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Soft-Core Baryon-Baryon One-Boson-Exchange Models. II. Hyperon-Nucleon Potential*

P.M.M. Maessen, Th.A. Rijken, J.J. de Swart
Institute for Theoretical Physics, University of Nijmegen, Nijmegen, the Netherlands

Abstract

The results of the Nijmegen soft-core potential model are presented for the low energy YN-interactions. The YN-version of the model is obtained by a straightforward extension of the NN-model through the application of SU(3). The potentials are due to the dominant parts of the $\pi, \eta, \eta', \rho, \omega, \phi, \delta, \varepsilon$, and $S^\star$ Regge trajectories. This gives the traditional One-Boson-Exchange potentials. In addition to these, the $J = 0$ contributions from the tensor f, f', A_2 and pomeron trajectories are included in the potentials. The latter give potentials of the gaussian-type. Also the form factors from Regge poles are gaussian, which guarantees that the potentials have a soft behavior near the origin. The multichannel Schrödinger equation is solved in configuration space for the (partially) nonlocal potentials. We work on the particle basis and include the Coulomb interaction exactly. The meson-baryon coupling constants are calculated via SU(3), using the coupling constants of the NN-analysis as input. Charge Symmetry Breaking in the Λp and Λn channels is included. An excellent description is achieved of the available low-energy data ($\chi^2/d.o.f. \approx$ 0.58 for 35 YN-data). In particular, we were able to fit the inelastic capture ratio at rest perfectly. We have $r_R = 0.471$, where experimentally the average value is $r_R = 0.468 \pm 0.010$. The obtained values for the adjustable mixing angles and $F/(F + D)$-ratio's agree very well with the literature. We find $\alpha_{PV} = 0.355$ and $\alpha_V^m = 0.275$. For the scalar-meson mixing-angle we obtain $\theta_S = 40.895\,^0$, which lies in between the ideal mixing-angles for the scalar $q^2\bar{q}^2$- and $q\bar{q}$-states. In the Λp-system we find a cusp at the $\Sigma^+ n$-threshold, but there is on the second Riemann sheet no pole in the vincinity causing this cusp. The predictions of the total cross sections up to the pion production threshold are given and compared to the experimental data.

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I. INTRODUCTION

For Nucleon-Nucleon NN scattering we have shown (Ref. [1], henceforth referred to as I) that a soft-core One-Boson-Exchange OBE model, based on Regge-pole theory, gives an excellent description of the wealthy and precise NN-data. In I only 13 free parameters were used. Moreover most of these parameters are coupling constants, mixing angles or $F/(F+D)$-ratio's and hence rather physical parameters, for which the fitted values can be checked against those found in other reactions. The Nucleon-Nucleon soft-core model of I can be fully derived in the context of the analytical S-matrix theory (Ref. [2]). In this framework the consequences of the Regge poles for low energy scattering and the corresponding (relativistic) Lippmann-Schwinger equations can be worked out in a consistent manner. The derivation of the Lippmann-Schwinger equation in this approach has been carried out recently in full detail also for the unequal mass case (Ref. [3]). This is useful in extending the model to Baryon-Baryon scattering. In this paper we describe the model for Hyperon-Nucleon YN-scattering and discuss the results. Because of the composite nature of the mesons in QCD, the proper description of the OBE-potentials is in principle in terms of Regge-trajectories. The large N-expansion in QCD strongly supports this viewpoint (Ref. [4]). This is also the case in the Bethe-Salpeter approach to the $Q\bar{Q}$-system, where any reasonable interaction leads to Regge poles. Therefore, in I and in this paper the OBE-potentials are treated as the dominant parts of the meson Regge trajectories. This includes also the $J = 0$ contributions from the tensor trajectories (f,f' and A_2). In elastic scattering however, the most important exchange at higher energies is pomeron-exchange. Therefore in I the traditional OBE-model was extended by including the pomeron. An excellent agreement was found between the pomeron parameters determined from the high energy and the low energy NN-data.

The reasons for a combined study of the NN- and YN-interactions are the same as that for our former hard-core model description of the NN- and YN-data (Refs. [5–7]):

- To test the assumption of SU(3)-symmetry. For example we want to investigate the properties of the scalar mesons ($\varepsilon(760)$, $S^\star(975)$, $\delta(980)$, $\kappa(1000)$). Furthermore we want to check whether in a combined NN- and YN- analysis the contribution to the interaction from the $J = 0$ components of the pomeron, f, f', and A_2 can be included.
- The determination of the $F/(F+D)$-ratio's.
- To extract, in spite of the scarce experimental YN-data, information about scattering lengths, effective ranges, the existence of resonances etc.

This program appears to be feasible, because a good theoretical description of the YN-interactions can indeed be given by using SU(3) and the meson-nucleon coupling constants from the NN-analysis.

The interactions in the model are described in terms of the following exchanges:

1. The pseudoscalar-meson nonet $\pi,\ \eta,\ \eta',\ K$ with the $\eta-\eta'$ mixing angle $\theta_P = -23.0^0$ from the Gell-Mann-Okubo mass formula.
2. The vector-meson nonet $\rho,\ \phi,\ K^\star,\ \omega$ with the $\phi-\omega$ ideal mixing angle $\theta_V = 37.56^0$.

3. The scalar-meson nonet $\delta, S^\star, \kappa, \varepsilon$ with a free $S^\star - \varepsilon$ mixing angle θ_S to be determined in a fit to the YN-data.

4. The 'diffractive' contribution from the pomeron P, f, f', and A_2. These interactions will give repulsive contributions to the potentials in all channels of a gaussian type and can perhaps partly justify the use of hard-cores in our earlier work.

The Baryon-Baryon-Meson vertices are described by coupling constants and form factors, which correspond to the Regge residues. The form factors are taken to be of the gaussian-type, like the residue functions in many Regge pole models for high energy scattering. Note that also in nonrelativistic quark models a gaussian behavior of the form factors is most natural. These form factors evidently guarantee a soft behavior of the potentials in configuration space at small distances.

The physical nature of pomeron-exchange can be understood in the framework of QCD as a two-gluon (or multigluon) exchange effect. Low and Nussinov (Ref. [8]) have shown that at high energies pomeron-exchange and two-gluon-exchange are equivalent. In Ref. [9] it has been demonstrated that this particular QCD-picture of the pomeron leads to a good description of the various diffractive processes. By extrapolating from the higher energies to low energies, using a Regge-pole model, the (multi-) gluon-exchange potential between two baryons was found in Ref. [2]. In NN the inclusion of the pomeron improved in particular the electric ω-coupling considerably (Ref. [1]). The role of the pomeron for low energy πN- and KN-scattering has been demonstrated convincingly using finite-energy sum rules (FESR) (Ref. [10]). In fact, it has been shown that the background amplitude, which remains after the subtraction of the known resonances, produces the pomeron contribution at high energies via FESR (Ref. [11]).

In this work we check whether a strong repulsion from the pomeron and the tensor mesons together with gaussian form factors can give also a high quality description of the YN-data. By high quality we understand here a YN-fit with low χ^2 and such that, while keeping the constraints forced on the potentials by the NN-fit, the free parameters with a clear physical significance, like e.g. the $F/(F+D)$-ratio's α_{PV} and α_V^m, assume realistic values.

In this paper we treat in detail the following YN-reactions:

(i) The coupled channels $\Lambda p \to \Lambda p, \Sigma^+ n, \Sigma^0 p$;

(ii) The coupled channels $\Sigma^- p \to \Sigma^- p, \Sigma^0 n, \Lambda n$;

(iii) The single channel $\Sigma^+ p \to \Sigma^+ p$.

It turns out that starting from the soft-core OBE-model of Ref. [1], we are indeed able to achieve a very good description of the YN-data and at the same time maintain values for the free parameters which are very consistent with other findings. As an application of the obtained potentials, the properties of the hyperfragments might be predicted. The determined value for the $F/(F+D)$-ratio α_{PV} for the pseudoscalar mesons is the same as that found in the weak interactions (Refs. [12,13]). Also the value obtained for the magnetic ratio of the vector mesons is in perfect agreement with non-static SU(6) (Ref. [14]). Like in previous work of the Nijmegen group on YN (e.g. Refs. [6,7]), we use SU(3)-symmetry for the coupling constants. SU(3)-breaking is introduced by

- using the physical masses of the mesons and baryons in the potentials and Schrödinger equation (see Refs. [6,7,15]),
- allowing for meson-mixing within a nonet ($\eta - \eta'$, $\omega - \phi$, $\varepsilon - S^\star$),
- including Charge Symmetry Breaking (CSB) (Ref. [16]) due to $\Lambda\Sigma$-mixing, which for example introduces a One-Pion-Exchange potential in the ΛN channel,
- taking into account the Coulomb-interaction.

In order to include the Coulomb interaction exactly, and to account as much as possible for the mass differences between the baryons, we solve the multichannel Schrödinger equation on the physical particle basis. However, the nuclear potentials are calculated on the isospin basis, in order to limit the number of different form factors (see section 7).

In NN we have fitted all partial waves with only one form factor parameter. For YN it appears to be impossible to use only one form factor for all channels. This would either introduce unobserved bound states in the model or make a fit to the YN-data impossible. Therefore we have to use several ones. (Note however that for avoiding unobserved bound states, the freedom to adjust form factors is much less powerful than the freedom of changing hard cores. For instance, changing the form factors does not change the volume integral of the potentials). In our approach, we have introduced the form factors *per channel*. For several reasons we did not choose to differentiate between the different kind of mesons w.r.t. the form factor. First, that would introduce more parameters in the model for both NN and YN. Secondly, it is believed that the nature of the short range potentials is at present, at least quantitatively, poorly understood. Probably any OBE-model includes effectively also forces which are not of the OBE-type. It is clearly allowed to parameterize these unknown short range forces for the different YN-channels independently.

The contents of this paper are as follows. In section 2 we define the OBE-potentials for the Lippmann-Schwinger equation. In section 3 the OBE-potentials in momentum space for pseudo-scalar, vector, scalar, and diffractive exchanges are given and discussed. In section 4 we systematically describe the Fourier transformation to configuration space for the potentials of section 3. In section 5 we outline the treatment of the multichannel Schrödinger equation with the nonlocal central potentials derived in section 3. In section 6 we discuss the form factor assignments in the context of SU(3). In section 7 the results for the coupling constants, $F/(F+D)$-ratio's, and mixing angles are discussed and compared with the literature and the nonrelativistic quark model. In section 8 we present the results of the fit to the YN-data. Here the results for the Λp-, $\Sigma^- p$-, and $\Sigma^+ p$-data are shown and discussed in detail. At several points in sections 7 and 8 the results are compared with the former hard-core models for YN-scattering of the Nijmegen group, Refs. [6,7].

