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The Nijmegen *NN* Phase Shift Analysis*

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Abstract

The Nijmegen group has now completed the phase-shift analysis of all *pp* scattering data below $T_{\rm lab} = 350$ MeV. This is a continuation of the Nijmegen 0–30 MeV analysis, Both in the *pp* and *np* analysis, a low value for the πNN coupling constant g_0^2 was found, indicating a large charge-independence breaking. In this contribution we report on the present status of the Nijmegen phase-shift analysis. The incorporation of the magnetic-moment interaction proved to be important and changed the numerical results. We find now for the $pp\pi^0$ coupling constant $g_0^2 = 13.53 \pm 0.14$. Some comments are made on the possible CIB sources.

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I. INTRODUCTION

The Nijmegen group has now completed the multienergy (ME) phase-shift analysis (PSA) of all pp scattering data below $T_{\rm lab} = 350$ MeV [1]. This is a continuation of the Nijmegen 0–30 MeV pp analysis [2]. Results of the phase-shift analysis of all published nucleon-nucleon (NN) scattering data for 0–30 MeV have already been published [2–4]. In the pp analysis a low value for the πNN coupling constant g_0^2 was found [5], indicating a large charge-independence Breaking (CIB) in the NN interaction. Subsequently, this CIB was investigated in an np and a combined np and pp analysis for $T_{\rm lab} = 0 - 30$ MeV [4]. Here this large CIB was confirmed. Because of the current interest in this particular result, we will restrict ourselves mainly to the CIB in this contribution. We review the phase-shift analysis, the potential tail used, the results, and briefly make some comments on the theoretical aspects of CIB.

In the most recent phase-shift analysis of the pp data for $T_{\rm lab} = 0 - 350$ MeV [1] the magnetic-moment interaction has been incorporated. In contrast to earlier reports in the literature, where effects on the analysis for a single energy or for a restricted energy range were studied (see, e.g., Refs. [6,7]), the inclusion of the magnetic-moment interaction proved to be rather relevant in pp. Because of its long range it influences the value of g_0^2. Another, but less important, difference with the analysis of Ref. [5] is a slightly different parametrization, resulting in a drop in χ^2. Also, the pp dataset has been updated a little. These last two modifications hardly changed g_0^2. With the magnetic-moment interaction the obtained value for the $pp\pi^0$ coupling constant is $g_0^2 \equiv g_{pp\pi^0}^2 = 13.53 \pm 0.14$ or $f_0^2 \equiv f_{pp\pi^0}^2 = (74.8 \pm 0.8) \times 10^{-3}$, where $f_0^2 = (m_c/2M_p)^2 g_0^2$. Here m_c is the charged-pion mass. The new value for g_0^2, though higher than the earlier reported value of $g_0^2 = 13.1 \pm 0.1$ [5], still means a rather large CIB in the NN interaction. The new value of (g_0^2 is about four standard deviations smaller than the charged-pion coupling constant $g_c^2 \equiv g(pn\pi^+)g(np\pi^-)/2 = 14.3 \pm 0.8$ [8], which means an unexpected large CIB. For excellent reviews on charge independence and charge symmetry see, e.g., Ref. [9].

The ME fit to the pp data, using 28 parameters, reached $\chi^2 = 1561.80$ for $N_{df} = 1394$. If we compare this to the ME analysis without the magnetic-moment interaction, where we find $g_0^2 = 13.18 \pm 0.14$ and $\chi^2 = 1578.97$, one sees a clear effect of the magnetic-moment interaction both on the $pp\pi^0$ coupling constant and on χ^2.

There are no new results on the low-energy and deuteron parameters. Therefore, we do not discuss these here, but refer to the papers of the Nijmegen group, quoted above.

II. THE NIJMEGEN PHASE-SHIFT ANALYSIS

In the Nijmegen phase-shift analysis the present knowledge of the nucleon-nucleon interaction is exploited as much as possible. For large distances, the electromagnetic interactions (Coulomb, vacuum polarization, and magnetic moments) are well known and can be incorporated in a model-independent way in the phase-shift analysis. The same holds for the OPE part of the nuclear interaction. Although this cannot be said for the intermediate-range part of the nuclear interaction, different potential models like in Refs. [10,11] fortunately do agree very well for $r \geq 1.4$ fm. This rather remarkable feature allows the use of such a potential for $r \geq 1.4$ fm without introducing to much model dependence. The short-range part of the nuclear interaction ($r < 1$ fm), however, is largely unknown, which is reflected in large differences between potential models.

