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Polarization in hyperon-nucleon scattering

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I. INTRODUCTION

Recently a polarization experiment has been done with the elastic reactions [1]:

$$\begin{aligned}\Sigma^- + p &\rightarrow \Lambda + n \\ &\rightarrow \Sigma^0 + n\end{aligned}$$

involving a low energetic polarized Σ^--beam incident on an unpolarized proton target. Polarization phenomena of nucleon-nucleon scattering have been discussed thoroughly in the literature [2]. But the polarization phenomena in scattering of two non-identical spin-$\frac{1}{2}$ baryons, e.g. hyperon-nucleon scattering, have not been discussed in detail yet. This is in particular the case for inelastic reactions. Gardner and Welton [3] consider the polarization of the Λ in elastic Λ-proton scattering. However, they restrict themselves to the case of incoming and outgoing S-waves only, and so neglect also $^3S_1 \rightarrow {}^3D_1$ transitions. Downs and Schrils [4] study the possible differences between the polarizations of the scattered particle and the recoil target, due to singlet-triplet transitions, which are absent in nucleon-nucleon scattering (up to electromagnetic corrections) because of the Pauli-principle. Their arguments are only valid for elastic scattering with no initial polarizations. Deloff [5] reviews the applications of these results to ΛN elastic scattering.

In this report we present the polarization formulae for the scattering of two spin-$\frac{1}{2}$ particles in the case of a polarized beam and an unpolarized target. These formulae are expressed in terms of the scattering matrix, which is written on the singlet-triplet basis in spin-space.

We specialize those formulae for low energy scattering in terms of the partial wave amplitudes, considering only S and P waves together with $^3S_1 \rightarrow {}^3D_1$ transitions and neglect possible singlet-triplet transitions, which amounts to neglecting $^1P_1 \rightarrow {}^3P_1$ amplitudes. We list the familiar experimental quantities and propose a new one: the asymmetry $\overline{\mathcal{A}}_n$ between the averaged polarization along the normal, of the particles scattered to the right and to the left. The latter quantity can give additional information about the $^3S_1 \rightarrow {}^3D_1$ transition[1].

The second issue of this report is the derivation of some theorems of a more general nature, due to the conditions of invariance under rotation, parity and time-reversal. Here the consequences of the presence or absence of singlet-triplet transitions will be discussed at several points. Firstly, we proof that the averaged polarization along the normal equals the averaged polarization along the normal if no initial polarization of the beam was present. So, the averaged polarization resulting from a completely unpolarized scattering can also be measured if the beam is polarized. Secondly, we proof that the left-right asymmetry $\boldsymbol{\varepsilon}^{fi}$ in the angular distribution of the scattering of a polarized beam incident on an unpolarized target equals the polarization of the scattered particle of the inverse reaction, where the latter has no initial polarization. Obviously, here time reversal invariance was employed. For example, from the left-right asymmetry in the angular distribution of the reactions

$$\begin{aligned}\Sigma^- + p &\rightarrow \Sigma^0 + n \\ &\rightarrow \Lambda + n\end{aligned}$$

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[1] In the case of only s-waves plus $^3S_1 \rightarrow {}^3D_1$ the depolarization $\overline{\mathcal{D}}$ and the asymmetry $\overline{\mathcal{A}}_n$ are the only independent polarization quantities.

with a polarized Σ^--beam[2], one infers the polarization from the inverse reactions

$$\begin{aligned} \Sigma^0 + n &\to \Sigma^- + p \\ \Lambda + n &\to \end{aligned}$$

if no initial polarization was present.

Finally we write down the relations between the transition matrix elements following from time reversal invariance and discuss the number of independent amplitudes in several cases.

The plan of this report is as follows. In section II the density matrix formalism is shortly reviewed. In section III the differential cross section and the polarization are calculated for the case of a polarized beam and an unpolarized target in terms of the scattering matrix M in spin-space. In spin-space we work in particular on the 'singlet-triplet' basis. In section IV the formulae for cross sections and polarizations are given in terms of the partial waves taking into account only S- and P-waves together with the ${}^3S_1 \to {}^3D_1$ coupling. Section V describes the well-known measurable quantities and proposes a new one: $\overline{\mathcal{A}}_n$. In Appendix A some symmetry properties of the M-matrix and the spin matrices are derived from invariance under rotations and parity. Furthermore some theorems are proven concerning the polarization. Appendix B discusses the implications of time reversal invariance for the M-matrix. The equality of the left-right asymmetry of $i \to f$ and the final polarization if no initial polarization was present for $f \to i$, is proven. Finally the relations between the M-matrix elements are stated and from them, and other results and considerations, we conclude to the number of independent ampitudes in several cases.

II. GENERAL DENSITY MATRIX FORMALISM

A general asymptotic wave function for the scattering $1 + 2 \to 1' + 2'$ in the center of mass may be written as:

$$\sum_{j=1}^{4} \psi_j^{\rm as}(x)\xi_j \simeq \sum_{j=1}^{4} a_j \left[\xi_j e^{ik_i z}\eta_i + \left(\frac{v_i}{v_f}\right)^{1/2} \sum_{\ell=1}^{4} M_{\ell j}(\theta,\phi)\xi_\ell \frac{e^{ik_f r}}{r}\eta_f\right] , \tag{2.1}$$

where v_i and v_f are the relative velocities of the initial and final particles and the ξ_j provide a basis in the composite spin space. The η_i and η_f indicate the different channels. If one takes as a spin basis the column vectors and considers the m-th component, then

$$\psi_m^{\rm as}(r) \simeq a_m e^{ik_i z}\eta_i + \left(\frac{v_i}{v_f}\right)^{1/2} \sum_{j=1}^{4} M_{mj}(\theta,\phi) a_j \frac{e^{ik_f r}}{r}\eta_f \ . \tag{2.2}$$

The density matrix ρ_i for a beam of N particles is defined by

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[2]Note that it is clear that when there is no initial polarization, there will be no left-right asymmetry in the angular distribution; this because of rotational invariance!

$$(\rho_i)_{jk} = \sum_{n=1}^{N} a_j(n) a_k^*(n) \ . \tag{2.3}$$

Similarly, the density matrix ρ_f of the final system is

$$(\rho_f)_{jk} = \sum_{n=1}^{N} a_j^{(f)}(n) a_k^{(f)^*}(n) \ . \tag{2.4}$$

Note that ρ is a hermitean matrix.

In view of eq. (2.2) we define

$$a_m^{(f)}(n) = \sum_{j=1}^{4} M_{mj}(\theta,\phi) a_j(n) \ , \tag{2.5}$$

then it follows immediately

$$\rho_f = M \rho_i M^\dagger \ . \tag{2.6}$$

The expectation value of an operator $\mathcal{O}$ in spin space, averaged over an ensemble, is then given by

$$\overline{\langle \mathcal{O} \rangle} = \frac{\sum_{n,k,\ell} a_k^*(n) a_\ell(n) \langle \xi_k | \mathcal{O} | \xi_\ell \rangle}{\sum_{n,k,\ell} a_k^*(n) a_\ell(n) \langle \xi_k | \xi_\ell \rangle} = \frac{\mathrm{Tr}\,(\rho \mathcal{O})}{\mathrm{Tr}\,(\rho)} \ . \tag{2.7}$$

We can express every hermitean 4×4 matrix in terms of 16 independent **hermitean** matrices S^μ ($\mu = 1, \ldots, 16$) with the property

$$\mathrm{Tr}\,(S^\mu S^\nu) = 4 \delta_{\mu\nu} \ , \tag{2.8}$$

therefore

$$\rho = \frac{1}{4} \sum_{\nu=1}^{16} \mathrm{Tr}\,(\rho S^\nu) S^\nu \ . \tag{2.9}$$

The polarization $\mathbf{P}_{1,2}$ is defined as the expectation value of $\boldsymbol{\sigma}_{1,2}$ averaged over the ensemble

$$\mathbf{P}_{1,2} = \overline{\langle \boldsymbol{\sigma}_{1,2} \rangle} = \frac{\mathrm{Tr}\,(\rho \boldsymbol{\sigma}_{1,2})}{\mathrm{Tr}\,(\rho)} \ . \tag{2.10}$$

If one takes as the 16 S^μ the direct product matrices

$$\mathbb{1} \otimes \mathbb{1} \ , \quad \boldsymbol{\sigma}_1 \otimes \mathbb{1} \ , \quad \boldsymbol{\sigma}_2 \otimes \mathbb{1} \ , \quad \boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2 \ ,$$

then it follows obviously from eq. (2.9) that in the case of a polarized beam incident or an unpolarized target

$$\rho_i = \frac{1}{4} \mathrm{Tr}\,(\rho_i) [1 + \mathbf{P}_i \cdot \boldsymbol{\sigma}_1] \ , \tag{2.11}$$

where $\mathbf{P}_i$ is the polarization of the beam (particle 1). If one takes $\mathrm{Tr}\,(\rho_i) = 1$ then eq. (2.11) reduces to

$$\rho_i = \frac{1}{4}\left[1 + \mathbf{P}_i \cdot \boldsymbol{\sigma}_1\right] \ . \tag{2.12}$$

In the case of an unpolarized beam and unpolarized target one has, of course, a density matrix that is a multiple of the unit matrix[3].