II. DEFINITION OF THE POTENTIALS FOR THE LIPPMANN-SCHWINGER EQUATION

We consider the hyperon-nucleon reactions

$$Y(p_1, s_1) + N(p_2, s_2) \rightarrow \; Y'(p_1', s_1') + N'(p_2', s_2') \; . \tag{1}$$

Like in Ref. [6], whose conventions we will follow in this paper, we will also refer to Y and Y' as particles 1 and 3 and to N and N' as particles 2 and 4. The four momentum of particle i is $p_i = (E_i, \mathbf{p}_i)$ where $E_i = \sqrt{\mathbf{p}_i^2 + M_i^2}$ and M_i is the mass. The transition amplitude matrix M is related to the S-matrix via

$$\langle f|S|i\rangle = \langle f|i\rangle - i(2\pi)^4\delta^4(P_f - P_i)\langle f|M|i\rangle \ , \tag{2}$$

where $P_i = p_1 + p_2$ and $P_f = p_1' + p_2'$ represent the total four momentum for the initial state $|i\rangle$ and the final state $|f\rangle$. The latter refer to the two-particle states, which we normalize in the following way

$$\langle \mathbf{p}_1', \mathbf{p}_2'|\mathbf{p}_1, \mathbf{p}_2\rangle = (2\pi)^3 2E(\mathbf{p}_1)\delta^3(\mathbf{p}_1' - \mathbf{p}_1)\cdot(2\pi)^3 2E(\mathbf{p}_2)\delta^3(\mathbf{p}_2' - \mathbf{p}_2) \ . \tag{3}$$

Three-dimensional integral equations for the amplitudes $\langle f|M|i\rangle$ can be derived in various ways. See for example Refs. [6,17–20]. In Ref. [3] is given a derivation based on two-particle unitarity and the analyticity properties of the amplitudes, using the N/D-formalism. The equation obtained with this method is

$$\begin{aligned} &M_{fi}(\mathbf{q}_f, \mathbf{q}_i; s) = W_{fi}(\mathbf{q}_f, \mathbf{q}_i; s) + \\ &\frac{1}{(2\pi)^3}\sum_n \int d^3k_n W_{fn}(\mathbf{q}_f, \mathbf{k}_n; s) G_0(\mathbf{k}_n, s) M_{ni}(\mathbf{k}_n, \mathbf{q}_i; s) \ , \end{aligned} \tag{4}$$

where $\mathbf{q}_i$ and $\mathbf{q}_f$ denote the initial and final state momenta, and

$$G_0(\mathbf{k}; s) = \frac{1}{2}\frac{E_1(\mathbf{k}) + E_2(\mathbf{k})}{E_1(\mathbf{k})E_2(\mathbf{k})}\left[s - (E_1(\mathbf{k}) + E_2(\mathbf{k}))^2 + i\varepsilon\right]^{-1} \ , \tag{5}$$

with $s = (E_1(\mathbf{p}) + E_2(\mathbf{p}))^2$. This follows from Eq.(4.27) in Ref. [3]. This same equation has been derived by Gersten, Verhoeven, and de Swart (Ref. [17]) in the context of an approach which uses the Bethe-Salpeter equation. Also in Ref. [3] it is shown that the pseudopotential $\langle f|W|i\rangle$ corresponds in the pole-approximation to the Feynman-amplitudes for OBE-exchanges with form factors at the BBM-vertices. In order to arrive at a Lippmann-Schwinger equation, we choose a new Green-function $g(\mathbf{k}; s)$ which satisfies a dispersion relation in $\mathbf{p}^2(s)$ rather than in s. Then we get, like in Refs. [3,6,20],

$$g(\mathbf{k}_n; s) = \frac{-1}{2[E_1(\mathbf{k}_n) + E_2(\mathbf{k}_n)]}(\mathbf{k}_n^2 - \mathbf{q}_n^2 - i\varepsilon)^{-1} \ , \tag{6}$$

where $\mathbf{q}_n$ is the on-energy-shell momentum. This Green-function is eventually used in the integral equation Eq.(4) instead of $G_0(\mathbf{k}_n; s)$. So the corrections to $\langle f|W|i\rangle$ due to the transformation of the Green-functions are neglected. They are of higher order in the couplings and are usually discarded in an OBE-approach. With the substitution of g for G, Eq.(5) becomes identical to Eq.(2.19) of Ref. [6]. From now on we follow section II of Ref. [6] in detail. The transformation to the non-relativistic normalization of the two-particle states leads to states with

$$(\mathbf{p}_1', s_1'; \mathbf{p}_2', s_2' | \mathbf{p}_1, s_1; \mathbf{p}_2, s_2) = (2\pi)^6 \delta^3(\mathbf{p}_1' - \mathbf{p}_1)\delta^3(\mathbf{p}_2' - \mathbf{p}_2)\delta_{s_1', s_1}\delta_{s_2', s_1} \ . \tag{7}$$

For these states we define the T-matrix by

$$(f|T|i) = \{4M_{34}(E_3 + E_4)\}^{-\frac{1}{2}} \langle f|M|i\rangle \{4M_{12}(E_1 + E_2)\}^{-\frac{1}{2}} \ , \tag{8}$$

and get from Eq. (4) the Lippmann-Schwinger equation

$$\begin{aligned} &(3,4|T|1,2) = (3,4|V|1,2) + \\ &\frac{1}{(2\pi)^3}\sum_n \int d^3k_n (3,4|V|n_1,n_2)\frac{2M_{n_1,n_2}}{\mathbf{q}_n^2 - \mathbf{k}_n^2 + i\varepsilon}(n_1,n_2|T|1,2) \ , \end{aligned} \tag{9}$$

and where analogously to Eq. (8) the potential V is defined as

$$(f|V|i) = \{4M_{34}(E_3 + E_4)\}^{-\frac{1}{2}} \langle f|W|i\rangle \{4M_{12}(E_1 + E_2)\}^{-\frac{1}{2}} \ . \tag{10}$$

Using rotational invariance and parity conservation we expand the T-matrix, which is a 4×4-matrix in Pauli-spinor space, into a complete set of Pauli-spinor invariants (Refs. [6,15])

$$T = \sum_{i=1}^{8} T_i(\mathbf{q}_f^2, \mathbf{q}_i^2, \mathbf{q}_i{\cdot}\mathbf{q}_f) P_i \ . \tag{11}$$

Introducing

$$\mathbf{q} = \frac{1}{2}(\mathbf{q}_f + \mathbf{q}_i), \qquad \mathbf{k} = \mathbf{q}_f - \mathbf{q}_i, \qquad \mathbf{n} = \mathbf{q}_i \times \mathbf{q}_f = \mathbf{q} \times \mathbf{k} \tag{12}$$

we choose for the operators P_i in spin-space

$$\begin{array}{ll} P_1 = 1 & P_2 = \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2} \\ P_3 = (\boldsymbol{\sigma_1}\cdot\mathbf{k})(\boldsymbol{\sigma_2}\cdot\mathbf{k}) - \frac{1}{3}(\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2})\mathbf{k}^2 & P_4 = \frac{i}{2}(\boldsymbol{\sigma_1}+\boldsymbol{\sigma_2})\cdot\mathbf{n} \\ P_5 = (\boldsymbol{\sigma_1}\cdot\mathbf{n})(\boldsymbol{\sigma_2}\cdot\mathbf{n}) & P_6 = \frac{i}{2}(\boldsymbol{\sigma_1}-\boldsymbol{\sigma_2})\cdot\mathbf{n} \\ P_7 = (\boldsymbol{\sigma_1}\cdot\mathbf{q})(\boldsymbol{\sigma_2}\cdot\mathbf{k}) + (\boldsymbol{\sigma_1}\cdot\mathbf{k})(\boldsymbol{\sigma_2}\cdot\mathbf{q}) & \\ P_8 = (\boldsymbol{\sigma_1}\cdot\mathbf{q})(\boldsymbol{\sigma_2}\cdot\mathbf{k}) - (\boldsymbol{\sigma_1}\cdot\mathbf{k})(\boldsymbol{\sigma_2}\cdot\mathbf{q}) & \end{array} \tag{13}$$

Here we follow Refs. [6,15], except that we have chosen here P_3 to be a purely 'tensor-force' operator.

In the OBEP-approximation we consider only second-order irreducible diagrams contributing to the kernel i.e. $W = M^{irr(2)}$. Similarly to Eq. (11) we expand the potentials V. Again following Ref. [6], we neglect the potential forms P_7 and P_8, and also the dependence of the potentials on $\mathbf{k}\cdot\mathbf{q}$. Consequently the potentials can be expanded as follows

$$V = \sum_{i=1}^{6} V_i(\mathbf{k}^2, \mathbf{q}^2) P_i \ . \tag{14}$$

In the following we will exploit this decomposition extensively.

III. ONE-BOSON-EXCHANGE POTENTIALS IN MOMENTUM SPACE

In this section we extend the NN-potentials given in Ref. [1] to the YN-channels. The local interaction Hamilton densities for the different couplings are
a) Pseudoscalar-meson exchange

$$\mathcal{H}_{PV} = i\frac{f_P}{m_{\pi^+}}\bar{\psi}\gamma_\mu\gamma_5\psi\partial^\mu\phi_P \ , \tag{15}$$

b) Vector-meson exchange

$$\mathcal{H}_V = ig_V\bar{\psi}\gamma_\mu\psi\phi_V^\mu + \frac{f_V}{4\mathcal{M}}\bar{\psi}\sigma_{\mu\nu}\psi(\partial^\mu\phi_V^\nu - \partial^\nu\phi_V^\mu) \ , \tag{16}$$

c) Scalar-meson exchange

$$\mathcal{H}_S = g_S\bar{\psi}\psi\phi_S \ , \tag{17}$$

where $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2i$ and m_{π^+} and $\mathcal{M}$ are scaling masses, chosen to be the charged pion and the proton mass, respectively. Note that the vertices for 'diffractive'-exchange have the same Lorentz structure as those for scalar-meson-exchange.

Including form factors $f(\mathbf{x}' - \mathbf{x})$, the interaction densities are modified to

$$H_X(\mathbf{x}) = \int d^3x'\, f(\mathbf{x}' - \mathbf{x})\mathcal{H}_X(\mathbf{x}') \ , \tag{18}$$

where $X = PV$, V, or S. Because of this 'convolutive' form, the potentials in momentum space are the same as for point interactions, except that the coupling constants are multiplied by the Fourier tranform of the form factors.

In the derivation of the V_i we use the following approximations, which can be justified for low energy scattering.

1. We make the expansion $E(p) \approx \left[\mathbf{k}^2/4 + \mathbf{q}^2 + M^2\right]^{\frac{1}{2}} \approx M + \mathbf{k}^2/8M + \mathbf{q}^2/2M$ and keep only terms up to first order in $\mathbf{k}^2/M$ and $\mathbf{q}^2/M$. This except for the form factors where the full $\mathbf{k}^2$-dependence is kept throughout the calculations. Notice that the gaussian form factors suppress the high $\mathbf{k}^2$-contributions strongly.

2. In the meson propagators $((p_1 - p_3)^2 + m^2) \approx (\mathbf{k}^2 + m^2)$.

3. When two different hyperons are involved at a BBM-vertex (e.g. Λ and Σ) their average mass is used in the potentials and the non-zero component of the momentum transfer is accounted for by using an effective mass in the meson propagator (for details see Ref. [21] and Ref. [15]).

Due to the approximations we get only a linear dependence on $\mathbf{q}^2$ for V_1. In the following, we write

$$V_1(\mathbf{k}^2, \mathbf{q}^2) = V_{1a}(\mathbf{k}^2) + V_{1b}(\mathbf{k}^2)\mathbf{q}^2 \ . \tag{19}$$

The OBE-potentials are now obtained in the standard way (see e.g. Refs. [1,6]) by evaluating the YN-interaction in Born-approximation. We write the potentials V_i of Eqs. (14) and (19) in the form

$$V_i(\mathbf{k}^2, \mathbf{q}^2) = \sum_X \Omega_i^{(X)}(\mathbf{k}^2) \cdot \Delta^{(X)}(\mathbf{k}^2, m^2, \Lambda^2) \ , \tag{20}$$

where $X = P, V, S$, and D (P = pseudo-scalar, V = vector, S = scalar, and D = diffractive). Furthermore

$$\Delta^{(X)}(\mathbf{k}^2, m^2, \Lambda^2) = \frac{1}{\mathbf{k}^2 + m^2} \cdot e^{-\mathbf{k}^2/\Lambda^2} \tag{21}$$

for $X = P, V, S$, and

$$\Delta^{(D)}(\mathbf{k}^2, m^2, \Lambda^2) = \frac{1}{\mathcal{M}^2} e^{-\mathbf{k}^2/(4m_P^2)} \tag{22}$$

for $X = D$. In the latter expression $\mathcal{M}$ is a universal scaling mass, which is again taken to be the proton mass. The mass parameter m_P controls the $\mathbf{k}^2$-dependence of the pomeron-, f-, f'-, A_2-, and $K^{\star\star}$-potentials. For the non-strange mesons we find, using the approximations (1-5), the following contributions to the different $\Omega_i^{(X)}$:

a) pseudoscalar-meson exchange:

$$\begin{aligned} \Omega_2^{(P)} &= g_{13}^P g_{24}^P \left(\frac{\mathbf{k}^2}{12 M_Y M_N} \right) \\ \Omega_3^{(P)} &= -g_{13}^P g_{24}^P \left(\frac{1}{4 M_Y M_N} \right) \end{aligned} \tag{23}$$

b) vector-meson exchange:

$$
\begin{aligned}
\Omega^{(V)}_{1a} &= \left\{ g^V_{13} g^V_{24} \left(1 - \frac{\mathbf{k}^2}{8 M_Y M_N}\right) - g^V_{13} f^V_{24} \frac{\mathbf{k}^2}{4 \mathcal{M} M_N} \right. \\
& \qquad\qquad \left. - f^V_{13} g^V_{24} \frac{\mathbf{k}^2}{4 \mathcal{M} M_Y} + f^V_{13} f^V_{24} \frac{\mathbf{k}^4}{16 \mathcal{M}^2 M_Y M_N} \right\} \\
\Omega^{(V)}_{1b} &= g^V_{13} g^V_{24} \left(\frac{3}{2 M_Y M_N}\right) \\
\Omega^{(V)}_{2} &= -\frac{2}{3} \mathbf{k}^2\, \Omega^{(V)}_3 \\
\Omega^{(V)}_{3} &= \left\{ (g^V_{13} + f^V_{13} \frac{M_Y}{\mathcal{M}})(g^V_{24} + f^V_{24} \frac{M_N}{\mathcal{M}}) - f^V_{13} f^V_{24} \frac{\mathbf{k}^2}{8 \mathcal{M}^2} \right\} / (4 M_Y M_N) \\
\Omega^{(V)}_{4} &= -\left\{ 12 g^V_{13} g^V_{24} + 8 (g^V_{13} f^V_{24} + f^V_{13} g^V_{24}) \frac{\sqrt{M_Y M_N}}{\mathcal{M}} \right. \\
& \qquad\qquad \left. - f^V_{13} f^V_{24} \frac{3 \mathbf{k}^2}{\mathcal{M}^2} \right\} / (8 M_Y M_N) \\
\Omega^{(V)}_{5} &= -\left\{ g^V_{13} g^V_{24} + 4 (g^V_{13} f^V_{24} + f^V_{13} g^V_{24}) \frac{\sqrt{M_Y M_N}}{\mathcal{M}} \right. \\
& \qquad\qquad \left. + 8 f^V_{13} f^V_{24} \frac{M_Y M_N}{\mathcal{M}^2} \right\} / (16 M_Y^2 M_N^2) \\
\Omega^{(V)}_{6} &= -\left\{ (g^V_{13} g^V_{24} + f^V_{13} f^V_{24} \frac{\mathbf{k}^2}{4 \mathcal{M}^2}) \frac{(M_N^2 - M_Y^2)}{4 M_Y^2 M_N^2} \right. \\
& \qquad\qquad \left. - (g^V_{13} f^V_{24} - f^V_{13} g^V_{24}) \frac{1}{\sqrt{\mathcal{M}^2 M_Y M_N}} \right\}
\end{aligned} \tag{24}
$$

c) scalar-meson exchange:

$$
\begin{aligned}
\Omega^{(S)}_{1a} &= -g^S_{13} g^S_{24} \left(1 + \frac{\mathbf{k}^2}{8 M_Y M_N}\right) \\
\Omega^{(S)}_{1b} &= g^S_{13} g^S_{24} \frac{1}{2 M_Y M_N} \\
\Omega^{(S)}_{4} &= -g^S_{13} g^S_{24} \frac{1}{2 M_Y M_N} \\
\Omega^{(S)}_{5} &= g^S_{13} g^S_{24} \frac{1}{16 M_Y^2 M_N^2} \\
\Omega^{(S)}_{6} &= -g^S_{13} g^S_{24} \frac{(M_N^2 - M_Y^2)}{4 M_Y M_N}
\end{aligned} \tag{25}
$$

d) ‘diffractive-exchange’ (pomeron, f, f', A_2):
The Ω_i^D are the same as for scalar-meson-exchange Eq.(25), but with $\pm g_{13}^S g_{24}^S$ replaced by $\mp g_{13}^D g_{24}^D$.

In the expressions for Ω^P, Ω^V, and Ω^S given above, M_Y and M_N denote the mean hyperon and nucleon mass, respectively $M_Y = (M_1 + M_3)/2$ and $M_N = (M_2 + M_4)/2$, and m denotes the mass of the exchanged meson. The form factor mass Λ will be discussed in section VI.

In deriving these formulae for the $\Omega' s$ we used $1/M_N^2 + 1/M_Y^2 \approx 2/M_N M_Y$, which holds to a good approximation since the mass differences between the baryons are not large.

For the strangeness carrying exchanges $(K, K^\star, \kappa, K^{\star\star})$ we have a complete symmetric appearance of M_Y and M_N and an additional minus sign. (For the details see Ref. [6].) Therefore the resulting potentials can be obtained from those given in Eqs.(23), (24), and (25), by replacing both M_Y and M_N by $(M_Y M_N)^{\frac{1}{2}}$ and adding a minus sign. Furthermore we get non-neglegible contributions from the second part of the vector-meson propagator $(k_\mu k_\nu / m^2)$ of the $K^\star$ meson giving

$$-V_i^{K^\star} = V_i^{(V)} - \frac{(M_3 - M_1)(M_4 - M_2)}{m^2} V_i^{(S)} \ , \tag{26}$$

where in $V_i^{(S)}$ the vector meson coupling constants have to be used, and M_Y and M_N have to be replaced by $(M_Y M_N)^{\frac{1}{2}}$.

IV. ONE-BOSON-EXCHANGE POTENTIALS IN CONFIGURATION SPACE

In configuration space we describe the YN-interactions by potentials of the general form

$$\begin{aligned} V = \{ & V_C(r) + V_\sigma(r) \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} + V_T(r) S_{12} + V_{SO}(r) \mathbf{L} \cdot \mathbf{S} + V_Q(r) Q_{12} \\ & + V_{ASO}(r) \frac{1}{2}(\boldsymbol{\sigma_1} - \boldsymbol{\sigma_2}) \cdot \mathbf{L} - \frac{1}{2}(\boldsymbol{\nabla}^2 \phi(r) + \phi(r) \boldsymbol{\nabla}^2) \} \cdot P \end{aligned} \tag{27}$$

where

$$S_{12} = 3 \frac{(\boldsymbol{\sigma_1} \cdot \mathbf{r})(\boldsymbol{\sigma_2} \cdot \mathbf{r})}{r^2} - (\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}) \tag{28}$$

$$Q_{12} = \frac{1}{2} \{ (\boldsymbol{\sigma_1} \cdot \mathbf{L})(\boldsymbol{\sigma_2} \cdot \mathbf{L}) + (\boldsymbol{\sigma_2} \cdot \mathbf{L})(\boldsymbol{\sigma_1} \cdot \mathbf{L}) \} \tag{29}$$

The exchange operator $P = 1$ for hypercharge $Y = 0$ exchange and $P = -P_x P_\sigma$ for $Y \neq 0$ exchange $(K, K^\star, \kappa, K^{\star\star})$ where P_x and P_σ are the space and spin exchange operators (for a discussion see Ref. [6]). The Fourier transformation of the nonlocal operators having a linear $\mathbf{q}^2$-dependence is given in Ref. [1], Eqs. (10) and (11).

The potentials in Eq. (27) are related to those in Eqs.(20-25), by the Fourier-Bessel transforms

$$
\begin{array}{lcll}
V_C(r) & = & (1/2\pi^2) & \int dk\, k^2 j_0(kr)\{V_{1a}(k^2) - \frac{1}{4}k^2 V_{1b}(k^2)\} \\
V_\sigma(r) & = & (1/2\pi^2) & \int dk\, k^2 j_0(kr) V_2(k^2) \\
V_T(r) & = & -(1/6\pi^2) & \int dk\, k^4 j_2(kr) V_3(k^2) \\
V_{SO}(r) & = & (1/2\pi^2 r) & \int dk\, k^3 j_1(kr) V_4(k^2) \\
V_Q(r) & = & -(1/2\pi^2 r^2) & \int dk\, k^4 j_2(kr) V_5(k^2) \\
V_{ASO}(r) & = & (1/2\pi^2 r) & \int dk\, k^3 j_1(kr) V_6(k^2) \\
\phi(r) & = & (1/2\pi^2) & \int dk\, k^2 j_0(kr) V_{1b}(k^2)
\end{array}
\tag{30}
$$

where k runs from 0 to ∞ and the j_l are the spherical Bessel functions of the first kind (Ref. [22]). In the above form the relation between the potentials in momentum and configuration space is explicit and the behavior of the potentials at $r = 0$ is clearly exhibited. Due to the gaussian form factors the k-integrals exist for all r.

With the momentum space potentials of section 3, these Fourier transforms can be carried out analytically (Ref. [1]). For this we use the ϕ-functions of Ref. [1], Eqs. (12) etc., which are defined in terms of the integrals (30) by taking $V_i = \exp(-k^2/\Lambda^2)/(k^2+m^2) = \Delta(k^2, m^2, \Lambda^2)$ which gives

$$
\begin{array}{rcll}
\frac{m}{4\pi}(-m^2)^n \phi_C^n(r) & = & (1/2\pi^2) & \int dk\, k^2 j_0(kr)(k^2)^n \Delta(k^2, m^2, \Lambda^2) \\
-\frac{m^3}{4\pi}(-m^2)^n \phi_T^n(r) & = & -(1/6\pi^2) & \int dk\, k^4 j_2(kr)(k^2)^n \Delta(k^2, m^2, \Lambda^2) \\
\frac{m^3}{4\pi}(-m^2)^n \phi_{SO}^n(r) & = & (1/2\pi^2 r) & \int dk\, k^3 j_1(kr)(k^2)^n \Delta(k^2, m^2, \Lambda^2)
\end{array}
\tag{31}
$$

Introducing the auxiliary functions $\Delta_X^n(\Lambda, r)$ by

$$
\begin{array}{rcll}
\frac{m}{4\pi}(-m^2)^{n+1} \Delta_C^{n+1}(\Lambda, r) & = & (1/2\pi^2) & \int dk\, k^2 j_0(kr)(k^2)^n \exp(-k^2/\Lambda^2) \\
-\frac{m^3}{4\pi}(-m^2)^{n+1} \Delta_T^{n+1}(\Lambda, r) & = & -(1/6\pi^2) & \int dk\, k^4 j_2(kr)(k^2)^n \exp(-k^2/\Lambda^2) \\
\frac{m^3}{4\pi}(-m^2)^{n+1} \Delta_{SO}^{n+1}(\Lambda, r) & = & (1/2\pi^2 r) & \int dk\, k^3 j_1(kr)(k^2)^n \exp(-k^2/\Lambda^2)
\end{array}
\tag{32}
$$

one can readily show, by making the substitution $k^2 \rightarrow (k^2 + m^2) - m^2$, that

$$
\phi_X^{n+1}(r) = \phi_X^n(r) + \Delta_X^{n+1}(\Lambda, r) \tag{33}
$$

for $X = C, T$, and SO. Also one has

$$
\Delta_X^{n+1}(\Lambda, r) = -\frac{1}{2}(\frac{\Lambda}{m})^2(\Lambda\frac{d}{d\Lambda})\Delta_X^n(\Lambda, r)\ . \tag{34}
$$

The functions $\phi_C^0(r)$, $\phi_T^0(r)$, and $\phi_{SO}^0(r)$ are given in Ref. [1], Eqs.(13),(17), and (20). They read

$$
\begin{aligned}
\phi^0_C(r) &= \exp(m^2/\Lambda^2)\left[e^{-mr}\text{erfc }(-\frac{\Lambda r}{2}+\frac{m}{\Lambda}) - e^{mr}\text{erfc }(\frac{\Lambda r}{2}+\frac{m}{\Lambda})\right]/2mr \\
\phi^0_T(r) &= \Big\{\exp(m^2/\Lambda^2)\Big[[1+mr+\frac{1}{3}(mr)^2]e^{-mr}\text{erfc }(-\frac{\Lambda r}{2}+\frac{m}{\Lambda}) \\
&\quad -[1-mr+\frac{1}{3}(mr)^2]e^{mr}\text{erfc}(\frac{\Lambda r}{2}+\frac{m}{\Lambda})\Big] \\
&\quad -\frac{4}{\sqrt{\pi}}(\frac{\Lambda r}{2})\left[1+\frac{2}{3}(\frac{\Lambda r}{2})^2\right]\exp\left[-\left(\frac{\Lambda r}{2}\right)^2\right]\Big\}/2(mr)^3 \qquad (35) \\
\phi^0_{SO}(r) &= \Big\{\exp(m^2/\Lambda^2)\Big[(1+mr)e^{-mr}\text{erfc}(-\frac{\Lambda r}{2}+\frac{m}{\Lambda}) \\
&\quad -(1-mr)e^{mr}\text{erfc}(\frac{\Lambda r}{2}+\frac{m}{\Lambda})\Big] \\
&\quad -\frac{4}{\sqrt{\pi}}(\frac{\Lambda r}{2})\exp\left[-\left(\frac{\Lambda r}{2}\right)^2\right]\Big\}/2(mr)^3\ .
\end{aligned}
$$

