\end{aligned} \tag{D2}$$

[31] The above is different from what is exposed in [22] (ch 17.2). The difference is the commutator part in (D2) and this term is non-zero for theories with couplings containing

derivatives and higher spin fields, carefully excluded in the treatment of [22]. Therefore (D2) could be seen as an extension of what is written in [22].

Returning to Haag's theorem we see that if the last term in (D2) is absent there is an unitary operator connecting $\mathbf{\Phi}_\alpha(x)$ and $\Phi_\alpha(x)$ and therefore they are both free fields. Such theories can then be considered as trivial, although they can still be useful as effective theories.

In our application we use various interaction Lagrangians (for the overview see section III) to be used in order to describe the various exchange (and resonance (paper II)) processes. Whether or not the non-vanishing commutator part in (D2) is present depends on the process under consideration. In the vector meson exchange diagrams (section IV C) and in the spin-3/2 exchange and resonance diagrams (paper II) those commutator parts are non-vanishing. If we include pair suppression in the way we do in paper II also in the spin-1/2 exchange and resonance diagrams the commutator parts will be non-vanishing. So, if we take the model as a whole (all diagrams) then it is most certainly not trivial in the sense of the Haag theorem.

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[25] By frame dependent we mean: dependent on a vector n^μ.
[26] If we include the n^μ-vector in the θ-function of the TOP, which would not make a difference, then we can make the replacement $\delta_0^\mu \to n^\mu$. This, to make the result more general.
[27] With 'propagator' we mean the $\Delta^+(x-y)$ and not the Feynman propagator $\Delta_F(x-y)$.
[28] Obviously these quasi particle may not appear as initial or final states, since they are not physical particles. However, since we use Kadyshevsky diagrams as input for an integral equation we allow for external quasi particles.
[29] Note that in [16] the out-field is used. Then

$$\mathbf{J}_\rho(x) = i\frac{\delta S}{\delta\phi_\rho(x)}S^\dagger \ .$$

Also, we take a minus sign in the definition of the current.

[30] Here $x \leq y$ means either $(x-y)^2 \geq 0$ and $x^0 < y^0$ or $(x-y)^2 < 0$. So, the point x is in the past of or is spacelike separated from the point y.

[31] We have included the n^μ-vector in the first line of (D2), which causes no effect. The reason for this inclusion is that we can keep the surface σ general, though space-like.