From eq. (2.1) one can easily calculate the differential cross section

$$\frac{d\sigma(i)}{d\Omega} = \frac{\sum_{mnp} M_{mn} a_n(i) a_p^*(i) M_{pm}^\dagger}{\sum_n |a_n(i)|^2} \ . \tag{2.13}$$

For an ensemble of N scatterings one gets then by virtue of eq. (2.3)

$$\frac{d\sigma}{d\Omega} = \frac{\mathrm{Tr}\,(M\rho_i M^\dagger)}{\mathrm{Tr}\,(\rho_i)} = \frac{\mathrm{Tr}\,(\rho_f)}{\mathrm{Tr}\,(\rho_i)} \ . \tag{2.14}$$

In the case of an **unpolarized beam** and unpolarized target, eq. (2.14) reduces to

$$\left[\frac{d\sigma}{d\Omega}\right]_{\mathrm{u}} = \frac{1}{4}\,\mathrm{Tr}\,(MM^\dagger) \ . \tag{2.15}$$

In the case of a **polarized beam** incident on an unpolarized target we have (cfr eq. (2.11))

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \mathrm{Tr}\,\left[M\frac{1}{4}(1 + \mathbf{P}_i \cdot \boldsymbol{\sigma}_1)M^\dagger\right] \\ &= \left[\frac{d\sigma}{d\Omega}\right]_{\mathrm{u}} (1 + \mathbf{P}_i \cdot \boldsymbol{\varepsilon}) \end{aligned} \tag{2.16}$$

where the so-called left-right asymmetry $\boldsymbol{\varepsilon}$ is defined by

$$\boldsymbol{\varepsilon} = \frac{\mathrm{Tr}\,(M^\dagger M \boldsymbol{\sigma}_1)}{\mathrm{Tr}\,(MM^\dagger)} \ . \tag{2.17}$$

The final polarizations are according to the definition (2.10) given by

$$\mathbf{P}_{1,2}^f = \frac{\mathrm{Tr}\,(\rho_f \boldsymbol{\sigma}_{1,2})}{\mathrm{Tr}\,\rho_f} \ , \tag{2.18}$$

hence using eq. (2.14)

$$\frac{d\sigma}{d\Omega} \cdot \mathbf{P}_{1,2}^f = \frac{\mathrm{Tr}\,(M\rho_i M^\dagger \boldsymbol{\sigma}_{1,2})}{\mathrm{Tr}\,\rho_i} \ . \tag{2.19}$$

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[3]Here and everywhere else we suppose that there are no correlations between the spins of the particles in the beam and those of the particles in the target.

When one deals with an unpolarized beam and an unpolarized target, eq. (2.18) becomes simply

$$[\mathbf{P}^f_{1,2}]_{\mathrm{u}} = \frac{\mathrm{Tr}\,(MM^\dagger \boldsymbol{\sigma}_{1,2})}{\mathrm{Tr}\,(MM^\dagger)} \ . \tag{2.20}$$

The difference between eqs (2.17) and (2.20) should be noted!

We shall proof in Appendix B that from time-reversal invariance the following relation holds (see Eqn. (A14))

$$\boldsymbol{\varepsilon}^{fi} = [\mathbf{P}^{if}_1]_{\mathrm{u}} \ .$$

Here and in the following the superscript fi denotes the reaction $i \to f$: $1+2 \to 1'+2'$, and the superscript if denotes the reaction $f \to i$: $1'+2' \to 1+2$. Also for the reaction $i \to f$ (inverse reaction $f \to i$) we call 1 ($1'$) the beam- and 2 ($2'$) the target-particle.

Equation (A14) expresses the fact that the left-right asymmetry, which can be observed in an experiment with a **polarized** beam, incident on an unpolarized target equals the polarization of the scattered beam in an experiment investigation the inverse reaction with an **unpolarized** beam and unpolarized target.

So, for elastic scattering the observed left-right asymmetry reveals immediately the polarization of an experiment with no initial polarization.

III. POLARIZED BEAM INCIDENT ON AN UNPOLARIZED TARGET

1. For the reaction $1+2 \to 1'+2'$ a coordinate system in the center of mass frame is chosen in the following way (figure 1):

The initial momentum $\mathbf{k}_i$ points in the positive z-direction; the beam is polarized along the x-direction with polarization P_i; the final momentum $\mathbf{k}_f$ lies in the direction (θ, ϕ). The normal to the reaction plane is defined as

$$\hat{n} = \frac{\mathbf{k}_i \times \mathbf{k}_f}{|\mathbf{k}_i \times \mathbf{k}_f|} = (-\sin\phi, \cos\phi, 0) \ . \tag{3.1}$$

2. The initial density matrix is then according to eq. (2.12) given by

$$\rho_i = \frac{1}{4}[1 + P_i \sigma_{1_x}] \ . \tag{3.2}$$

As quantization axis of the spin serves the z-axis. If $\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ denote the usual spin basis for spin-$\frac{1}{2}$ particles, we then choose as **triplet-singlet** basis in the composite spin space the following set

$$\left.\begin{array}{l} \alpha(1)\alpha(2) \\ \frac{1}{\sqrt{2}}\ [\beta(1)\alpha(2) + \alpha(1)\beta(2)] \\ \beta(1)\beta(1) \end{array}\right\} \text{Triplet}$$
$$\frac{1}{\sqrt{2}}\ [\alpha(1)\beta(2) - \beta(1)\alpha(2)] \quad \text{Singlet} \tag{3.3}$$

The matrix ρ_i expressed on this basis can then easily be calculated

$$\rho_i = \frac{1}{4}\begin{bmatrix} 1 & P_i/\sqrt{2} & 0 & -P_i/\sqrt{2} \\ P_i/\sqrt{2} & 1 & P_i/\sqrt{2} & 0 \\ 0 & P_i/\sqrt{2} & 1 & P_i/\sqrt{2} \\ -P_i/\sqrt{2} & 0 & P_i/\sqrt{2} & 1 \end{bmatrix} \tag{3.4}$$

3. The scattering matrix $M(\theta,\phi)$ has the following property (see Appendix A, eqs (A1)-(A5))

$$M^{fi}(\theta,\phi)^{S_1S_2}_{m'm} = \mathcal{M}^{fi}(\theta)^{S_1S_2}_{m'm} e^{i(m-m')\phi} \ ,$$

where

$$\mathcal{M}^{fi}(\theta) \equiv M^{fi}(\theta,0) \ ,$$

and S_1, S_2 refer to the total spin of the basis (0 or 1) and m', m to the z-component of this total spin.

It is also proven in Appendix A that the relation holds

$$\mathcal{M}^{fi}(\theta)^{S_2\ S_1}_{\rho\ \lambda} = (-1)^{S_2-S_1+\rho-\lambda}\mathcal{M}^{fi}(\theta)^{S_2S_1}_{-\rho-\lambda} \ .$$

Note that it follows from this relation that

$$\mathcal{M}^{fi}(\theta)^{1\ 0}_{0\ 0} = \mathcal{M}^{fi}(\theta)^{0\ 1}_{0\ 0} = 0 \ .$$

The scattering matrix therefore reads explicitly

$$M^{fi}(\theta,\phi) = \begin{bmatrix} \mathcal{M}_{11}(\theta) & \mathcal{M}_{10}(\theta)e^{-i\phi} & \mathcal{M}_{1-1}(\theta)e^{-2i\phi} & \mathcal{M}_{1S}(\theta)e^{-i\phi} \\ \mathcal{M}_{01}(\theta)e^{i\phi} & \mathcal{M}_{00}(\theta) & \mathcal{M}_{0-1}(\theta)e^{-i\phi} & 0 \\ \mathcal{M}_{-11}(\theta)e^{2i\phi} & \mathcal{M}_{-10}(\theta)e^{i\phi} & \mathcal{M}_{-1-1}(\theta) & \mathcal{M}_{-1S}(\theta)e^{i\phi} \\ \mathcal{M}_{S1}(\theta)e^{i\phi} & 0 & \mathcal{M}_{S-1}(\theta)e^{-i\phi} & \mathcal{M}_{SS}(\theta) \end{bmatrix} \tag{3.5}$$

where the indices fi as well as the total spin quantum numbers are suppressed. The index S refers to singlet states. Straightforward evaluation of eq. (2.14) using the relations (3.5) and (A4) yields for the angular distribution

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \frac{1}{2}\Big[& |\mathcal{M}_{11}|^2 + |\mathcal{M}_{10}|^2 + |\mathcal{M}_{1-1}|^2 + |\mathcal{M}_{01}|^2 + \tfrac{1}{2}|\mathcal{M}_{00}|^2 + \\ & \tfrac{1}{2}|\mathcal{M}_{SS}|^2 + |\mathcal{M}_{1S}|^2 + |\mathcal{M}_{S1}|^2 + P_i\sqrt{2}\ \mathrm{Im}\ \Big\{\mathcal{M}_{10}\mathcal{M}^*_{11} - \\ & \mathcal{M}_{10}\mathcal{M}^*_{1-1} + \mathcal{M}_{00}\mathcal{M}^*_{01} + \mathcal{M}_{11}\mathcal{M}^*_{1S} + \mathcal{M}_{1-1}\mathcal{M}^*_{1-S} - \\ & \mathcal{M}_{SS}\mathcal{M}^*_{S1}\Big\}\sin\phi\Big] \end{aligned} \tag{3.6}$$