For $\Delta^1_C(\Lambda,r)$, $\Delta^1_T(\Lambda,r)$, and $\Delta^1_{SO}(\Lambda,r)$ we have (compare Ref. [1], Eqs. (14),(18), and (21)) [1].

$$
\begin{aligned}
\Delta^1_C(\Lambda,r) &= -\frac{1}{2\sqrt{\pi}}\ \left(\frac{\Lambda}{m}\right)^3\exp\left[-\left(\frac{\Lambda r}{2}\right)^2\right] \\
\Delta^1_T(\Lambda,r) &= -\frac{1}{6\sqrt{\pi}}\ \left(\frac{\Lambda}{m}\right)^5\exp\left[-\left(\frac{\Lambda r}{2}\right)^2\right]\left(\frac{\Lambda r}{2}\right)^2 \\
\Delta^1_{SO}(\Lambda,r) &= -\frac{1}{4\sqrt{\pi}}\ \left(\frac{\Lambda}{m}\right)^5\exp\left[-\left(\frac{\Lambda r}{2}\right)^2\right]
\end{aligned}
\qquad (36)
$$

It is clear from Eq.(30) that for $V_\sigma(r)$ and $\phi(r)$ the basic integrals needed for the Fourier transformation are the same as for $V_C(r)$, i.e. $\phi^n_C(r)$. Similarly the basic integrals for $V_{ASO}(r)$ and $V_Q(r)$ are readily given in terms of $\phi^n_{SO}(r)$ and $\phi^n_T(r)$, respectively. For instance, Eq. (22) of Ref. [1] concerning the Fourier transformation of the quadratic spin-orbit potential can readily be obtained from the formulae given in this paper.

For the 'diffractive' exchanges, only gaussian integrals occur in the course of the Fourier transformation and they can be evaluated in terms of the Δ^n_X-functions or directly (see Ref. [1], Eqs.(25-28)).

Using these Fourier-Bessel transforms one can perform the Fourier transformation in a straightforward manner. The results are

a) pseudoscalar-meson exchange

$$
V_{PS}(r)\ =\ \frac{m}{4\pi}\left(g^P_{13}g^P_{24}\frac{m^2}{4M_YM_N}\left\{\tfrac{1}{3}(\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2})\phi^1_C+S_{12}\phi^0_T\right\}\right)P\ . \qquad (37)
$$

---

[1]Note that in Eq.(15) of Ref. [1], there is an erroneous m^2 in front of ϕ^1_C.

b) vector-meson exchange

$$
\begin{aligned}
V_V(r) \;=\; & \frac{m}{4\pi}\Bigg(\left\{ g^V_{13}g^V_{24}\left[\phi^0_C + \frac{m^2}{2M_Y M_N}\phi^1_C - \frac{3}{4M_Y M_N}(\boldsymbol{\nabla}^2\phi^0_C + \phi^0_C\boldsymbol{\nabla}^2)\right]\right. \\
& + \left.\left[g^V_{13}f^V_{24}\frac{m^2}{4\mathcal{M}M_N} + f^V_{13}g^V_{24}\frac{m^2}{4\mathcal{M}M_Y}\right]\phi^1_C + f^V_{13}f^V_{24}\frac{m^4}{16\mathcal{M}^2 M_Y}\phi^2_C\right\} \\
& + \frac{m^2}{4M_Y M_N}\left\{\left[(g^V_{13}+f^V_{13}\frac{M_Y}{\mathcal{M}}) + (g^V_{24}+f^V_{24}\frac{M_N}{\mathcal{M}})\right]\phi^1_C\right. \\
& \qquad + \left. f^V_{13}f^V_{24}\frac{m^2}{8\mathcal{M}^2}\phi^2_C\right\}\frac{2}{3}(\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}) \\
& - \frac{m^2}{4M_Y M_N}\left\{\left[(g^V_{13}+f^V_{13}\frac{M_Y}{\mathcal{M}}) + (g^V_{24}+f^V_{24}\frac{M_N}{\mathcal{M}})\right]\phi^0_T\right. \\
& \qquad \left. + f^V_{13}f^V_{24}\frac{m^2}{8\mathcal{M}^2}\phi^1_T\right\} S_{12} \\
& - \frac{m^2}{M_Y M_N}\left\{\left[\frac{3}{2}g^V_{13}g^V_{24} + (g^V_{13}f^V_{24}+f^V_{13}g^V_{24})\frac{\sqrt{M_Y M_N}}{\mathcal{M}}\right]\phi^0_{SO}\right. \\
& \qquad + \left.\frac{3}{8}f^V_{13}f^V_{24}\frac{m^2}{\mathcal{M}^2}\phi^1_{SO}\right\}\mathbf{L}\cdot\mathbf{S} \\
& + \frac{m^4}{16M_Y^2 M_N^2}\left\{\left[g^V_{13}g^V_{24} + 4(g^V_{13}f^V_{24}+f^V_{13}g^V_{24})\frac{\sqrt{M_Y M_N}}{\mathcal{M}}\right.\right. \\
& \qquad \left.\left. + 8f^V_{13}f^V_{24}\frac{M_Y M_N}{\mathcal{M}^2}\right]\right\}\frac{3}{(mr)^2}\phi^0_T Q_{12} \\
& - \frac{m^2}{M_Y M_N}\left\{\left[(g^V_{13}g^V_{24} - f^V_{13}f^V_{24}\frac{m^2}{\mathcal{M}^2})\frac{(M_N^2-M_Y^2)}{4M_Y M_N}\right.\right. \\
& \qquad \left.\left. -(g^V_{13}f^V_{24} - f^V_{13}g^V_{24})\frac{\sqrt{M_Y M_N}}{\mathcal{M}}\right]\phi^0_{SO}\right\}\cdot\ \frac{1}{2}(\boldsymbol{\sigma_1}-\boldsymbol{\sigma_2})\cdot\mathbf{L}\Bigg)P\ .
\end{aligned}
\tag{38}
$$

c) scalar-meson exchange

$$
\begin{aligned}
V_S(r) \;=\; & -\frac{m}{4\pi}\Bigg(g^S_{13}g^S_{24}\Bigg\{ (\phi^0_C - \frac{m^2}{4M_Y M_N}\phi^1_C) \\
& \qquad + \frac{m^2}{2M_Y M_N}\phi^0_{SO}\mathbf{L}\cdot\mathbf{S} + \frac{m^4}{16M_Y^2 M_N^2}\frac{3}{(mr)^2}\phi^0_T Q_{12} \\
& \qquad + \frac{m^2}{M_Y M_N}\left[\frac{(M_N^2-M_Y^2)}{4M_Y M_N}\right]\phi^0_{SO}\cdot\frac{1}{2}(\boldsymbol{\sigma_1}-\boldsymbol{\sigma_2})\cdot\mathbf{L} \\
& \qquad + \frac{1}{4M_Y M_N}(\boldsymbol{\nabla}^2\phi^0_C + \phi^0_C\boldsymbol{\nabla}^2)\Bigg\}\Bigg) P\ .
\end{aligned}
\tag{39}
$$

d) diffractive exchange

$$
\begin{aligned}
V_D(r) \;=\; & \frac{m_P}{4\pi}\Bigg(g^D_{13}g^D_{24}\frac{4}{\sqrt{\pi}}\frac{m_P^2}{\mathcal{M}^2}\cdot\Bigg\{\Bigg[1+\frac{m_P^2}{2M_YM_N}(3-2m_P^2r^2) \\
& \qquad +\frac{m_P^2}{M_YM_N}\mathbf{L}\cdot\mathbf{S}+\Big(\frac{m_P}{2M_YM_N}\Big)^2 Q_{12} \\
+ \; & \frac{m_P^2}{M_YM_N}\left[\frac{(M_N^2-M_Y^2)}{4M_YM_N}\right]\cdot\frac{1}{2}(\boldsymbol{\sigma_1}-\boldsymbol{\sigma_2})\cdot\mathbf{L}\Bigg]\,e^{-m_P^2r^2} \\
& \qquad +\frac{1}{4M_YM_N}(\boldsymbol{\nabla}^2e^{-m_P^2r^2}+e^{-m_P^2r^2}\boldsymbol{\nabla}^2)\Bigg\}\Bigg)P\;.
\end{aligned}
\tag{40}
$$

In these formulae m is the average mass in the meson isospin-multiplet.

V. MULTICHANNEL SCHRÖDINGER EQUATION

We have to solve the Lippmann-Schwinger for the YN-reactions:

$$
\begin{array}{lcll}
\Sigma^+p & \rightarrow & \Sigma^+p & \text{(I)} \\
(\Lambda p,\Sigma^+n,\Sigma^0p) & \rightarrow & (\Lambda p,\Sigma^+n,\Sigma^0p) & \text{(II)} \\
(\Lambda n,\Sigma^0n,\Sigma^-p) & \rightarrow & (\Lambda n,\Sigma^0n,\Sigma^-p) & \text{(III)}
\end{array}
\tag{41}
$$

So, reaction (I) involves only a single two-particle channel, but for (II) and (III) the state vectors $|\psi)$ are three component vectors and the potentials V are 3×3 matrices in the two-particle channel space. For system (II) we have on the physical particle basis

$$
|\psi)_{II}=\begin{pmatrix}|\psi)_{\Lambda p}\\ |\psi)_{\Sigma^+n}\\ |\psi)_{\Sigma^0p}\end{pmatrix}\quad,\quad V_{II}=\begin{pmatrix}V_{\Lambda\Lambda} & V_{\Lambda\Sigma^+} & V_{\Lambda\Sigma^0}\\ V_{\Sigma^+\Lambda} & V_{\Sigma^+\Sigma^+} & V_{\Sigma^+\Sigma^0}\\ V_{\Sigma^0\Lambda} & V_{\Sigma^0\Sigma^+} & V_{\Sigma^0\Sigma^0}\end{pmatrix}
\tag{42}
$$

and likewise for system (III) we have

$$
|\psi)_{III}=\begin{pmatrix}|\psi)_{\Lambda n}\\ |\psi)_{\Sigma^0n}\\ |\psi)_{\Sigma^-p}\end{pmatrix}\quad,\quad V_{III}=\begin{pmatrix}V_{\Lambda\Lambda} & V_{\Lambda\Sigma^0} & V_{\Lambda\Sigma^-}\\ V_{\Sigma^0\Lambda} & V_{\Sigma^0\Sigma^0} & V_{\Sigma^0\Sigma^-}\\ V_{\Sigma^-\Lambda} & V_{\Sigma^-\Sigma^0} & V_{\Sigma^-\Sigma^-}\end{pmatrix}
\tag{43}
$$

where $V_{\Lambda\Sigma^+}=(\Lambda p|V|\Sigma^+n)$, etc.

In the case of exact isospin symmetry of the potential the matrices V_{II} and V_{III} are connected by an isospin rotation.