The Nijmegen phase-shift analysis is a multienergy (ME) analysis. In distinction to other ME phase-shift analyses [12–15], in the Nijmegen ME analysis the energy dependence of the phase shifts is described via a (simple) parametrization of the P matrix. The details of this procedure have been described in Refs. [2,16,17]. The unknown short-range interaction is described phenomenologically for each partial wave, by parametrizing a P matrix

$$P(b; k^2) = b \left(\frac{d\chi}{dr} \chi^{-1} \right)_{r=b} , \tag{1}$$

which is a boundary condition for the radial wave function $\chi(r)$ at $r = b$, where 1 fm $< b < 2$ fm. The energy dependence of the P matrix is indicated by k, the relativistic momentum in the center of mass (c.m.) system. This boundary condition implements the connection between the unknown inner region and the outer region. The interactions in the latter region are incorporated by solving the Schrödinger equation for the potential tail outside $r = b$. In that case, the electromagnetic and OPE potential tails are taken into account exactly. For the realistic intermediate-range forces different NN models (e.g., [10,11]) have been analyzed. The higher partial waves ($J \geq 3$ for 0–30 MeV analysis, $J \geq 5$ for 0–350 MeV analysis) are either given by the OPE phase parameters alone, or by adding for the intermediate partial waves ($5 \leq J \leq 8$) the non-OPE contributions due to the heavy-boson exchanges (HBE) of the Nijmegen [10] or Paris [11] potential. For $J \geq 9$ the OPE amplitudes are calculated in CDWBA.

The optimal-polynomial-theory (OPT) predictions for the non-OPE contributions to some of the intermediate partial-wave parameters were studied [18], but with only a few lower waves as input, some of the predictions for the intermediate partial waves ($5 \leq J \leq 8$) were not successful. This was in particular the case for the mixing parameters ϵ_J and some coupled waves.

III. THE POTENTIAL TAIL

The potential tail plays a prominent role in the PSA, and since the pp analysis is at present the most important source of information for the determination of g_0^2, we review this part of the interaction for pp here. The potential tail V_L, which describes the long-range interactions used in the PSA, is

$$V_L = V_{NUC} + V_{EM} , \tag{2}$$

where V_{NUC} contains as its longest-range part the OPE potential. The electromagnetic potential V_{EM} contains the Coulomb interaction with relativistic corrections $\tilde{V}_C$ [19], the vacuum polarization [20], and recently also the magnetic part V_{MM} of the photon-exchange potential [21], derived in [19]. So,

$$V_{EM} = \tilde{V}_C + V_{MM} + V_{VP} , \tag{3}$$

with

$$\tilde{V}_C = \frac{\alpha'}{r} - \frac{1}{2M_p^2}\left[\left(\Delta + k^2\right)\frac{\alpha}{r} + \frac{\alpha}{r}\left(\Delta + k^2\right)\right] \ ,$$

$$V_{MM} = -\frac{\alpha}{4M_p^2 r^3}\left[\mu_p^2 S_{12} + (6+8\kappa)\mathbf{L}\cdot\mathbf{S}\right] \ , \tag{4}$$

where $\alpha' = 2k\eta'/M_p$ with $\eta' = \alpha/v_{\rm lab}$, $\mu_p = 2.7928$ is the proton magnetic moment, and κ the anomalous magnetic moment. The contribution of V_{MM} to the scattering amplitude is calculated in CDWBA by regularizing the $1/r^3$ dependence with the standard electromagnetic dipole form factor.

The longest-range part of the nuclear interaction, one-pion exchange (OPE), is included in the form

$$V_{OPE} = \frac{1}{3}\frac{f_{pp\pi^0}}{4\pi}\frac{M_p}{E}\left(\frac{m}{m_c}\right)^2 \frac{e^{-mr}}{r} \times \left[(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + S_{12}\left(1 + \frac{3}{(mr)} + \frac{3}{(mr)^2}\right)\right] \ , \tag{5}$$

where m is the π^0 mass and m_c is a scaling mass (introduced to make $f_{pp\pi^0}$ dimensionless) chosen to be the charged-pion mass [8]. $E = \sqrt{k^2 + M_p^2}$, with k the relativistic c.m. momentum. No form factor is included here. The latter represents a short-range effect ($r \leq 1.4$ fm). The coupling constant $f_0^2 \equiv f_{pp\pi^0}^2$ is the important free parameter to be determined in the PSA. Notice that in the analysis described here, the pion-nucleon coupling is determined at the one-pion pole.

With $V_{NUC} = V_{OPE}$ for $b = 1.8$ fm a reasonable fit was reached. Introducing besides OPE also intermediate-range forces, a better fit to the data was achieved for $b = 1.4$ fm. In that case the non-OPE (HBE) contributions of modern potential models [10,11] were used. In order to allow for an adjustment of the HBE potential of the used model, to remedy possible imperfections of the intermediate forces, factors $f_{\rm med}^{\rm singlet}$ and $f_{\rm med}^{\rm triplet}$, respectively, for the spin-singlet and spin-triplet waves were introduced. So,

$$V_L = V_{EM} + V_{OPE} + V_{HBE}^N(f_{\rm med}^{\rm singlet/triplet}) \ , \tag{6}$$

where V_{HBE}^N is the Nijmegen soft-core potential from the heavier bosons. Similarly, the parametrized Paris potential [11] was used in the analysis, but this gave a slightly higher χ^2. This way also different potential tails from (other) good NN potential models can be tested.