The total cross section is then easily obtained by integrating over $d\Omega$

$$\begin{aligned} \sigma_{\rm tot} &= \frac{1}{4}\int d\Omega \left[\sum_{m'm}|\mathcal{M}_{m'm}|^2 + \sum_m\{|\mathcal{M}_{mS}|^2 + |\mathcal{M}_{Sm}|^2\} + |\mathcal{M}_{SS}|^2\right] \\ &= \frac{1}{4}\int d\Omega \left[\sum_{m'm}|M_{m'm}(\theta,\phi)|^2 + \sum_m\{|M_{mS}(\theta,\phi)|^2 + |M_{Sm}(\theta,\phi)|^2\} + |M_{SS}(\theta,\phi)|^2\right] \\ &= \sigma_S + \sigma_{St} + \sigma_t \end{aligned} \tag{3.7}$$

where the singlet cross section is defined as

$$\sigma_S = \frac{1}{4}\int d\Omega\ |M_{SS}(\theta,\phi)|^2\ , \tag{3.8}$$

and the singlet-triplet transition cross section as

$$\sigma_{St} = \frac{1}{4}\int d\Omega \sum_m \{|M_{mS}(\theta,\phi)|^2 + |M_{Sm}(\theta,\phi)|^2\}\ , \tag{3.9}$$

and the triplet cross section as

$$\sigma_t = \frac{1}{4}\int d\Omega \sum_{m'm} |M_{m'm}(\theta,\phi)|^2\ , \tag{3.10}$$

i.e. the statistical factors are included. In the above formulae m and m' run over 1, 0, and -1. Evaluation of eq. (2.19) gives the polarizations of the jth particle after the scattering $(j = 1, 2)$

$$\begin{aligned}\frac{d\sigma}{d\Omega}P^f_{x_j} = \frac{1}{2}\Big[&\sqrt{2}\,\mathrm{Im}\ \Big\{\mathcal{M}_{11}\mathcal{M}^*_{01} - \mathcal{M}_{1-1}\mathcal{M}^*_{01} + \mathcal{M}_{10}\mathcal{M}^*_{00} + (-1)^{j+1}\ \cdot\\ &\cdot(\mathcal{M}_{S1}\mathcal{M}^*_{11} + \mathcal{M}_{S1}\mathcal{M}^*_{1-1} - \mathcal{M}_{1S}\mathcal{M}^*_{SS})\Big\}\sin\phi\\ &+P_i\ \Big\{\mathrm{Re}\ (\mathcal{M}_{00}\mathcal{M}^*_{11} + \mathcal{M}_{SS}\mathcal{M}^*_{11} - \mathcal{M}_{01}\mathcal{M}^*_{10})\\ &+\mathrm{Re}\ (\mathcal{M}_{00}\mathcal{M}^*_{1-1} - \mathcal{M}_{SS}\mathcal{M}^*_{1-1} + \mathcal{M}_{01}\mathcal{M}^*_{10})\cos 2\phi\\ &-2\mathrm{Re}\ (\mathcal{M}_{S1}\mathcal{M}^*_{1S})\sin^2\phi\cdot(-1)^{j+1}\\ &-2\mathrm{Re}\ (\mathcal{M}_{S1}\mathcal{M}^*_{10}(-1)^{j+1} + \mathcal{M}_{1S}\mathcal{M}^*_{01})\cos^2\phi\Big\}\Big]\end{aligned} \tag{3.11}$$

$$\begin{aligned}\frac{d\sigma}{d\Omega}P^f_{y_j} = \frac{1}{2}\Big[&-\sqrt{2}\,\mathrm{Im}\ \Big\{\mathcal{M}_{11}\mathcal{M}^*_{01} - \mathcal{M}_{1-1}\mathcal{M}^*_{01} + \mathcal{M}_{10}\mathcal{M}^*_{00} + (-1)^{j+1}\cdot\\ &\cdot(\mathcal{M}_{S1}\mathcal{M}^*_{11} + \mathcal{M}_{S1}\mathcal{M}^*_{1-1} - \mathcal{M}_{1S}\mathcal{M}^*_{SS})\Big\}\cos\phi\\ &+P_i\ \Big\{\mathrm{Re}\ (\mathcal{M}_{00}\mathcal{M}^*_{1-1} - \mathcal{M}_{SS}\mathcal{M}^*_{1-1} + \mathcal{M}_{01}\mathcal{M}^*_{10})\sin 2\phi\\ &+2\mathrm{Re}\ (\mathcal{M}_{S1}\mathcal{M}^*_{1S})\sin\phi\cos\phi(-1)^{j+1}\\ &-2\mathrm{Re}\ (\mathcal{M}_{S1}\mathcal{M}^*_{10}(-1)^{j+1} + \mathcal{M}_{1S}\mathcal{M}^*_{01})\cos\phi\sin\phi\Big\}\Big]\end{aligned} \tag{3.12}$$

$$\begin{aligned}\frac{d\sigma}{d\Omega}P^f_{z_j} = \frac{P_i}{\sqrt{2}}\,\mathrm{Re}\ &\Big\{\mathcal{M}_{10}\mathcal{M}^*_{11} + \mathcal{M}_{10}\mathcal{M}^*_{1-1} - \mathcal{M}_{SS}\mathcal{M}^*_{01} - \mathcal{M}_{11}\mathcal{M}^*_{1S} +\\ &\mathcal{M}_{1-1}\mathcal{M}^*_{1S} + \mathcal{M}_{00}\mathcal{M}^*_{S1}(-1)^{j+1}\Big\}\cos\phi\end{aligned} \tag{3.13}$$

In the direction of the normal one finds then

$$\frac{d\sigma}{d\Omega}\mathbf{P}^f_j\cdot\hat{\mathbf{n}} = \frac{1}{2}\Big[-\sqrt{2}\,\mathrm{Im}\,\{\mathcal{M}_{11}\mathcal{M}^*_{01} - \mathcal{M}_{1-1}\mathcal{M}^*_{01} + \mathcal{M}_{10}\mathcal{M}^*_{00} +$$
$$(-1)^{j+1}(\mathcal{M}_{S1}\mathcal{M}^*_{11} + \mathcal{M}_{S1}\mathcal{M}^*_{1-1} - \mathcal{M}_{1S}\mathcal{M}^*_{SS})\Big\} +$$
$$P_i\mathrm{Re}\,\Big\{\mathcal{M}_{00}\mathcal{M}^*_{1-1} - \mathcal{M}_{00}\mathcal{M}^*_{11} - \mathcal{M}_{SS}\mathcal{M}^*_{1-1} -$$
$$2\mathcal{M}_{01}\mathcal{M}^*_{10} + 2\mathcal{M}_{S1}\mathcal{M}^*_{1S}(-1)^{j+1}\Big\}\sin\phi\Big] \tag{3.14}$$

Putting $P_i = 0$ gives, of course, only polarization along the normal (cfr. Appendix A).

The difference of the polarization along the normal of the scattered particle and the recoil target is then given by

$$\frac{d\sigma}{d\Omega}\mathbf{P}^f_1\cdot\hat{\mathbf{n}} - \frac{d\sigma}{d\Omega}\mathbf{P}^f_2\cdot\hat{\mathbf{n}} = -\sqrt{2}\,\mathrm{Im}\,(\mathcal{M}_{S1}\mathcal{M}^*_{11} + \mathcal{M}_{S1}\mathcal{M}^*_{1-1}$$
$$-\ \mathcal{M}_{1S}\mathcal{M}^*_{SS}) + 2P_i\,\mathrm{Re}\,(\mathcal{M}_{S1}\mathcal{M}^*_{1S})\sin\phi\ . \tag{3.15}$$

IV. S AND P WAVES

The scattering matrix $M_{m',m}(\theta,\phi)$ (eq. (2.3)) can be written as

$$M_{m',m}(\theta,\phi) = \sum_{j\ell\ell'}\sqrt{4\pi(2\ell+1)}\,i^{\ell-\ell'}C^{\ell'}_{m-m'}\,{}^{s}_{m'}\,{}^{j}_{m}C^{\ell}_{0}\,{}^{s}_{m}\,{}^{j}_{m}\,\cdot$$
$$\times Y^{(\ell')}_{m-m'}(\theta,\phi)\left\langle f\ell'\left|\frac{s_j-1}{2ik_i}\right|\ell i\right\rangle \tag{4.1}$$

This formula can be simplified for singlet states:

$$M_{SS}(\theta,\phi) = \sum_{\ell}\sqrt{4\pi(2\ell+1)}Y^{(\ell)}_0(\theta,\phi)\left\langle f\ell\left|\frac{s_j-1}{2ik_i}\right|\ell i\right\rangle \tag{4.2}$$

Specializing to S and P waves only, the notation for

$$\left\langle f\ell'\left|\frac{s_j-1}{2ik_i}\right|\ell i\right\rangle$$

is given in table I.

We have calculated the terms occurring in the amplitude $M_{m'm}(\theta)$ for ℓ and j values of interest to us, apart from the factor $\left\langle\left|\frac{s_j-1}{2ik_i}\right|\right\rangle$, in table II.