Expressing the potentials on the physical particle basis in the potential matrix elements on the isospin basis, we find for (II)

$$
V_{II}=\begin{pmatrix}
V_{\Lambda\Lambda} & \sqrt{\frac{2}{3}}V_{\Lambda\Sigma} & -\sqrt{\frac{1}{3}}V_{\Lambda\Sigma}\\
\sqrt{\frac{2}{3}}V_{\Lambda\Sigma} & \frac{1}{3}(2V_{\Sigma\Sigma}(\frac{1}{2})+V_{\Sigma\Sigma}(\frac{3}{2})) & \frac{\sqrt{2}}{3}(-V_{\Sigma\Sigma}(\frac{1}{2})+V_{\Sigma\Sigma}(\frac{3}{2}))\\
-\sqrt{\frac{1}{3}}V_{\Lambda\Sigma} & \frac{\sqrt{2}}{3}(-V_{\Sigma\Sigma}(\frac{1}{2})+V_{\Sigma\Sigma}(\frac{3}{2})) & \frac{1}{3}(V_{\Sigma\Sigma}(\frac{1}{2})+2V_{\Sigma\Sigma}(\frac{3}{2}))
\end{pmatrix}
\tag{44}
$$

and for (III)

$$V_{III} = \begin{pmatrix} V_{\Lambda\Lambda} & \sqrt{\frac{1}{3}}V_{\Lambda\Sigma} & -\sqrt{\frac{2}{3}}V_{\Lambda\Sigma} \\ \sqrt{\frac{1}{3}}V_{\Lambda\Sigma} & \frac{1}{3}(V_{\Sigma\Sigma}(\frac{1}{2}) + 2V_{\Sigma\Sigma}(\frac{3}{2})) & \frac{\sqrt{2}}{3}(-V_{\Sigma\Sigma}(\frac{1}{2}) + V_{\Sigma\Sigma}(\frac{3}{2})) \\ -\sqrt{\frac{2}{3}}V_{\Lambda\Sigma} & \frac{\sqrt{2}}{3}(-V_{\Sigma\Sigma}(\frac{1}{2}) + V_{\Sigma\Sigma}(\frac{3}{2})) & \frac{1}{3}(2V_{\Sigma\Sigma}(\frac{1}{2}) + V_{\Sigma\Sigma}(\frac{3}{2})) \end{pmatrix} \tag{45}$$

In Table I we give the channels and states relevant to this work and in anticipation to the further discusions, we also give here the SU(3)-contents of the potentials on the isospin basis.

The multi-channel Lippmann-Schwinger equation for the components of the state vector $\tilde{\psi}_i(\mathbf{q}_i)$ corresponding to Eq.(9) reads

$$\tilde{\psi}_i(\mathbf{q}_i) = \tilde{\phi}_i(\mathbf{q}_i) + 2M_{n_1,n_2} \sum_n \int \frac{d^3k_n}{(2\pi)^3} \frac{1}{\mathbf{q}_n^2 - \mathbf{k}_n^2 + i\varepsilon} (i_1, i_2|V|n_1, n_2)\tilde{\psi}_n(\mathbf{q}_n) \tag{46}$$

The multi-channel Schrödinger equation in configuration space is derived from the Lippmann-Schwinger equation through the standard Fourier transformation (see for example Ref. [1]). We find

$$\begin{aligned} \sum_j \Big[(-2\mu_i)^{-1}\delta_{i,j}\boldsymbol{\nabla}^2 + V_{i,j}(r) - \{\boldsymbol{\nabla}^2\phi_{i,j}(r) + \phi_{i,j}(r)\boldsymbol{\nabla}^2\} \\ +M_i\delta_{i,j}\Big]\,\psi_j(r) = E_i\psi_i(r)\ . \end{aligned} \tag{47}$$

Here μ_i stands for the reduced mass in channel i, i.e. $M_{i,j} = \mu_i\delta_{i,j}$, $M_i = M_{i_1} + M_{i_2}$ is the total rest mass for channel i, and E is the total energy in the center of mass. By $V_{i,j}(r)$ is meant the local part of the potentials, i.e. all terms in Eq.(27) but the last one.

We make the partial wave projection of Eq.(27) and write for the partial wave $\psi_i^{LSJ}(r) = (u_{l,i}/r)\cdot Y_m^l$. Introducing the center of mass momentum $k_i^2 = 2\mu_i(E-M)_i$, and defining $f_{i,j}(r) = \mu_{i,i}\phi_{i,j}(r)$, we get from Eq.(47) for the radial Schrödinger equation

$$\begin{aligned} (\delta_{i,j} + 2f_{i,j})\left[u''_{l,j} - \frac{l(l+1)}{r^2}u_{l,j}\right] + 2f'_{i,j}u'_{l,i} \\ + \left[k_i^2\delta_{i,j} - (2\mu_i V_{i,j} + f''_{i,j})\right]u_{l,j} = 0 \end{aligned} \tag{48}$$

which we put into the form

$$N\left[u'' - \frac{l(l+1)}{r^2}u\right] + N'u' + Au + \frac{1}{2}N''u = 0\ . \tag{49}$$

We now transform away the u'-term in the standard way, by writing

$$u = Tv\ , \tag{50}$$

where the transformation matrix T satisfies the differential equation

$$2NT' + N'T = 0\ , \tag{51}$$

with the boundary condition: $T \to 1$ for $r \to \infty$. As a result $v(r)$ satisfies an ordinary local multi-channel Schrödinger equation

$$v_l'' + \left[k^2 - 2M_{red}W - \frac{l(l+1)}{r^2}\right] v_l = 0 \ , \tag{52}$$

where the 'potential' W in this equation is given by

$$2M_{red}W = (NT)^{-1}(2M_{red}V)T + \frac{1}{2}(NT)^{-1}(N'T') - [(NT)^{-1}k^2T - k^2] \tag{53}$$

and $(M_{red})_{i,j} = \mu_i \delta_{i,j}$. This is the multi-channel analogon of Eq.(39) of Ref. [1].

The differential equation for the transformation T has to be solved numerically. After T is computed we can solve the coupled-channel Schrödinger equation. For further details of handling the Schrödinger equation for coupled channels we refer to Refs. [15,23].

VI. FORM FACTORS AND SU(3)

The states and SU(3)-irreps for the NN-channels and in particular the YN-channels, are displayed in Table II. The partial connection between the YN- and NN-channels via SU(3) is evident from this table.

We assume that SU(3) symmetry is broken only kinematically, i.e. via the physical masses of the mesons and baryons. In Ref. [7] the SU(3)-irreps in the BB-channels form the basis for the parametrization of the short-range interaction with 'hard cores'. The same scheme we follow here except that the role of the 'hard cores' is taken over by the form factors. The behavior of these form factors is controlled by Λ, the so-called cut-off mass (see Eq.(21)). Thus for the S-waves the cut-off masses of the $\{27\}$ and the $\{10^\star\}$ are in principle fixed in the NN-fit. In YN we then have as free form factor parameters the $\{8_s\}$-, $\{8_a\}$-, and the $\{10\}$-cut-off masses. However, because of the various SU(3)-symmetry-breaking mechanisms included in the model, we are forced to calculate the nuclear potentials on the isospin basis. On this level, we not only have to deal with states belonging to a single SU(3)-irrep, but also with states belonging to a combination of two irreps (see Table II). In the latter case, we then allow for a separate form factor. Altogether, this would imply four different form factors to be used:

- Λ_{27} for ${}^1S_0(\Sigma N; I = 3/2)$
- Λ_{10} for ${}^3S_1(\Sigma N; I = 3/2)$
- Λ_{27+8_s} for ${}^1S_0(\Lambda N, \Sigma N; I = 1/2)$
- $\Lambda_{10^\star+8_a}$ for ${}^3S_1(\Lambda N, \Sigma N; I = 1/2)$.

Note that in the ΛN- and ΣN isospin $I = \frac{1}{2}$ channels the SU(3)-irreps have different weigths (see Table I). Therefore it is justified to use different form factor masses for these cases. However, in order to obtain a good fit this was unnecessary.

The S-waves are by far the dominant waves for fitting the low energy YN-data, which are mainly total cross sections. The P-waves are not very important for the latter and so from these data the experimental P-wave information is meagre. Also from the measured low energy differential cross sections there is no evidence for large P-waves. It appeared that we could use the same form factor for states with different L, but in isomorphic SU(3)-irreps, without giving any problems with the data at higher energies.

It turned out that imposing a form factor assignment on the basis of strict SU(3)-symmetry leads to a certain friction between the $\Sigma^+ p$ and the $\Sigma^- p$ channels for the $^3S_1$-wave. The capture ratio at rest and the $\Sigma^- p$ cross sections are very sensitive to the potential in the $\{10\}$. In order to fit these data we would have to allow for a cut-off mass Λ in the $\{10\}$ such that the model would produce an unobserved bound state in the $^3S_1$ $\Sigma^+ p$ state. To avoid this, we take a separate form factor for the $^3S_1$ $\Sigma^+ p$ and $^3S_1$ $\Sigma^- p$ $I = \frac{3}{2}$ states. This means that we introduce an SU(2)-breaking for the irrep $\{10\}$. This breaking could not be explained by refining our CSB treatment, i.e. by including the $\pi - \eta$ and $\rho - \omega$ mixing. These effects turned out to be very small. Also, introducing a breaking of the π^0-coupling [24] could not solve this.

The amount of SU(2)-breaking needed could be varied also by allowing a medium strong SU(3)-breaking in the $\{27\}$-form factor, i.e. not taking the value of Λ_{27} as found in NN. This could be justified by arguing that in the YN-channels $Y \neq 0$ exchange occurs, which introduces some medium strong SU(3)-breaking effects also in the form factor masses. However this way we can not eliminate the SU(2)-breaking completely. The $\Sigma^- p$-data force the Λ_{10} in a region where the $^3S_1$ is very sensitive to a possible SU(2)-breaking. We now have two choices: (i) we make the SU(2)-breaking in the $\{10\}$ as small as possible at the cost of an SU(3)-breaking for Λ_{27}, or (ii) we keep strict SU(3) for Λ_{27} and introduce a larger SU(2)-breaking for the Λ_{10}. In principle we have no clear reason to prefer (i) or (ii). The model (coupling constant's etc.) and the fit to the data do not depend on the choice made here. In the following we will work with choice (i). The results for choice (ii) are very similar. Although (ii) would be more appealing (one parameter less and strict SU(3) in the Λ_{27}), it appears that then the $^3S_1$ -wave in the $\Sigma^+ p$ becomes more repulsive and this seems to be unfavorable for the Σ-hyperfragments (Ref. [25]).

In Table III we finally summarize the form factor prescription as used in the calculations (choice (i), see above) and give the form factor masses which have emerged from the YN-fit. As one sees from the table, our form factor scheme is finally very simple: (i) for $\Sigma^+ p$ a $^1S_0$ - and a $^3S_1$ -form factor, (ii) for $\Lambda N - \Sigma N$ a $^1S_0$ - and a $^3S_1$ -form factor.

VII. COUPLING CONSTANTS, $F/(F+D)$ RATIO'S, AND MIXING ANGLES

The OBE-coupling constants we employ here for the description of the YN-channels are obtained from the NN-analysis of Ref. [1] using SU(3) relations. The SU(3) relations are assumed to hold for the pseudo-vector couplings of the pseudo-scalar mesons, for the Pauli-Dirac couplings of the vector mesons, for the coupling of the scalar mesons, and for the pomeron and tensor meson contributions. We have analysed the low energy YN-data (see e.g. Refs. [6,7,15,23] for a description) for all YN- channels simultaneously. An excellent solution was found which appears qualitatively even better than the Nijmegen hard-core potentials, Refs. [6,7]. The handling of SU(3) for the pseudoscalar and vector mesons has been discussed in Refs. [7,13]. Here we discuss briefly the treatment of the scalar mesons and the 'diffractive' exchange.

In the scalar meson nonet the physical ε- and $S^\star$-meson are described in terms of the SU(3)-singlet $\varepsilon 0$ and -octet state $S_0^\star$ using a single mixing angle θ_S

$$\begin{aligned} |\varepsilon\rangle &= \cos\theta_S |S_0^\star\rangle - \sin\theta_S |\varepsilon 0\rangle \\ |S^\star\rangle &= \sin\theta_S |S_0^\star\rangle + \cos\theta_S |\varepsilon 0\rangle \end{aligned} \tag{54}$$

With this convention, the ideal mixing angle for the scalar mesons in the $q^2\bar{q}^2$-picture (Ref. [26]) is $\theta_S = 35.3^0$, and in the $q\bar{q}$-picture is $\theta_S = 54.7^0$ (Ref. [27]). The convention Eq. (54) differs from that in Ref. [7]. There the same convention is used as for the vector and pseudo-scalar mesons (for definitions see Refs. [5,6]). For the vector and pseudo-scalar mesons we stick to the conventions used in Refs. [5,6]. Using these expressions for meson-mixing the couplings can readily be expressed in terms of the singlet coupling g_1, the octet coupling g_8, the $F/(F+D)$-ratio α_S, and the mixing angle θ_S.

For the 'diffractive' exchanges we take the 'bare' pomeron as an SU(3)-singlet. The tensor nonet contains the f_0 and the f_0' which are, respectively, the SU(3)-singlet and octet state. Exact SU(3) and unitarity cause a strong mixing between the 'bare' pomeron and f_0. We describe this system by P_0, which is obviously a SU(3)-singlet. Medium strong SU(3)-breaking then gives mixing of P_0 and f_0', leading to the physical pomeron P and f. In the NN-analysis the combination

$$g_P^2 = g_{PNN}^2 + g_{fNN}^2 = g_1^2 + \frac{1}{3}(4\alpha_D - 1)^2 g_8^2 \tag{55}$$

and also $g_8 = g_{A_2NN}$ has been fixed. From the expression for g_P^2 one sees that g_1 and α_D can be written in terms of g_P, g_{A_2NN} and an angle that we call ψ_D. One has

$$g_1 = \cos(\psi_D) g_P \quad , \quad (4\alpha_D - 1)/\sqrt{3} = \sin(\psi_D) g_P / g_{A_2NN} \tag{56}$$

So, in the YN-analysis we have for the diffractive contributions one extra free parameter, the angle ψ_D. Another possible relevant free parameter would be θ_D, but since we have used the same mass m_P for all diffractive exchanges, we have no SU(3)-breaking due to these exchanges and so the results are independent of θ_D. In Ref. [3] a natural kinematical SU(3)-breaking is suggested in $\Delta(\mathbf{k}^2, m^2)$ by using in Eq.(22) $M_Y M_N$ instead of $\mathcal{M}^2$ in the denominator. However, a recent analysis of the pomeron-couplings (Ref. [28]) suggests that there is almost no SU(3)-breaking of this kind.

In Tables IV and V we have listed the coupling constants. All figures refer to rationalized couplings i.e. they should be understood as $g/\sqrt{4\pi}$. Furthermore 'CSB' refers to the couplings due to charge-symmetry-breaking and are given by $g_{\Lambda\Lambda M}(\text{CSB}) = -0.0271 g_{NNM} g_{\Lambda\Sigma M}$.

The value found for α_{PV} agrees very well with the determination in weak interactions (Refs. [12,13]). The advantage of our combined NN- and YN-analysis, in contrast to most other analyses, is that α_{PV} enters in many coupling constants simultaneously ($NN\eta_8$, $\Sigma\Sigma\pi$, $\Lambda\Sigma\pi$, ΛNK, ΣNK, $\Lambda\Lambda\eta_8$, $\Sigma\Sigma\eta_8$) and is vital to several measured cross sections and ratio's. Also, the value obtained for α_V^m is in full accordance with relativistic SU(6) (Ref. [14]). Note that here α_V^e has not been fitted, but is theoretical input. Another important free parameter is the scalar mixing angle θ_S. The fit appears to be rather sensitive to this parameter, but there is still some room for variation by making at the same time adjustments for ψ_D and Λ_{10}. We obtained $\theta_S \approx 40.9^0$, a value in between ideal mixing for the scalar $q^2\bar{q}^2$- and the scalar $q\bar{q}$-states. In the region where the data can be fitted successfully the $\Sigma^- p$ elastic and inelastic cross sections depend rather steeply on θ_S. A recent determination of θ_S (Ref. [29]), based on a study of $\Gamma(S^\star \to \pi\pi)$, gave $\theta_S = 32^0$. For the angle ψ_D we obtained the value 15.5^0, which means that the pomeron is dominantly an SU(3)-singlet as is also found in high energy scattering.

The singlet couplings g_1 can readily be calculated using the values of the tables and the formulas (this paper and Refs. [5,6]) for the meson mixings.

In Table VI we give the singlet and octet coupling constants, mixing angles and $F/(F+D)$-ratio's.

For a discussion of the NNM-coupling constants we refer to Ref. [1]. In the discussion of the YYM- and YNM-coupling constants we shall restrict ourselves mainly to the pseudo-scalar mesons, because there is independent information on them available from the application of the Goldberger-Treiman relation and the analysis of KN-scattering. We compare in the following our values with those in the literature, i.e. as given in the compilations of coupling constants, Refs. [12,13].

In Refs. [12,13] are given $g^2_{\Sigma\Sigma\pi}/4\pi = 13 \pm 2$ and $g^2_{\Lambda\Sigma\pi}/4\pi = 11 \pm 1$. We have $g^2_{\Sigma\Sigma\pi}/4\pi = 11.09$ and $g^2_{\Lambda\Sigma\pi}/4\pi = 11.42$, which agree very well with those from the literature. To a certain extend, this is no surprise because (i) we have used SU(3) for the PV-coupling, and (ii) we found a perfect $F/(F+D)$-ratio. Pilkuhn showed (Ref. [30]) using the Goldberger-Treiman relation in combination with superconvergence that one can expect that only the pseudo-vector couplings satisfy SU(3) very well.

From $K^\pm N$ forward dispersion relations one has determined the YNK-coupling constants. Typically one finds (Ref. [12]) $g^2_{\Lambda NK}/4\pi = 13.9 \pm 2.6$ and $g^2_{\Sigma NK}/4\pi = 0.9 \pm 0.4$. We have $g^2_{\Lambda NK}/4\pi = 16.0$ and $g^2_{\Sigma NK}/4\pi = 1.44$, again in good agreement. These values are also perfectly compatible with the determinations of the effective coupling $g^2_Y \equiv (g^2_{\Lambda NK} + 0.84 g^2_{\Sigma NK})/4\pi$. In Ref. [12], for example, one quotes $g^2_Y = 16.6 \pm 0.7$, which is also close to our determination $g^2_Y = 17.2$. The conclusion we draw from this comparison is that the use of SU(3) for the PV-coupling has led to a seemingly perfect realistic description of the pseudo-scalar meson exchange in the present model.

We close this section by making some remarks on the couplings of the vector mesons in the context of the naive nonrelativistic quark model (henceforth referred to as QM). For similar observations pertinent to model D (Ref. [6]) and F (Ref. [7]) see Ref. [31]. Because $g_{NN\phi}/\sqrt{4\pi} = -0.31$ is fairly small and since we use ideal mixing, it is clear that the QM relations

$$g_{\Sigma\Sigma\omega} = g_{\Lambda\Lambda\omega} = (2/3) g_{NN\omega},\ g_{\Sigma\Sigma\phi} = g_{\Lambda\Lambda\phi},\ g_{NN\phi} = 0 \tag{57}$$

are indeed satisfied approximately in Table IV. To illustrate further how close we obey QM-relations for the electric couplings we look at

$$g_{NN\phi} = -\sin\theta_V g_1 + \cos\theta_V (4\alpha^e_V - 1) g_8/\sqrt{3}. \tag{58}$$

Imposing the QM restriction $g_{NN\phi} = 0$ one gets that

$$\frac{g_1}{g_8\sqrt{6}}\sqrt{2}\tan\theta_V = 1. \tag{59}$$

From Table VI we find for this combination 1.26, which is rather close to the QM-value. Model D and F give here 2.34 and 1.75, respectively (Ref. [31]).

For the magnetic couplings $G^m = (f+g)$ one would have $\alpha_V^m = 2/5$ following static SU(6), whereas we have the relativistic SU(6)-value (Ref. [14]). The QM would require, using $\alpha_V^m = 2/5$, that $G_1^m/G_8^m\sqrt{6} = 1/5$ and we have for this combination 0.308. Models D and F give here, respectively, 0.423 and 0.255.

Usually OBE-models have larger relative strengths for the tensor couplings of vector mesons than the QM (see e.g. Refs. [12,13]). In this respect the present soft-core model is closer to the QM than other models. Note that $(f/g)_{NN\rho} = 4.2$, where the QM predicts for this ratio 3.7. For the other ratio's we have $(f/g)_{NN\omega} = 0.31$, $(f/g)_{\Lambda NK^\star} = 2.2$, and $(f/g)_{\Sigma NK^\star} = -1.9$. These values are closer to the QM than those for the models D and F.

VIII. RESULTS OF THE CALCULATIONS

A. Determination of the free parameters

The values for the eight free parameters in the soft-core model are determined in the YN-analysis in a parameter search to a selected set of 35 best low-energy YN-data (Table VII, the data are from Refs. [32]– [37]). The fitted parameters and the values obtained are given in Tables IV and V.

The lowest total $\chi^2 = 15.7$ reached for the 35 data and 8 free parameters i.e. $\chi^2/d.o.f. = 0.58$. In Table VII we compare the calculated model values with the experimental ones. In calculating the cross sections the P-wave contributions have been included. (Note here that the ${}^1P_1 - {}^3P_1$ transitions are not included in our calculations.) In counting the number of free parameters we could have been less conservative and say that α_{PV} and α_V^m are fixed during the search. After that we noticed that we could fix these parameters on the 'theoretical' ideal values, we did not allow them to vary. So actually we have effectively 6 free parameters. The potentials in the different SU(3)-irreps are shown in Fig. 1. Combining these figures with Table I gives a qualitative picture of the potentials in the different isospin channels.

B. Σ^+p, Σ^-n scattering

The Σ^+p 'total' cross sections [35] (for the definition see Ref. [23]) are compared with the experimental values in Table VII and Fig 2.

The calculated cross sections agree very well with the experimental data. The angular distribution at $p_{\Sigma^+} = 170$ MeV/c is shown in Fig. 3. This distribution has $\chi^2 = 5.0$ for seven data points. The spin singlet Coulomb interference is about three times larger in magnitude than the triplet Coulomb interference, giving in total a destructive Coulomb-interference result. In the overall result for the angular distribution this is partially compensated by the

$^1S_0$ -$^1P_1$ interference term. The scattering lengths and effective ranges in the S and P waves are given in Table VIII. The $^1S_0$ scattering lengths and effective ranges are similar to those of model D (Ref. [6]) and model F (Ref. [7]). The $^3S_1$ low energy parameters are similar to those of model D. In model F there is more repulsion. It was found by Yamamoto and Bando (Ref. [25]) and by Dover and Gal (Ref. [31]) that model D is favored over model F as far as the ΣN potential well depths are concerned. So the present soft-core model seems to meet the requirements from the potential well depths calculations.

In Table IX the nuclear bar phase shifts for $\Sigma^+ p$ are listed. We note the great difference between the $^3S_1$ -phases from the soft-core model and the models D and F. This difference is caused by the attraction in the potential for $r < 0.5$ fm for the $\{10\}$ (cfr. Fig. 2). In D and F this attraction was masked by the hard cores and so only the medium and long range repulsion was visible. Changing the cut-off Λ in the $\{10\}$ from 1230.0 MeV/c to 1000.0 MeV/c the scattering length a_t ($^3S_1$) ranges from 0.31 fm to 0.70 fm. This way the sign-change in the $^3S_1$ phase shift could be shifted to higher energies, while keeping a good fit to the $\Sigma^+ p$ total cross sections through an adjustment of the cut-off in the $\{27\}$. However, then the fit to the $\Sigma^- p$ cross sections tends to deteriorate, in particular if we make Λ_{10} too small. Note that in this case the $^3S_1$ -wave is more repulsive and this makes this solution perhaps less attractive for the calculation of the Σ-hyperfragments (Ref. [25]).

The P-waves are not fitted in the soft-core model to the angular distributions, they are straightforward predictions. This in contrast to the P-waves in model D and F. The triplet P-waves are similar to those from D and F, which is not surprising because all models fit the NN phase shifts and both the NN and the ΣN triplet P-waves are in the $\{27\}$. The $^1P_1$ phase shifts are rather moderate. This in contrast to the behavior of the $^1P_1$ in D and F.