Working out the formulae (3.13) and (3.14) we obtain the following results

(i) Angular distribution:

$$\frac{d\sigma}{d\Omega} = A_1 + A_2\cos\theta + A_3\cos^2\theta + P_i\Big[A_4\sin\theta\sin\phi + A_5\sin\theta\cos\theta\sin\phi\Big]\ , \tag{4.3}$$

where the coefficients A_i are:

${}^{2S+1}L_j \to {}^{2S+1}L'_j$	notation
${}^1S_0 \to {}^1S_0$	S
${}^1P_1 \to {}^1P_1$	P
${}^3S_1 \to {}^3S_1$	S_1
${}^3S_1 \to {}^3D_1$	D
${}^3P_0 \to {}^3P_0$	P_0
${}^3P_1 \to {}^3P_1$	P_1
${}^3P_2 \to {}^3P_2$	P_2

TABLE I. Note, that we take the ${}^3S_1 \to {}^3D_1$ coupling into account, but not the ${}^3P_2 \to {}^3F_2$. Not considering singlet-triplet coupling means omitting ${}^1P_1 \to {}^3P_1$ for these waves.

$$
\begin{aligned}
A_1 &= \frac{1}{4}\left[|S|^2 + 3|S_1|^2 + 3|D|^2 + |P_0|^2 + \frac{9}{4}|P_1|^2 + \frac{13}{4}|P_2|^2 \right. \\
&\quad \left. + \mathrm{Re}\ \left(-2P_0 - \frac{9}{2}P_1\right) P_2^*\right] \\
A_2 &= \frac{1}{4}\ \mathrm{Re}\ \left[6SP^* + 2S_1(P_0^* + 3P_1^* + 5P_2^*) + \sqrt{2}D(2P_0^* - 3P_1^* + P_2^*)\right] \\
A_3 &= \frac{1}{16}\ \left[36|P|^2 + 9|P_1|^2 + 21|P_2|^2 + \mathrm{Re}\ \{(24P_0 + 54P_1)P_2^*\}\right] \\
A_4 &= \frac{1}{8}\ \mathrm{Im}\ \left[2S_1(2P_0^* + 3P_1^* - 5P_2^*) + \sqrt{2}D(4P_0^* - 3P_1^* - P_2^*)\right] \\
A_5 &= \frac{3}{4}\ \mathrm{Im}\ \left[(2P_0^* + 3P_1^*)P_2\right]
\end{aligned}
\tag{4.4}
$$

The total cross section becomes:

$$
\sigma_{\mathrm{tot}} = 4\pi\left[A_1 + \tfrac{1}{3}A_3\right]\ . \tag{4.5}
$$

(ii) Polarizations:

$$
\begin{aligned}
\frac{d\sigma}{d\Omega}P_x^f &= \{X_1\sin\theta + X_2\sin\theta\cos\theta\}\sin\phi \\
&\quad + P_i\left[\left\{X_3 + X_4\cos\theta + X_5\cos^2\theta + X_6\cos^3\theta\right\}\right. \\
&\quad \left. + \left\{X_7\sin^2\theta + X_8\sin^2\theta\cos\theta\right\}\cos 2\phi\right]
\end{aligned}
\tag{4.6}
$$

where the coefficients X_i are:

$$
\begin{aligned}
X_1 &= \frac{1}{4}\,\mathrm{Im}\ \left[S_1(2P_0^* + 3P_1^* - 5P_2^*) + \sqrt{2}D(-P_0^* + 3P_1^* - 2P_2^*)\right] \\
X_2 &= \frac{3}{4}\,\mathrm{Im}\ \left[(2P_0^* + 3P_1^*)P_2\right] = A_5
\end{aligned}
$$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -\frac{1}{4}\sqrt{2}(3\cos^2\theta-1) & -\frac{3}{2}\sin\theta\cos\theta & -\frac{3}{4}\sqrt{2}\sin^2\theta \\ -\frac{3}{2}\sin\theta\cos\theta & \frac{1}{2}\sqrt{2}(3\cos^2\theta-1) & \frac{3}{2}\sin\theta\cos\theta \\ -\frac{3}{4}\sqrt{2}\sin^2\theta & \frac{3}{2}\sin\theta\cos\theta & -\frac{1}{4}\sqrt{2}(3\cos^2\theta-1) \end{bmatrix}$
${}^3S_1 \to {}^3S_1$	${}^3S_1 \to {}^3D_1$
$\begin{bmatrix} -\frac{1}{2}\sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & -\frac{1}{2}\sqrt{2} \end{bmatrix}$	$\begin{bmatrix} 0 & -\frac{1}{2}\sqrt{2}\sin\theta & 0 \\ 0 & \cos\theta & 0 \\ 0 & \frac{1}{2}\sqrt{2}\sin\theta & 0 \end{bmatrix}$
${}^3D_1 \to {}^3S_1$	${}^3P_0 \to {}^3P_0$
$\begin{bmatrix} \frac{3}{2}\cos\theta & 0 & 0 \\ \frac{3}{4}\sqrt{2}\sin\theta & 0 & -\frac{3}{4}\sqrt{2}\sin\theta \\ 0 & 0 & \frac{3}{2}\cos\theta \end{bmatrix}$	$\begin{bmatrix} \frac{3}{2}\cos\theta & \frac{1}{2}\sqrt{2}\sin\theta & 0 \\ -\frac{3}{4}\sqrt{2}\sin\theta & 2\cos\theta & \frac{3}{4}\sqrt{2}\sin\theta \\ 0 & -\frac{1}{2}\sqrt{2}\sin\theta & \frac{3}{2}\cos\theta \end{bmatrix}$
${}^3P_1 \to {}^3P_1$	${}^3P_2 \to {}^3P_2$
$[1]$	$[3\cos\theta]$
${}^1S_0 \to {}^1S_0$	${}^1P_1 \to {}^1P_1$

TABLE II. Angular dependent factors in Eqn. (4.1) for different partial waves.

$$
\begin{aligned}
X_3 &= \frac{1}{8}\,\Big[4|S_1|^2 - |D|^2 + \mathrm{Re}\,\Big\{4S_1\cdot S^* - \sqrt{2}D(S_1^* - S^*) + 3(P_0 - P_2)(P_1^* - P_2^*)\Big\}\Big] \\
X_4 &= \frac{1}{4}\,\mathrm{Re}\,\Big[S_1(6P^* + 2P_0^* + 3P_1^* + 7P_2^*) + \frac{1}{4}\sqrt{2}\,D(6P^* - 4P_0^* \\
&\quad + 3P_1^* - 5P_2^*) + 3S(P_1^* + P_2^*)\Big] \\
X_5 &= \frac{3}{8}\,\Big[-|D|^2 + 3|P_2|^2 + \mathrm{Re}\,\Big\{\sqrt{2}D(S_1^* - S^*) + 6P(P_1^* + P_2^*) \\
&\quad + P_0(P_1^* + 3P_2^*) + 5P_1P_2^*\Big\}\Big] \\
X_6 &= \frac{9}{16}\sqrt{2}\,\mathrm{Re}\,\Big[D(-2P^* + P_1^* + P_2^*)\Big] \\
X_7 &= \frac{3}{8}\,\Big[|D|^2 + \mathrm{Re}\,\Big\{-\sqrt{2}D(S_1^* - S^*) - (P_0 - P_2)(P_1^* - P_2^*)\Big\}\Big] \\
X_8 &= \frac{9}{16}\sqrt{2}\,\mathrm{Re}\,\Big[D(2P^* - P_1^* - P_2^*)\Big] = -X_6
\end{aligned}
\tag{4.7}
$$

$$
\frac{d\sigma}{d\Omega}P_y^f = \Big\{Y_1\sin\theta + Y_2\sin\theta\cos\theta\Big\}\cos\phi + P_i\Big\{Y_3\sin^2\theta + Y_4\sin^2\theta\cos\theta\Big\}\sin 2\phi \tag{4.8}
$$

where the coefficients Y_i are:

$$
\begin{array}{lll}
Y_1 = -X_1 & \qquad & Y_2 = -X_2 \\
Y_3 = X_7 & \qquad & Y_4 = X_8
\end{array}
\tag{4.9}
$$

$$
\frac{d\sigma}{d\Omega}P_z^f = P_i\Big\{Z_1\sin\theta + Z_2\sin\theta\cos\theta + Z_3\sin\theta\cos^2\theta\Big\}\cos\phi\ , \tag{4.10}
$$

where the coefficients Z_i are:

$$
\begin{aligned}
Z_1 &= \frac{1}{4}\mathrm{Re}\,\Big[(-2S_1 + \sqrt{2}D)(P_0^* - P_2^*) - 3S(P_1^* - P_2^*)\Big] \\
Z_2 &= \frac{3}{4}\Big[|D|^2 + \mathrm{Re}\,\Big\{-\sqrt{2}D(S_1^* - S^*) - 3P(P_1^* - P_2^*) \\
&\quad -(P_1 + P_2)(P_0^* - P_2^*)\Big\}\Big] \\
Z_3 &= \frac{9}{8}\sqrt{2}\,\mathrm{Re}\,\Big\{D(2P^* - P_1^* - P_2^*)\Big\} = -2X_6
\end{aligned}
\tag{4.11}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Omega}\mathbf{P}^f\cdot\hat{\mathbf{n}} &= -\Big\{X_1\sin\theta + X_2\sin\theta\cos\theta\Big\} + P_i\Big\{-X_3 + X_7 - \\
&\quad -(X_4 + X_6)\cos\theta - (X_5 + X_7)\cos^2\theta\Big\}\sin\phi\ .
\end{aligned}
\tag{4.12}
$$

V. MEASURABLE QUANTITIES

1. Besides the total cross section (4.5) and the differential cross section (4.1) one defines usually two more experimental quantities related to the angular distribution:

(i) the 'forward-backward' ratio:

$$\frac{F-B}{F+B} = \left\{ \int_0^{2\pi} d\phi \int_0^1 d\cos\theta \frac{d\sigma}{d\Omega} - \int_0^{2\pi} d\phi \int_{-1}^0 d\cos\theta \frac{d\sigma}{d\Omega} \right\} / \sigma_{\rm tot} \tag{5.1}$$