C. ΛN scattering

The low-energy parameters of the S-waves are given in Table X. In the soft-core model we have clearly $|a_s| > |a_t|$. Compared to models D and F $|a_s|$ and $|a_t|$ have become larger, respectively, smaller. The effective range r_s is reduced whereas r_t is about the same. The consequences of this for the hypertriton $^3_\Lambda H$ are unclear. The effects of the changes in the scattering lengths and effective ranges can compensate each other to certain extend (Ref. [38]). Note that in our ΛN calculations we get different values for the low energy parameters in the charge +1 and 0 states, notably because of the inclusion of charge-symmetry-breaking potentials.

The fit to the low-energy Λp data is even better than in models D and F. In Table VII it is shown that the 6 Rehovoth-Heidelberg data have $\chi^2 = 1.0$ and the 6 Maryland data have $\chi^2 = 2.6$.

In Fig. 4 the Λp elastic total cross sections up to $p_\Lambda = 1$ GeV/c are drawn for the experimental data and the fit with the soft-core model ($p_\Lambda < 0.4$ GeV), respectively, soft-core model predictions ($p_\Lambda > 0.4$ GeV). The calculated elastic cross sections above 0.6 GeV/c are very well compatible with the Berkeley-71 data (Ref. [40]).

In Table XI the Λp nuclear bar phase shifts are given and in Table XII we show the elastic and inelastic Λp cross sections for p_Λ from 650 MeV/c to 1 GeV/c.

The large cusp of 42.5 mb at the $\Sigma^+ n$ threshold in Fig. 4 is due to the enhancement in the $^3S_1$ waves which is caused by the coupling of the ΛN and ΣN channels and the rather strong interaction in the $^3S_1$ -wave ΣN channel. This cusp is also seen in Ref. [41] and [42] where a large peak was found in the ΛN invariant mass at $E_{c.m.} = 2128.7 \pm 0.2$ MeV, and 2129.0 ± 0.4 MeV, the $\Sigma^+ n$ threshold being located at 2128.97 MeV. To analyze this cusp we made, like in Ref. [7], a multichannel effective range approximation (ERA) around the $\Sigma^0 p$ threshold i.e.

$$p^{L+1/2}(\bar{K}^J)^{-1}p^{L+1/2} = -A^{-1} + \frac{1}{2}(p^2 - p_0^2)^{1/2}R(p^2 - p_0^2)^{1/2} \ . \tag{60}$$

Here $\bar{K}^J$ is the mutilated K^J matrix, where as before (Refs. [7,23]) we cut out the $^3D_1$ ΣN waves, A^{-1} is the inverse scattering length matrix, R the effective range matrix, $p^{L+1/2}$ and $(p^2 - p_0^2)^{1/2}$ are diagonal matrices with elements $p_i^{L+1/2}$ and $(p_i^2 - p_0^2)^{1/2}$, where p_{0i} denotes the momentum at the $\Sigma^0 p$ threshold energy. In Table XIII we give the results for the effective range approximation.

The radius of convergence of ERA is 5 MeV, as determined by the pion-exchange cut. We did not find any poles within this region on the second Riemann sheet. We see that the situation in the soft-core model is quite different from for example model F, where we found poles at $E = 2131.77 \pm i\, 2.39$ MeV on the 2^{nd} sheet, which again indicates that we now have a weaker interaction in the $^3S_1$ waves.

The ΛN P-wave scattering lengths and effective ranges are given in Table X. Comparing these values and the P-wave phase shifts in Table XI with those of the D and F model, it appears that these are qualitatively similar to those of model F and not like in model D. This seems favorable for $\Lambda - N$ well depths calculations (Ref. [43]).

D. $\Sigma^- p$ scattering

1. $\Sigma^- p \rightarrow \Sigma^- p$

The fit to the total cross sections (see for the precise definition e.g. Ref. [23]) of the Heidelberg group (Ref. [35]) is given in Table VII and shown in Fig. 5. The data are well described ($\chi^2 = 2.6$ for six data points).

In Fig. 6 we compare the predicted angular distribution at $p_{\Sigma^-} = 160$ MeV/c with the Heidelberg data (Ref. [35]). The curve of the soft-core model for this distribution is intermediate to the ones obtained for model D and F.

In Table XIV we give the total nuclear cross sections for $\Sigma^- p$ elastic scattering up to $p_{\Sigma^-} = 600$ MeV/c. Here also like in D and F the scattering is mainly given by the $^3S_1$-wave for $p_{\Sigma^-} \leq 300$ MeV/c. At the higher energies the P-waves dominate the total nuclear cross sections. The contributions of the higher L waves are always very small.

2. $\Sigma^- p \rightarrow \Sigma^0 n$

The calculated total cross sections are compared to the measurents of the Heidelberg group (Ref. [32]) in Table VII and Figure 7.

The result fits the data excellently, this of course with the exception of the datum at $p_{\Sigma^-} = 110$ MeV/c.

In Table XIV we give the total nuclear cross sections for $\Sigma^- p \rightarrow \Sigma^0 n$ up to $p_{\Sigma^-} = 600$ MeV/c. Here also like in D and F the scattering is dominated by the $^3S_1$ wave for $p_{\Sigma^-} \leq 250$ MeV/c. At the higher energies the P-waves dominate the total nuclear cross sections. The contributions of the higher L-waves are always very small. In Fig. 8 we compare the calculated total cross sections in the momentum region $150 \leq p_{\Sigma^-} \leq 600$ MeV/c with the unpublished data of the Massachusetts group (Ref. [37]). In contrast to the result of model D the soft-core model prediction agrees very well with these data.

3. $\Sigma^- p \rightarrow \Lambda n$

In Table VII and Fig. 9 we compare the calculated total cross sections with the measured values of the Heidelberg group (Ref. [32]). A good fit is obtained which is comparable to that for model D and much better than for model F.

In Fig. 10 we compare the calculated angular distribution for $p_{\Sigma^-} = 160$ MeV/c with the Heidelberg data (Ref. [32]). The curve is rather flat, showing a rather small forward-backward ratio. This is like in model F, whereas the data tend to indicate a greater forward-backward ratio.

In Table XIV we give the total nuclear cross sections for $\Sigma^- p \rightarrow \Lambda n$ up to $p_{\Sigma^-} = 600$ MeV/c. Below $p_{\Sigma^-} \simeq 200$ MeV/c is for 65% provided by the $^3S_1 - {}^3S_1$ transition and for 25% by the $^3S_1 - {}^3D_1$ transition. For $p_{\Sigma^-} \geq 350$ MeV/c these transitions account for 60% of the total cross sections. The other half is provided essentially by the P-waves.

In Fig. 11 we compare the calculated total cross sections in the momentum region $150 \leq p_{\Sigma^-} \leq 600$ MeV/c with the unpublished data of the Massachusetts group (Ref. [37]). In contrast to the result of model D the soft-core model prediction agrees rather well with these data.

Finally we mention the excellent value that we were able to reach for the inelastic capture ratio at rest $r_R = 0.471$ (cfr. Table VII) which is almost equal to the averaged experimental value (Refs. [15,36,37]).

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TABLES

	Space-spin antisymmetric states $^1S_0,\ ^3P,\ ^1D_2,...$	
$NN \to NN$	$I=1$	$V_{NN}(I=1) = V_{27}$
$\Lambda N \to \Lambda N$		$V_{\Lambda\Lambda}\left(I=\frac{1}{2}\right) = \left(9V_{27} + V_{8s}\right)/10$
$\Lambda N \to \Sigma N$	$I=\frac{1}{2}$	$V_{\Lambda\Sigma}\left(I=\frac{1}{2}\right) = \left(-3V_{27} + 3V_{8s}\right)/10$
$\Sigma N \to \Sigma N$		$V_{\Sigma\Sigma}\left(I=\frac{1}{2}\right) = \left(V_{27} + 9V_{8s}\right)/10$
$\Sigma N \to \Sigma N$	$I=\frac{3}{2}$	$V_{\Sigma\Sigma}\left(I=\frac{3}{2}\right) = V_{27}$
	Space-spin symmetric states $^3S_1,\ ^1P_1,\ ^3D,...$	
$NN \to NN$	$I=0$	$V_{NN}(I=0) = V_{10^\star}$
$\Lambda N \to \Lambda N$		$V_{\Lambda\Lambda}\left(I=\frac{1}{2}\right) = \left(V_{10^\star} + V_{8a}\right)/2$
$\Lambda N \to \Sigma N$	$I=\frac{1}{2}$	$V_{\Lambda\Sigma}\left(I=\frac{1}{2}\right) = \left(V_{10^\star} - V_{8a}\right)/2$
$\Sigma N \to \Sigma N$		$V_{\Sigma\Sigma}\left(I=\frac{1}{2}\right) = \left(V_{10^\star} + V_{8a}\right)/2$
$\Sigma N \to \Sigma N$	$I=\frac{3}{2}$	$V_{\Sigma\Sigma}\left(I=\frac{3}{2}\right) = V_{10}$

TABLE I. SU(3)-contents of the various potentials on the isospin basis.

Y	I	Channels	States	SU(3) irreps
	1		$^1S_0,\ ^3P,\ ^1D_2,...$	$\{27\}$
2		NN		
	0		$^3S_1,\ ^1P_1,\ ^3D,...$	$\{10^\star\}$
	$\frac{1}{2}$		$^1S_0,\ ^3P,\ ^1D_2,...$	$\{27\}\oplus\{8_s\}$
		$\Lambda N, \Sigma N$		
			$^3S_1,\ ^1P_1,\ ^3D,...$	$\{10^\star\}\oplus\{8_a\}$
1				
	$\frac{3}{2}$		$^1S_0,\ ^3P,\ ^1D_2,...$	$\{27\}$
		ΣN		
			$^3S_1,\ ^1P_1,\ ^3D,...$	$\{10\}$

TABLE II. Channels, states and SU(3) irreps in $Y=1$ and $Y=2$ Baryon-Baryon scattering.

$^1S_0$	Σ^+p	$\{27\}$	Λ_{27}	=	1020.0	MeV
$^3S_1$	Σ^+p	$\{10\}$	Λ_{10}	=	1230.0	MeV
	Σ^-p	$\{10\}$	Λ'_{10}	=	1270.5	MeV
$^1S_0$	$\Lambda N, \Sigma N$	$\{27\}+\{8_s\}$	Λ_{27+8_s}	=	820.0	MeV
$^3S_1$	$\Lambda N, \Sigma N$	$\{10^\star\}+\{8_a\}$	$\Lambda_{10^\star+8_a}$	=	1270.5	MeV

TABLE III. S-wave form factor masses used in this work.

M		NNM	$\Lambda\Lambda M$	$\Lambda\Sigma M$	$\Sigma\Sigma M$	ΛNM	ΣNM
π	g	3.69822	CSB	3.38318	3.32840	—	—
	f	0.27204	CSB	0.20261	0.19315	—	—
η	g	1.80574	-1.84905	—	4.45648	—	—
η'	g	1.95894	4.02537	—	1.56718	—	—
K	g	—	—	—	—	-3.98122	1.20749
	f	—	—	—	—	-0.96466	0.28336
ρ	g	0.89147	CSB	—	1.78295	—	—
	f	3.76255	CSB	3.14985	2.06940	—	—
ϕ	g	-0.31477	-1.53976	—	-1.53976	—	—
	f	-0.42414	-3.09542	—	1.90246	—	—
ω	g	2.94663	2.00666	—	2.00666	—	—
	f	0.90959	-1.14016	—	2.69485	—	—
$K^\star$	g	—	—	—	—	-1.54408	-0.89147
	f	—	—	—	—	-3.36708	1.69315

TABLE IV. Coupling constants for pseudo-scalar and vector meson $Y = 0$ and $Y = \pm 1$ exchanges.