From (4.1) we find

$$\frac{F-B}{F+B} = 2\pi A_2 / \sigma_{\rm tot} \ . \tag{5.2}$$

(ii) the 'polar to equatorial' ratio:

$$\frac{P-E}{P+E} = \left\{ \int_0^{2\pi} d\phi \left(\int_{-1}^{-\frac{1}{2}} + \int_{\frac{1}{2}}^1 \right) d\cos\theta \frac{d\sigma}{d\Omega} - \int_0^{2\pi} d\phi \int_{-\frac{1}{2}}^{\frac{1}{2}} d\cos\theta \frac{d\sigma}{d\Omega} \right\} / \sigma_{\rm tot} \ . \tag{5.3}$$

From (4.1) we find

$$\frac{P-E}{P+E} = 2\pi A_3 / \sigma_{\rm tot} \ . \tag{5.4}$$

(iii) the (averaged) left-right asymmetry:
The asymmetry with respect to the plane of the incident momentum and the direction of the initial polarization is (see also (2.17)):

$$\overline{\varepsilon} = \left\{ \int_0^{\pi} d\phi \int_{-1}^{+1} d\cos\theta \frac{d\sigma}{d\Omega} - \int_{\pi}^{2\pi} d\phi \int_{-1}^{+1} d\cos\theta \frac{d\sigma}{d\Omega} \right\} / \sigma_{\rm tot} \ . \tag{5.5}$$

Again from (4.1) we find

$$\overline{\varepsilon} = 2\pi A_4 / \sigma_{\rm tot} \ . \tag{5.6}$$

So, the coefficients A_1, A_2, A_3 and A_4 can be determined from $\sigma_{\rm tot}$, $\frac{F-B}{F+B}$, $\frac{P-E}{P+E}$, and $\overline{\varepsilon}$.

Remark about the angular distribution: one sees from (4.1) that the angular distribution becomes isotropic in the case of only S-waves and also if one has in addition the presence of $S-D$ transitions.

2. Here and from now on we restrict ourselves to the case of hyperon-nucleon scattering $Y+N \to Y'+N'$. So Y and Y' stand for one of the particles Λ, Σ^+, Σ^0 and Σ^-. Similarly N and N' for p or n. For the determination of the average polarization vector of a sample of hyperons at rest one can sometimes use the non-leptonic hyperon decay, $Y \to N + \pi$, as a polarization analyzer.

In section III we have expressed the polarization vector in the center-of-mass frame for the hyperon-nucleon system. Since we consider only low energetic hyperons, we may apply a parallel Galilei-transformation to each individual hyperon in order to transform it to its rest frame. The coordinate system in this rest frame is taken parallel to that described in section III.

Under this transformation the solid angle Ω and so, $d\sigma/d\Omega$ are invariant. Also the components of the polarization vector do not change under this operation. Therefore the components of the polarization vector of the sample at rest are the same as those of the same sample in the center-of-mass frame.

Now the average polarization vector of the sample, formed by the hyperons, which are scattered into the solid angle Ω, in the center-of-mass frame, and so also in the rest frame under the above stated conditions, is

$$\overline{\mathbf{P}} = \int_{\Omega} d\Omega\ \mathbf{P}^{(f)}(\Omega)\frac{d\sigma}{d\Omega}/\sigma_{\rm tot}\ , \tag{5.7}$$

then, we have for the decay angular distribution of the hyperons, taken at rest, with respect to the x-direction

$$W(\Theta_x) = \text{const.}\ (1 + \alpha\overline{P}_x \cos\Theta_x)\ , \tag{5.8}$$

where α denotes the decay parameter for the hyperons involved, and $\cos\Theta_x$ the direction cosine of the π-momentum with respect to the x-axis.

Analogous formulae hold for $W(\Theta_y)$ and $W(\Theta_z)$. So, from (5.7) one sees that when α is known and unequal to zero: the asymmetry in the decay enables us to determine $\mathbf{P}$. The experimental values are [6]:

asymmetry parameter	decay mode
$\alpha_\Lambda = 0.645 \pm 0.016$	$\Lambda \rightarrow p + \pi^-$
$= 0.71 \pm 0.18$	$\Lambda \rightarrow n + \pi^0$
$\alpha_{\Sigma^+} = -0.995 \pm 0.022$	$\Sigma^+ \rightarrow p + \pi^0$
$= +0.068 \pm 0.016$	$\Sigma^+ \rightarrow n + \pi^+$
$\alpha_{\Sigma^-} = -0.078 \pm 0.020$	$\Sigma^- \rightarrow n + \pi^-$

For Σ^0 one has almost exclusively the decay $\Sigma^0 \rightarrow \Lambda + \gamma$ and therefore cannot be discussed in the above stated terms.

From the quoted data one can conclude that the average polarization vector of a sample of Λ- or Σ^+-hyperons indeed can be determined by observation of the non-leptonic decay of these hyperons.

We work out the following distributions:
(i) The depolarization is defined as $\mathcal{D}(\Omega) = P_x(\Omega)/P_0$, then, the average depolatization is obtained from (5.6) as $\overline{\mathcal{D}} = \overline{P}_x/P_i$, where the solid angle Ω (see (5.6)) includes all directions. So from (4.3) follows

$$\begin{aligned}\overline{\mathcal{D}} = 2\pi \Big[&|S_1|^2 - \frac{1}{2}|D|^2 + \frac{3}{2}|P_2|^2 + \text{Re}\ \Big\{ S_1 \cdot S^* + \\ &+ P_1\left(P_0^* + \frac{1}{2}P_2^* + \frac{3}{2}P^*\right) + \frac{3}{2}P \cdot P_2^* \Big\}\Big]/\sigma_{\rm tot}\ . \end{aligned}\tag{5.9}$$

If one neglect the P waves, the angular distribution is isotropic and (4.3) and (5.8) reduce to

$$\overline{\mathcal{D}} = 2\pi \left[|S_1|^2 - \frac{1}{2}|D|^2 + \text{ Re } S_1 S^* \right] / \sigma_{\text{tot}} \ . \tag{5.10}$$

Note that in (5.9) two partial cross sections occur

$$\sigma({}^3S_1 \to {}^3S_1) = 3\pi |S_1|^2 \ , \qquad \sigma({}^3S_1 \to {}^3D_1) = 3\pi |D|^2 \ .$$

If one considers only S-waves and no $S-D$-coupling, the formulae (4.3) and (5.8) reduce to the Gardner and Welton formula

$$\mathcal{D} = \overline{\mathcal{D}} = \frac{2\{|S_1|^2 + \text{ Re } S_1 S^*\}}{|S|^2 + 3|S_1|^2} \ . \tag{5.11}$$

(ii) The average polarization in the direction of the normal can, of course, also be measured. From (4.8) it follows

$$\overline{P}^f_n = -\pi^2 X_1 / \sigma_{\text{tot}} \ , \tag{5.12}$$

which is pure interference (cfr. eq. (4.3)). Note also that $\overline{P}^f_n$ is independent of P_i, which is a general feature and will be proven in Appendix A.

(iii) An interesting quantity is the asymmetry between the averaged polarization along the normal, of the particles scattered to the right and to the left

$$\begin{aligned} \overline{\mathcal{A}}_n &\equiv \overline{P^f_n(r)} - \overline{P^f_n(\ell)} = \\ &= \int_0^{\pi} d\phi \int_{-1}^{+1} d\cos\theta (\mathbf{P}^f \cdot \hat{\mathbf{n}}) \frac{d\sigma/d\Omega}{\sigma_{\text{tot}}} \\ &\qquad - \int_{\pi}^{2\pi} d\phi \int_{-1}^{+1} d\cos\theta (\mathbf{P}^f \cdot \hat{\mathbf{n}}) \frac{d\sigma/d\Omega}{\sigma_{\text{tot}}} \\ &= \frac{8(-X_3 - \frac{1}{3}X_5 + \frac{2}{3}X_7)}{\sigma_{\text{tot}}} \ . \end{aligned} \tag{5.13}$$

In the case of only S-waves and $S-D$-transition, formula (5.12) becomes

$$\overline{\mathcal{A}}_n = -4 \ \frac{\left[|S_1|^2 - |D|^2 + \text{ Re } (S_1 S^*) + \frac{1}{2}\sqrt{2}\,\text{Re } (S_1^* - S^*) D \right]}{4\pi [3|S_1|^2 + |S|^2 + 3|D|^2]} \ . \tag{5.14}$$

Note that $\overline{\mathcal{A}}_n$ can be written as (cfr. (5.10))

$$\overline{\mathcal{A}}_n = -\frac{2}{\pi}\overline{\mathcal{D}} + \frac{2|D|^2 - \sqrt{2}\,\text{Re } \{(S_1^* - S^*)D\}]}{\sigma_{\text{tot}}} \ . \tag{5.15}$$

So, in this case two independent quantities can be measured: $\overline{\mathcal{D}}$ and $\overline{\mathcal{A}}_n$.

APPENDIX A

1. In this appendix we derive firstly a few symmetry properties both for the M-matrix and for the spin-matrices.
(i) From rotational invariance we will proof the following relation

$$M_{\alpha\beta}^{S_1S_2}(\theta,\phi)_{fi} = M_{\alpha\beta}^{S_1S_2}(\theta,0)_{fi}e^{i(\beta-\alpha)\phi}\ . \tag{A1}$$

Proof:

$$\begin{aligned}
M_{\alpha\beta}^{S_1S_2}(\theta,\phi)_{fi} &\equiv \langle f;k_f,\theta,\phi;S_1,\alpha|M|i;k_i,0,0;S_2,\beta\rangle \\
&= \langle f;k_f,\theta,\phi;S_1,\alpha|R^{-1}(\phi^0,0,0)MR(\phi^0,0,0)|i;k_i,0,0;S_2,\beta\rangle \\
&= \langle f;k_f,\theta,\phi+\phi^0;S_1,\alpha|M|i;k_i,0,0;S_2,\beta\rangle e^{-i(\beta-\alpha)\phi_0} \\
&= M_{\alpha\beta}^{S_1S_2}(\theta,\phi+\phi^0)_{fi}e^{-i(\beta-\alpha)\phi_0}
\end{aligned}$$

Putting $\phi = 0$ in the above expression yields (A1). Defining now

$$\mathcal{M}_{\alpha\beta}^{S_1S_2}(\theta)_{fi} \equiv M_{\alpha\beta}^{S_1S_2}(\theta,0)_{fi} \tag{A2}$$

gives then

$$M_{\alpha\beta}^{S_1S_2}(\theta,\phi)_{fi} \equiv \mathcal{M}_{\alpha\beta}^{S_1S_2}(\theta)e^{i(\beta-\alpha)\phi}\ . \tag{A3}$$

(ii) By virtue of rotational invariance we can derive another useful symmetry property

$$\mathcal{M}_{\alpha\beta}^{S_1S_2}(\theta)_{fi} = (-1)^{-S_1+S_2+\alpha-\beta}\mathcal{M}_{-\alpha-\beta}^{S_1S_2}(\theta)_{fi} \tag{A4}$$

by considering a rotation over π around the y-axis together with invariance under **parity**

$$\begin{aligned}
\mathcal{M}_{\alpha\beta}^{S_1S_2}(\theta)_{fi} &= M_{\alpha\beta}^{S_1S_2}(\theta,0)_{fi} \\
&= \langle f;k_f,\theta,0;S_1,\alpha|M|i;k_i,0,0;S_2,\beta\rangle \\
&= \langle f;k_f,\theta,0;S_1,\alpha|P^{-1}R^{-1}(0,\pi,0)MR(0,\pi,0)P|i;k_i,0,0;S_2,\beta\rangle \\
&= \sum_{\gamma\delta} d_{\alpha\gamma}^{S_1}(\pi)\langle f;k_f,\theta,0;S_1,\gamma|M|i;k_i,0,0;S_2,\delta\rangle d_{\delta\beta}^{S_2}(\pi) \\
&= (-1)^{-S_1+\alpha+S_2-\beta}\langle f;k_f,\theta,0;S_1,-\alpha|M|i;k_i,0,0;S_2,-\beta\rangle \\
&= (-1)^{-S_1+S_2+\alpha-\beta}\mathcal{M}_{-\alpha-\beta}^{S_1S_2}(\theta)_{fi}
\end{aligned}$$

From (A4) one can immediately conclude

$$\mathcal{M}_{0\ 0}^{0\ 1}(\theta) = \mathcal{M}_{0\ 0}^{1\ 0}(\theta) = 0 \tag{A5}$$

(iii) The symmetry relations of the σ-matrices are derived in a similar manner. From the commutation relations of $\boldsymbol{\sigma}_i$, $i = 1,2$

$$R(\pi,0,0)\begin{pmatrix}\sigma_{ix}\\ \sigma_{iy}\\ \sigma_{iz}\end{pmatrix}R(\pi,0,0)^{-1} = \begin{pmatrix}-\sigma_{ix}\\ -\sigma_{iy}\\ +\sigma_{iz}\end{pmatrix} \tag{A6}$$

we get

$$\left\langle \chi_\alpha^{(S_1)} \left| \begin{pmatrix} \sigma_{ix} \\ \sigma_{iy} \\ \sigma_{iz} \end{pmatrix} \right| \chi_\beta^{(S_2)} \right\rangle =$$

$$\left\langle \chi_\alpha^{(S_1)} \left| R^{-1}(\pi,0,0) \begin{pmatrix} -\sigma_{ix} \\ -\sigma_{iy} \\ \sigma_{iz} \end{pmatrix} R(\pi,0,0) \right| \chi_\beta^{(S_2)} \right\rangle$$

$$= -(-1)^{\alpha-\beta} \left\langle \chi_\alpha^{(S_1)} \left| \begin{pmatrix} \sigma_{ix} \\ \sigma_{iy} \\ -\sigma_{iz} \end{pmatrix} \right| \chi_\beta^{(S_2)} \right\rangle \tag{A7}$$

In a different notation they read therefore

$$(\sigma_{ix})_{\alpha\ \beta}^{S_1 S_2} = -(-1)^{\alpha-\beta} (\sigma_{ix})_{\alpha\ \beta}^{S_1 S_2} \tag{A8}$$

similar for y, and

$$(\sigma_{iz})_{\alpha\ \beta}^{S_1 S_2} = (-1)^{\alpha-\beta} (\sigma_{iz})_{\alpha\ \beta}^{S_1 S_2}\ . \tag{A9}$$

From (A8) it is clear that σ_{ix} and σ_{iy} have a checker-board pattern.

(iv) A rotation over an angle π around the y-axis gives us the next result. Again from the commutation relations it follows

$$R(0,\pi,0) \begin{pmatrix} \sigma_{ix} \\ \sigma_{iy} \\ \sigma_{iz} \end{pmatrix} R(0,\pi,0)^{-1} = \begin{pmatrix} -\sigma_{ix} \\ \sigma_{iy} \\ -\sigma_{iz} \end{pmatrix} \tag{A10}$$

therefore for the σ-matrices we get

$$\left\langle \chi_\alpha^{(S_1)} \left| \begin{pmatrix} -\sigma_{ix} \\ \sigma_{iy} \\ -\sigma_{iz} \end{pmatrix} \right| \chi_\beta^{(S_2)} \right\rangle =$$

$$= \Sigma_{\gamma\delta} d_{\gamma\alpha}^{S_1}(\pi) \left\langle \chi_\gamma^{(S_1)} \left| \begin{pmatrix} \sigma_{ix} \\ \sigma_{iy} \\ \sigma_{iz} \end{pmatrix} \right| \chi_\delta^{(S_2)} \right\rangle d_{\beta\delta}^{S_2}(\pi)$$

$$= (-1)^{S_2-S_1-\beta+\alpha} \left\langle \chi_{-\alpha}^{(S_1)} \left| \begin{pmatrix} \sigma_{ix} \\ \sigma_{iy} \\ \sigma_{iz} \end{pmatrix} \right| \chi_{-\beta}^{(S_2)} \right\rangle$$

$$= (-1)^{S_2-S_1-\beta+\alpha} \left\langle \chi_{-\beta}^{(S_2)} \left| \begin{pmatrix} \sigma_{ix} \\ -\sigma_{iy} \\ \sigma_{iz} \end{pmatrix} \right| \chi_{-\alpha}^{(S_1)} \right\rangle \tag{A11}$$

where use has been made in the last step of the fact that σ_{iy} is purely imaginary, the hermiticity of the $\boldsymbol{\sigma}_i$ and the reality of the spin basis. Rewritten in a different notation (A11) reads

$$\begin{pmatrix}\sigma_{ix}\\ \sigma_{iy}\\ \sigma_{iz}\end{pmatrix}\begin{matrix}S_1 & S_2\\ \alpha & \beta\end{matrix} = (-1)^{S_2-S_1-\beta+\alpha}\begin{pmatrix}-\sigma_{ix}\\ \sigma_{iy}\\ -\sigma_{iz}\end{pmatrix}\begin{matrix}S_1 & S_2\\ -\alpha & -\beta\end{matrix} \tag{A12}$$

and

$$(\boldsymbol{\sigma}_i)\begin{matrix}S_1 & S_2\\ \alpha & \beta\end{matrix} = -(-1)^{S_2-S_1-\beta+\alpha}(\boldsymbol{\sigma}_i)\begin{matrix}S_1 & S_2\\ -\alpha & -\beta\end{matrix} \tag{A13}$$

(v) From the explicit forms of the spin basis (3.3) it is obvious that the following relation holds

$$\boldsymbol{\sigma}_1^{S_1S_2} = (-1)^{S_1-S_2}\boldsymbol{\sigma}_2^{S_1S_2}\ . \tag{A14}$$

2. Having derived some symmetry properties of the M-matrix and the σ-matrices, we can show a few general features of the polarization:

(i) In the case of an unpolarized beam and an unpolarized target, we can easily demonstrate that the final polarization is in the direction of the normal, due to rotational symmetry around the z-axis and invariance under parity. Because of the rotational symmetry around the z-axis we may put $\phi = 0$ without loss of generality. The scattering is then in the $x-z$ plane and the normal points into the y-direction. The polarization was defined in Eqn. (2.10)

$$\left[\mathbf{P}^f_{1,2}\right]_u = \frac{\mathrm{Tr}\,[M(\theta,0)_{fi}M(\theta,0)^\dagger_{fi}\boldsymbol{\sigma}_{1,2}]}{\mathrm{Tr}\,[M(\theta,0)_{fi}M^\dagger(\theta,0)_{fi}]}$$