M		NNM	$\Lambda\Lambda M$	$\Lambda\Sigma M$	$\Sigma\Sigma M$	ΛNM	ΣNM
δ	g	1.27734	CSB	-0.42116	3.28416	—	—
$S^\star$	g	-0.83894	-2.56308	—	-3.11453	—	—
ε	g	4.76773	2.77698	—	2.14025	—	—
κ	g	—	—	—	—	-2.63359	-2.00682
A_2	g	0.44372	CSB	-0.01427	1.15196	—	—
f'	g	-1.10989	-2.05561	—	-2.21779	—	—
$P \bigoplus f$	g	2.74708	2.70161	—	2.86606	—	—
$K^{\star\star}$	g	—	—	—	—	-0.81469	-0.51408

TABLE V. Coupling constants for scalar meson and 'diffractive' $Y = 0$ and $Y = \pm 1$ exchanges.

mesons		{1}	{8}	$F/(F+D)$	angles
pseudoscalar	f	0.18455	0.27204	$\alpha_{PV} = 0.355^{\star)}$	$\theta_P = -23.00^0$
vector	g	2.52934	0.89147	$\alpha_V^e = 1.0$	$\theta_V = 37.50^0$
	f	0.97982	3.76255	$\alpha_V^m = 0.275^{\star)}$	
scalar	g	3.75548	1.27734	$\alpha_S = 1.28555$	$\theta_S = 40.895^{0\ \star)}$
diffractive	g	2.85507	0.44372	$\alpha_D = 1.02267$	$\psi_D = 15.50^{0\ \star)}$

TABLE VI. Coupling constants, $F/(F+D)$-ratio's, mixing angles etc. The values with $\star)$ have been determined in the fit to the YN-data. The other parameters are theoretical input or determined by the fitted parameters and the constraint from the NN-analysis.

$\Lambda p \to \Lambda p$		$\chi^2 = 1.0$	$\Lambda p \to \Lambda p$		$\chi^2 = 2.6$
p_Λ	σ_{exp}^{RH}	σ_{th}	p_Λ	σ_{exp}^{M}	σ_{th}
145	180±22	192.8	135	209.0±58	209.2
185	130±17	138.8	165	177.0±38	163.6
210	118±16	113.2	195	153.0±27	127.9
230	101±12	96.4	225	111.0±18	100.4
250	83± 9	82.4	255	87.0±13	79.2
290	57± 9	60.8	300	46.0±11	56.6
$\Sigma^+ p \to \Sigma^+ p$		$\chi^2 = 0.2$	$\Sigma^- p \to \Sigma^- p$		$\chi^2 = 2.6$
p_{Σ^+}	σ_{exp}	σ_{th}	p_{Σ^-}	σ_{exp}	σ_{th}
145	123±62	107.3	142.5	152±38	137.4
155	104±30	99.0	147.5	146±30	135.5
165	92±18	91.4	152.5	142±25	133.9
175	81±12	84.3	157.5	164±32	132.4
			162.5	138±19	131.0
			167.5	113±16	129.8
$\Sigma^- p \to \Sigma^0 n$		$\chi^2 = 6.5$	$\Sigma^- p \to \Lambda n$		$\chi^2 = 2.7$
p_{Σ^+}	σ_{exp}	σ_{th}	p_{Σ^-}	σ_{exp}	σ_{th}
110	396±91	178.3	110	174±47	219.1
120	159±43	157.5	120	178±39	191.7
130	157±34	140.5	130	140±28	169.6
140	125±25	126.3	140	164±25	151.4
150	111±19	114.2	150	147±19	136.2
160	115±16	103.8	160	124±14	123.4
$r_R^{exp} = 0.468 \pm 0.010$			$r_R^{th} = 0.471$		$\chi^2 = 0.1$

TABLE VII. Comparison of the calculated and experimental values for the 35 YN-data that were included in the fit. The superscipts RH and M denote, respectively, the Rehovoth-Heidelberg (Ref. [33]) and Maryland data (Ref. [34]). The laboratory momenta are in MeV/c, and the total cross sections in mb.

	$^1S_0$	$^3S_1$	$^3P_0$	$^3P_1$	$^3P_2$	$^1P_1$
a^C	-3.63	0.305	-2.33	1.63	-0.099	-0.97
r^C	3.27	-20.15	4.77	-7.66	10.60	13.74
a	-4.71	0.247	-2.12	1.49	-0.101	-0.88
r	3.36	-26.86	4.93	-8.33	26.30	14.56

TABLE VIII. $\Sigma^+ p$ and $\Sigma^- n$ S- and P-wave effective range parameters in units of fm. The superscript C denotes the presence of the Coulomb interaction.

p_{Σ^+}(MeV/c)	200	400	600	800	1000
T_{lab}(MeV)	16.7	65.5	142.8	244.0	364.5
$^1S_0$	36.63	27.13	12.82	-10.05	-13.59
$^3S_1$	-7.95	-10.46	6.51	40.70	51.17
$\varepsilon 1$	-1.83	-4.24	-2.31	3.74	5.80
$^3P_0$	4.42	7.53	0.80	-10.31	-22.35
$^1P_1$	1.71	4.27	4.10	1.06	-3.61
$^3P_1$	-3.14	-10.40	-18.46	-26.86	-34.90
$^3P_2$	0.54	3.14	6.61	10.81	15.91
$\varepsilon 2$	-0.35	-1.80	-2.90	-2.85	-1.67
$^3D_1$	0.27	1.14	0.81	-1.67	-7.25
$^1D_2$	0.28	1.64	3.93	6.59	8.52
$^3D_2$	-.43	-2.43	1.08	1.10	0.21
$^3D_3$	0.03	0.46	-5.11	-8.57	-12.91

TABLE IX. Σ^+p nuclear bar phase shifts in degrees.

		$^1S_0$	$^3S_1$	$^3P_0$	$^3P_1$	$^3P_2$	$^1P_1$
Λp	a	-2.73	-1.48	-0.033	0.016	-0.200	0.088
	r	2.87	3.04	-154	2887	11.62	-38.5
ΛN	a	-2.78	-1.41	-0.096	0.061	-0.20	0.062
	r	2.88	3.11	72.3	40.86	10.44	32.7
Λn	a	-2.86	-1.24	-0.156	0.108	-0.199	0.037
	r	2.91	3.33	62.6	-42.1	9.62	388

TABLE X. Λp and Λn S- and P-wave effective range parameters in units of fm.

p_Λ(MeV/c)	100	200	300	400	500	600	633.4
T_{lab}(MeV)	4.5	17.8	39.6	69.5	106.9	151.1	167.3
$^1S_0$	27.43	33.47	30.13	23.78	16.61	9.47	7.21
$^3S_1$	17.08	24.78	26.07	25.32	25.68	33.36	51.97
$\varepsilon 1$	0.20	1.15	2.91	5.61	9.83	18.31	27.78
$^3P_0$	0.03	0.12	-0.09	-1.30	-3.79	-7.30	-8.56
$^1P_1$	-0.06	-0.40	-1.25	-2.70	-4.70	-6.59	-7.74
$^3P_1$	-0.02	-0.26	-1.03	-2.47	-4.49	-6.69	-7.20
$^3P_2$	0.14	0.95	2.61	4.78	6.99	8.99	9.61
$\varepsilon 2$	0.00	-0.01	-0.07	-0.24	-0.50	-0.83	-0.98
$^3D_1$	0.00	0.05	0.31	1.06	2.79	6.69	8.51
$^1D_2$	0.00	0.04	0.28	0.90	1.93	3.27	3.76
$^3D_2$	0.00	0.07	0.38	1.10	2.29	3.89	4.51
$^3D_3$	0.00	0.03	0.19	0.57	1.18	1.92	2.18

TABLE XI. Λp nuclear bar fase shifts in degrees below the ΣN tresholds.

p_Λ(MeV/c)	T_{lab}(MeV)	$\Lambda p \to \Lambda p$	$\Lambda p \to \Sigma^+ n$	$\Lambda p \to \Sigma^0 p$
650	175.5	26.20	6.29	2.49
700	201.4	17.02	7.94	4.18
750	228.7	14.48	8.37	4.27
800	257.2	13.56	8.21	4.11
850	286.9	13.32	7.82	3.88
900	317.8	13.40	7.36	3.64
950	349.7	13.62	6.90	3.40
1000	382.6	13.92	6.47	3.19

TABLE XII. $\Lambda p \to \Lambda p, \Sigma^+ n, \Sigma^0 p$ total cross sections in mb above the ΣN tresholds.

	$\Lambda p \to \Sigma^0 p$		$\Lambda n \to \Sigma^- p$		$(\Lambda n \to \Sigma^- p)_C$	
	A^{-1}	R	A^{-1}	R	A^{-1}	R
11	12.33	57.22	9.73	35.00	9.74	35.16
12	-31.88	-95.73	-26.03	-53.15	-26.38	-54.82
13	-1.45	-6.29	-0.83	-3.57	-0.73	-2.64
14	1.07	3.92	1.00	2.18	1.11	2.92
22	92.20	112.86	80.85	37.96	82.52	41.66
23	1.08	15.04	0.16	8.50	-0.09	6.89
24	-0.83	-9.35	0.20	-7.06	-0.04	-8.75
33	0.24	-0.62	-0.34	-2.77	-0.27	-2.45
34	-0.78	-3.81	-0.74	-3.29	-0.69	-2.97
44	-0.27	-3.53	0.23	-1.04	-0.39	-0.71

TABLE XIII. Inverse-scattering-length and effective-range matrices at the $\Sigma^0 p$ and $\Sigma^- p$ tresholds. The order of the states (1-4) reads $\Lambda p(^3S_1)$, $\Lambda p(^3D_1)$, $\Sigma^+ n(^3S_1)$, $\Sigma^0 p(^3S_1)$, respectively, $\Lambda n(^3S_1)$, $\Lambda n(^3D_1)$, $\Sigma^0 n(^3S_1)$, $\Sigma^- p(^3S_1)$. The dimensions of the matrix elements are in fm$^{-1-l-l'}$ (A^{-1}) and fm$^{1-l-l'}$ (R). The subscript C denotes the presence of the Coulomb interaction in the $\Sigma^- p$ channel.

p_{Σ^-} (MeV/c)	T_{lab}(MeV)	$\Sigma^- p \to \Sigma^- p$	$\Sigma^- p \to \Sigma^0 n$	$\Sigma^- p \to \Lambda n$
50	1.0	547.9	575.1	757.3
100	4.2	176.5	207.6	254.4
150	9.4	138.1	116.7	136.1
200	16.6	123.6	74.9	87.5
250	25.8	108.6	51.1	62.2
300	37.0	91.0	37.2	47.1
350	50.1	74.0	29.0	37.3
400	65.0	59.7	23.9	30.4
450	81.8	48.7	20.3	25.3
500	100.2	40.4	17.5	21.5
550	120.3	33.9	15.3	18.5
600	141.9	29.1	13.5	16.1

TABLE XIV. $\Sigma^- p \to \Sigma^- p, \Sigma^0 n, \Lambda n$ total cross sections in mb above the ΣN tresholds.

FIGURES

FIG. 1. Potentials for channels in definite SU(3)-irrep.

FIG. 2. Calculated $\Sigma^+ p$ 'total' cross sections compared with experimental values of Ref. [35].

FIG. 3. Calculated $\Sigma^+ p$ differential cross sections compared with experimental values of Ref. [35].

FIG. 4. Calculated Λp elastic total cross sections compared with the Rehovoth-Heidelberg (Ref. [33]), Maryland (Ref. [34]), and Berkeley (Refs. [39,40]) data.

FIG. 5. Calculated $\Sigma^- p$ elastic 'total' cross sections compared with the experimental data (Ref. [35]).

FIG. 6. Calculated $\Sigma^- p$ differential cross sections compared with the experimental data (Ref. [35]).

FIG. 7. Calculated $\Sigma^- p \rightarrow \Sigma^0 n$ total cross sections compared with the experimental data (Ref. [32]).

FIG. 8. Calculated $\Sigma^- p \rightarrow \Sigma^0 n$ total cross sections compared with the Massachusetts data (Ref. [37]).

FIG. 9. Calculated $\Sigma^- p \rightarrow \Lambda n$ total cross sections compared with the experimental data (Ref. [32]).

FIG. 10. Calculated $\Sigma^- p \rightarrow \Lambda n$ differential cross sections compared with the experimental data (Ref. [32]).

FIG. 11. Calculated $\Sigma^- p \rightarrow \Lambda n$ total cross sections compared with the experimental data of the Massachusetts group (Ref. [37]).