Now, using (A4) and (A12) yields

$$\begin{aligned}
\mathrm{Tr}\,(MM^\dagger)\begin{pmatrix}P^f_x\\ P^f_y\\ P^f_z\end{pmatrix}_{1,2u} &= \mathcal{M}^{S_1S_2}_{\alpha\beta}(\theta)_{fi}\mathcal{M}^{S_3S_2}_{\gamma\beta}(\theta)^*_{fi}\begin{pmatrix}\sigma_{1,2x}\\ \sigma_{1,2y}\\ \sigma_{1,2z}\end{pmatrix}\begin{matrix}S_2 & S_1\\ \gamma & \alpha\end{matrix}\\
&= \frac{1}{2}\left[\mathcal{M}^{S_1S_2}_{\alpha\beta}(\theta)_{fi}\mathcal{M}^{S_3S_2}_{\gamma\beta}(\theta)^*_{fi}\begin{pmatrix}\sigma_{1,2x}\\ \sigma_{1,2y}\\ \sigma_{1,2z}\end{pmatrix}\begin{matrix}S_3 & S_1\\ \gamma & \alpha\end{matrix}\right.\\
&\qquad\left.+\ \mathcal{M}^{S_1S_2}_{-\alpha-\beta}(\theta)_{fi}\mathcal{M}^{S_3S_2}_{-\gamma-\beta}(\theta)^*_{fi}\begin{pmatrix}-\sigma_{1,2x}\\ \sigma_{1,2y}\\ -\sigma_{1,2z}\end{pmatrix}\begin{matrix}S_3 & S_1\\ -\gamma & -\alpha\end{matrix}\right]\\
&= \mathcal{M}^{S_1S_2}_{\alpha\beta}(\theta)_{fi}\mathcal{M}^{S_3S_2}_{\gamma\beta}(\theta)^*_{fi}\begin{pmatrix}0\\ \sigma_{1,2y}\\ 0\end{pmatrix}\begin{matrix}S_3 & S_1\\ \gamma & \alpha\end{matrix}
\end{aligned} \tag{A15}$$

where also is summed over S_1, S_2 amd S_3. In the above result we see that the final polarization is in the direction of the normal. If we have $\phi \neq 0$, then, of course the rotation over ϕ around the z-axis to the new situation would not affect the z-components of $[P^f]_u$: it is always zero.

Note that the use of invariance under parity is hidden in employing (A4).

(ii) The averaged final polarization along the normal is always independent of the initial polarization. In fact we will proof that

$$\overline{\mathbf{P}^f_{1,2} \cdot \hat{\mathbf{n}}} = \overline{[\mathbf{P}^f_{1,2}]_u \cdot \hat{\mathbf{n}}} \ , \tag{A16}$$

where the average is defined in equation (5.6).
Proof:
Inserting (2.12) into (2.19) and recalling (2.18) and (2.20) yields

$$\begin{aligned}
\frac{d\sigma}{d\Omega}\mathbf{P}^f_{1,2} \cdot \hat{\mathbf{n}} &= \mathrm{Tr}\left[M\frac{1}{4}(1+\mathbf{P}_i\cdot\boldsymbol{\sigma})M^\dagger\boldsymbol{\sigma}_{1,2}\right] \\
&= \left[\frac{d\sigma}{d\Omega}\right]_u \frac{\mathrm{Tr}\,[MM^\dagger\boldsymbol{\sigma}_{1,2}]}{\mathrm{Tr}\,[MM^\dagger]}\cdot\hat{\mathbf{n}} + \frac{1}{4}\mathrm{Tr}\left[M(\mathbf{P}_i\cdot\boldsymbol{\sigma}_1)M^\dagger\boldsymbol{\sigma}_{1,2}\right]\cdot\hat{\mathbf{n}} \\
&= \left[\frac{d\sigma}{d\Omega}\right]_u [\mathbf{P}^f_{1,2}]_u\cdot\hat{\mathbf{n}} + \frac{1}{4}\mathrm{Tr}\left[M(\mathbf{P}_i\cdot\boldsymbol{\sigma}_1)M^\dagger\boldsymbol{\sigma}_{1,2}\right]\cdot\hat{\mathbf{n}}
\end{aligned} \tag{A17}$$

The next step is to show that the latter term of (A17) vanishes by integration over ϕ. In our coordinate system we have $\mathbf{P}_i = P_i\hat{\mathbf{x}}$ and $\hat{\mathbf{n}} = -\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}}$. So using (A1)

$$\begin{aligned}
&\int_0^{2\pi} d\phi\,\mathrm{Tr}\,[M(\mathbf{P}_i\cdot\boldsymbol{\sigma}_1)M^\dagger\boldsymbol{\sigma}_{1,2}]\cdot\hat{\mathbf{n}} = \\
&\quad = P_i \sum_{S_1S_2S_3S_4}\int_0^{2\pi} d\phi \mathcal{M}^{S_1S_2}_{\alpha\beta}(\theta)(\sigma_{1x})^{S_2S_3}_{\beta\gamma}\mathcal{M}^{S_4S_3}_{\delta\gamma}(\theta)^* \cdot \\
&\quad \times \left[-(\sigma_{1,2x})\begin{matrix} S_4 & S_1 \\ \delta & \alpha\end{matrix}\ \sin\phi + (\sigma_{1,2y})\begin{matrix} S_4 & S_1 \\ \delta & \alpha\end{matrix}\ \cos\phi\right] e^{i(\beta-\alpha-\gamma+\delta)\phi} \ .
\end{aligned} \tag{A18}$$

Now $(\sigma_{1,2x})\begin{matrix} S_1 & S_2 \\ \mu & \nu\end{matrix}$ and $(\sigma_{1,2y})\begin{matrix} S_1 & S_2 \\ \mu & \nu\end{matrix}$ have only nonzero elements for $\mu-\nu=\pm1$ (cfr. (A8)). Therefore in the above expression $\beta-\gamma=\pm1$ and $\delta-\alpha=\pm1$. Since $\beta-\alpha-\gamma+\delta$ is even, it is obvious that the integral over ϕ in (A18) vanishes. Averaging equation (A17) yields immediately (A16).

(iii) Finally, it follows from (3.11) to (3.13) that in the case of no singlet-triplet coupling, i.e.

$$M^{S_1S_2}_{\alpha\beta}(\theta,\phi)_{fi} = 0 \qquad \text{if} \ \ S_1 \neq S_2 \tag{A19}$$

that

$$[\mathbf{P}^f_1]_u = [\mathbf{P}^f_2]_u \ . \tag{A20}$$

APPENDIX B

The quantization axis of the spin is chosen to point in the $\mathbf{k}_i$-direction. Moreover, we attribute phases to the spin states in such a way that for the time reversed state, the relation holds:

$$\mathcal{T}\chi_m^{(s)} = (-)^{s-m}\chi_{-m}^{(s)} \ . \tag{B1}$$

Then the condition of time reversal invariance for the T-matrix reads

$$\begin{aligned}\langle f;\mathbf{k}_f,s_1,\nu|T|i;\mathbf{k}_i,s_2,\mu\rangle &= (-)^{-S_1+\nu+S_2-\mu} \cdot \\ &\langle i;-\mathbf{k}_i,s_2,-\mu|T|f;-\mathbf{k}_f,s_1,-\nu\rangle \ .\end{aligned} \tag{B2}$$

Using invariance under the parity transformation, we can rewrite (B2)

$$\begin{aligned}\langle f;\mathbf{k}_f,s_1,\nu|T|i;\mathbf{k}_i,s_2,\mu\rangle &= (-)^{-S_1+\nu+S_2-\mu} \cdot \\ &\langle i;\mathbf{k}_i,s_2,-\mu|T|f;\mathbf{k}_f,s_1,-\nu\rangle \ .\end{aligned} \tag{B3}$$

In the coordinate system, where $\hat{k}_i$ points in the positive z-direction, we have the relation

$$\hat{\mathbf{k}}_f = R(\phi,\theta,-\phi)\hat{\mathbf{k}}_i \ . \tag{B4}$$

If we now change the spin quantization axis to $\hat{k}_f$, then the connection between the two spinbases is

$$\chi_\mu^{(s)} = D_{\rho\mu}^{s^\dagger}(\phi,\theta,-\phi)\chi_\rho^{(s)'} \ , \tag{B5}$$

where the summation over repeated indices is, of course, understood. In the following the prime refers to this new spin basis. Rewriting the r.h.s. of (B4) on the new spin basis, yields

$$\begin{aligned}&\langle f;\mathbf{k}_f,s_1,\nu|T|i;\mathbf{k}_i,s_2,\mu\rangle = (-)^{-S_1+\nu+S_2-\mu} \cdot \\ &D_{-\mu,\rho}^{S_2}(\phi,\theta,-\phi)'\langle i;\mathbf{k}_i,s_2,\rho|T|f;\mathbf{k}_f,s_1,\lambda\rangle' D_{\lambda,-\nu}^{S_1^\dagger}(\phi,\theta,-\phi) \ .\end{aligned} \tag{B6}$$

The relation between the M-matrix, defined in (2.1) and the T-matrix is

$$\begin{aligned}T_{fi} = \langle f|T|i\rangle = \left\langle f\left|\frac{S-1}{i}\right|i\right\rangle &= -\left(\frac{1}{m_i m_f}\right)^{1/2}\left(\frac{k_i}{k_f}\right)\langle f|M|i\rangle \\ \equiv -\left(\frac{1}{m_i m_f}\right)^{1/2}\left(\frac{k_i}{k_f}\right)^{1/2} M_{fi} \ .\end{aligned} \tag{B7}$$

From (B6) follows then the relation for the M-matrices

$$\begin{aligned}&\langle f;\mathbf{k}_f,s_1,\nu|T|i;\mathbf{k}_i,s_2,\mu\rangle \equiv M_{\nu\ \ \mu}^{S_1\ S_2}(\mathbf{k}_f,\mathbf{k}_i)_{fi} = \frac{k_f}{k_i}(-)^{-S_1+\nu+S_2-\mu} \\ &D_{-\mu,\rho}^{S_2}(\phi,\theta,-\phi)M_{\rho,\lambda}^{S_2S_1\prime}(\mathbf{k}_i,\mathbf{k}_f)_{if}D_{\lambda,-\nu}^{S_1}(\phi,\theta,-\phi) \ .\end{aligned} \tag{B8}$$

Due to (B4) we can rewrite (B8) in terms of the polar angles

$$
\begin{aligned}
&M^{S_1,S_2}_{\nu,\mu}(\theta,\phi)_{fi} = \frac{k_f}{k_i}(-)^{-S_1+\nu+S_2-\mu} D^{S_2}_{-\mu,\rho}(\phi,\theta,-\phi) \\
&\quad M^{S_2 S_1 \prime}_{\rho,\lambda}(\theta,\phi+\pi)_{if} D^{S_1^\dagger}_{\lambda,-\nu}(\phi,\theta,-\phi) = \frac{k_f}{k_i}(-)^{-S_1+\nu+S_2-\mu} \\
&\quad \left(D^{S_2}(\phi,\theta,-\phi) M^{S_2 S_1 \prime}(\theta,\phi+\pi)_{if} D^{S_1^\dagger}(\phi,\theta,-\phi)_{-\mu,-\nu}\right)
\end{aligned}
\tag{B9}
$$

For scattering of a polarized beam incident on an unpolarized target, we have derived for the angular distribution (cfr. (2.18))

$$
\frac{d\sigma^{fi}}{d\Omega} = \left[\frac{d\sigma^{fi}}{d\Omega}\right]_u (1 + \boldsymbol{\varepsilon}^{fi} \cdot \mathbf{P}_i) \ , \tag{B10}
$$

where the left-right asymmetry $\boldsymbol{\varepsilon}^{fi}$ is defined (cfr. (2.17))

$$
\boldsymbol{\varepsilon}^{fi} \equiv \frac{\mathrm{Tr}\,(M^\dagger_{fi} M_{fi} \boldsymbol{\sigma}_1)}{\mathrm{Tr}\,(M_{fi} M^\dagger_{fi})} \ . \tag{B11}
$$

The polarization of the scattered particle in the scattering of an unpolarized beam incident on an unpolarized target for the reaction $f \to i$ is defined

$$
[\mathbf{P}^{if}_1]_u = \frac{\mathrm{Tr}\,(M_{if} M^\dagger_{if} \boldsymbol{\sigma}_1)}{\mathrm{Tr}\,(M_{if} M^\dagger_{if})} \ . \tag{B12}
$$

Next we will proof that the following relation holds

$$
\boldsymbol{\varepsilon}^{fi} = [\mathbf{P}^{if}_1]_u \ . \tag{B13}
$$

Proof:
By using (B9) and the properties of the rotation matrices we get

$$
\begin{aligned}
&\mathrm{Tr}\ (M^\dagger(\theta,\phi)_{fi} M(\theta,\phi)_{fi} \boldsymbol{\sigma}_1) = \sum_{S_1 S_2 S_3} M^{S_2 \ S_1}_{\alpha \ \ \alpha}(\theta,\phi)^*_{fi} M^{S_2 \ S_3}_{\beta \ \ \gamma}(\theta,\phi)_{fi} (\boldsymbol{\sigma}_1)^{\ S_3 \ S_1}_{\ \gamma \ \ \alpha} \\
&= \left(\frac{k_f}{k_i}\right)^2 \sum_{S_1 S_2 S_3} (-)^{-S_1+S_3+\alpha-\gamma} \left[D^{S_1} M^{S_1 S_2 \prime}(\theta,\phi+\pi)_{if} D^{S_2 \dagger}\right]^*_{-\alpha,-\beta} \cdot \\
&\qquad \cdot \left[D^{S_3} M^{S_3 S_2 \prime}(\theta,\phi+\pi)_{if} D^{S_2 \dagger}\right]_{-\gamma,-\beta} (\boldsymbol{\sigma}_1)^{\ S_3 \ S_1}_{\ \gamma \ \ \alpha} = \\
&= \left(\frac{k_f}{k_i}\right)^2 \sum_{S_1 S_2 S_3} (-)^{-S_1+S_3+\alpha-\gamma} \left[D^{S_3} M^{S_3 S_2 \prime}(\theta,\phi+\pi)_{if} M^{S_2 S_1 \dagger \prime}(\theta,\phi+\pi)_{if} D^{S_1 \dagger}\right]_{-\gamma,-\alpha} \cdot \\
&\qquad \cdot (\boldsymbol{\sigma}_1)^{\ S_3 \ S_1}_{\ \gamma \ \ \alpha} \ ,
\end{aligned}
\tag{B14}
$$

where the arguments of the D^S are $(\phi,\theta,-\phi)$.

Combining (B5), (A13) yields

$$
\begin{aligned}
(\boldsymbol{\sigma}_1)^{\ S_3 \ S_1}_{\ \gamma \ \ \alpha} = -(-)^{S_1-S_3-\alpha+\gamma} (\boldsymbol{\sigma}_1)^{\ S_1 \ \ S_3}_{\ -\alpha \ -\gamma} + -(-)^{S_1-S_3-\alpha+\gamma} \\
\left[D^{S_1}(\phi,\theta,-\phi)(\boldsymbol{\sigma})'^{S_1 S_3} D^{S_3}(\phi,\theta,-\phi)^\dagger\right]_{-\alpha,-\gamma} \ .
\end{aligned}
\tag{B15}
$$

Inserting this expression into (B14) gives as a result

$$\begin{aligned}\mathrm{Tr}\ (M^\dagger(\theta,\phi)_{fi}M(\theta,\phi)_{fi}\boldsymbol{\sigma}_1) &= -\left(\frac{k_f}{k_i}\right)^2 \cdot \\ &\cdot \left[D^{S_3}M^{S_3S_2}(\theta,\phi+\pi)'_{if}M^{S_2S_1\dagger}(\theta,\phi+\pi)'_{if}D^{S_1\dagger}D^{S_1}(\boldsymbol{\sigma}_1)^{\ S_1S_3}D^{S_3\dagger}\right]^*_{-\alpha,-\alpha} \\ &= -\left(\frac{k_f}{k_i}\right)^2\ \mathrm{Tr}\ \left[M(\theta,\phi+\pi)'_{fi}\ M^\dagger(\theta,\phi+\pi)'_{fi}\boldsymbol{\sigma}'_1\right] \end{aligned} \tag{B16}$$

Using (A1) together with (A6) and the fact that Tr $(MM^\dagger\sigma_{iz}) = 0$ (cfr. equation (A15)), we can rewrite (B16)

$$\mathrm{Tr}\ (M^\dagger(\theta,\phi)_{fi}M(\theta,\phi)_{fi}\boldsymbol{\sigma}_1) = \left(\frac{k_f}{k_i}\right)^2\ \mathrm{Tr}\ (M(\theta,\phi)'_{if}M^\dagger(\theta,\phi)'_{if}\boldsymbol{\sigma}_1{}') \tag{B17}$$

It is obvious from relation (B9) and (A1) that

$$\mathrm{Tr}\ (M(\theta,\phi)_{fi}M^\dagger(\theta,\phi)_{fi}) = \left(\frac{k_f}{k_i}\right)^2\ \mathrm{Tr}\ (M(\theta,\phi)'_{if}M^\dagger(\theta,\phi)'_{if}) \tag{B18}$$

Combining (B11), (B12), (B17) and (B18) we can conclude to (B13)

$$\boldsymbol{\varepsilon}^{fi} = \left[\mathbf{P}_1^{if}\right]_u\ . \tag{B19}$$

Next we consider equation (B9). We rewrite this equation using relation (A1) and the properties of the rotation matrices and find

$$d^{S_1}_{\kappa\rho}(\theta)(-)^{S_1-\rho}\mathcal{M}^{S_1\ \ S_2}_{\rho\ \ \mu}(\theta)_{fi} = d^{S_2}_{\mu\rho}(\theta)(-)^{S_2-\rho}\mathcal{M}^{S_2\ \ S_1}_{\rho\ \ \kappa}(\theta)^1_{if} \tag{B20}$$

In the case of elastic scattering this leads to two non-trivial relations

$$\mathcal{M}^{11}_{11} - \mathcal{M}^{11}_{1-1} - \mathcal{M}^{11}_{00} = \sqrt{2}\cot\theta(\mathcal{M}^{11}_{01} + \mathcal{M}^{11}_{10}) \tag{B21}$$

and

$$\mathcal{M}^{10}_{10} = \mathcal{M}^{01}_{01} \tag{B22}$$

where in the above relations we have suppressed the argument θ of the matrix elements. Relation (B21) can be found e.g. in [2], but is here derived in a more direct manner employing space-time symmetries.

Finally we discuss the number of independent amplitudes on the basis of the relations among the singlet-triplet amplitudes due to rotation and parity invariance, eq. (A4), and time-reversal invariance, eq. (B21) and eq. (B22). So, we consider only the reduction of the number of independent amplitudes coming from space-time symmetries, but not an eventual further reduction due to other principles as e.g. charge-conjugation – or G-parity invariance, SU(3) symmetry.

A priori we have 16 amplitudes, among which there are only 8 independent over by eq. (A4). (On the 'helicity' basis the counting is even easier!) Therefore an **inelastic** reaction has in general 8 independent amplitudes. For instance in case of an inelastic hyperon-nucleon reaction 8 independent 'invariant' amplitudes have been constructed by Protopapadakis [7].

An **elastic** reaction has in general 6 independent amplitudes, because of a further reduction of 2 amplitudes, by eq. (B21) and eq. (B22).

If one has to do with identical particles such that the Pauli-principle is operative, as for instance in nucleon-nucleon scattering, one has only 5 independent amplitude (see Ref. [2]).

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