IV. THE G($pp\pi^0$) DETERMINATION

The coupling of the neutral pion to the proton can be described by either the pseudoscalar (PS) or the pseudovector (PV) phenomenological Lagrangians $\mathcal{L}_{PS}$ or $\mathcal{L}_{PV}$, where $\mathcal{L}_{PS} = g_0(4\pi)^{1/2}(\bar{\psi}i\gamma_5\psi)\phi$ and $\mathcal{L}_{PV} = (f_0/m_c)(4\pi)^{1/2}(\bar{\psi}i\gamma_\mu\gamma_5\psi)\partial^\mu\phi$. The couplings PV and PS lead to the same OPE potential for pp, provided that $g_0^2 = (2M_p/m_c)^2 f_0^2 = 180.78 f_0^2$, where M_p

denotes the proton mass. As is well known, the PV and PS couplings are not equivalent in higher order. For instance, the so-called pair terms are in the PV coupling stronly suppressed w.r.t. those generated by the PS coupling. This makes the PV coupling more likely in the quark model [22]. Also, the PV coupling is more natural from the chiral symmetry point of view [23]. Fortunately, the determination of the pion-nucleon coupling constant in the Nijmegen pp phase-shift analysis is not liable to higher-order corrections. It is only the lowest-order OPE which determines the f_0^2 value.

The coupling of the charged pions to the nucleons is described by the charged-pion coupling constant f_c, where $f_c^2 \equiv f(pn\pi^+)f(np\pi^-)/2$. For charge independence (i.e., SU(2)-isospin symmetry) of the pion-nucleon interaction $f_0^2 = f_c^2$. The charged-pion coupling constant f_c is determined with rather high accuracy in πN scattering, where the consensus seems to be $f_c^2 = (79 \pm 1) \times 10^{-3}$ [8]. On the other hand, the neutral-pion coupling f_0 was not known with the same precision and varies from analysis to analysis (see Table I for a list of determinations of f_0).

The Nijmegen group has succeeded in the determination of the $pp\pi^0$ coupling f_0, with an accuracy which is as good as that which has been achieved for the charged-pion coupling. This accurate determination has been done primarily in the pp phase-shift analysis [5]. Lateron the consequences of the value found for f_0 have been studied for the np phase-shift analysis [4]. Because of its importance, we will first briefly review the pp phase-shift analysis determination. The long-range pp interaction depends strongly on f_0. For the intermediate-range forces $r \geq b$, we studied the forces from the heavier and/or higher-order boson exchanges (HBE) for some modern potential models. The various choices considered are

(i) $V_{NUC} = V_{OPE}$. The change in V_{OPE} due to a form factor as in the Nijmegen potential [10] has no influence (here we used $b = 1.8$ fm).

(ii) $V_{NUC} = V_{OPE} + V_{HBE}^N$, where V_{HBE}^N is the non-OPE part of the Nijmegen potential [10].

(iii) $V_{NUC} = V_{OPE}^s + V_{HBE}^N$, where V_{OPE}^s is the static OPE potential.

(iv) $V_{NUC} = V_{OPE} + V_{HBE}^P$, where V_{HBE}^P is the non-OPE part of the Paris potential [11].

This defines the different potential tails that were studied.

For each potential, the P matrix parameters and f_0^2, where the latter affects all partial waves, have been fitted to the data. The results found in [5] are shown in Table II. For tail (i) $b = 1.4$ fm appeared not reasonable, so the HBE forces are not negligible outside 1.4 fm. Here $b = 1.8$ fm was used and also one extra P-matrix parameter was necessary. For the tails with the HBE forces, a lower χ^2 was obtained. Static or non-static OPE does not make a difference in the result.

To show the sensitivity of the pp data for the pion, also the pion mass has been determined as well as the coupling constant. The result found was $m = 134.7 \pm 2.1$ MeV [5], which is in complete agreement with the value given by [28]. In Fig. 1 the χ^2 surface is shown as a function of m and f_0^2. It shows a strong correlation and supports the value found for f_0^2. To check further the consistency of the pp analysis and the f_0 determination, we also studied whether indeed all partial waves need to have a value for the $pp\pi^0$ coupling as required by the overall fit. The following results were found for $V_{NUC} = V_{OPE} + V_{HBE}^N$ [5]:

(i) Introducing a different coupling for the spin singlets f_S and the spin triplets f_T, one obtained $f_S^2 = (74 \pm 2) \times 10^{-3}$ and $f_T^2 = (72.5 \pm 0.6) \times 10^{-3}$, which indicates the particular importance of the triplet waves.

(ii) Introducing a different coupling for the ^3P waves $f(^3P)$ and in all other partial waves $f(\text{rest})$, one found $f^2(\text{rest}) = (73.8 \pm 0.9) \times 10^{-3}$ and $f^2(^3P) = (72.2 \pm 0.6) \times 10^{-3}$.

Similar results were also found for the other potential tails.

The results for f_0^2 are in reasonable agreement with earlier (less accurate) determinations, except the value obtained by Kroll [27]. They deviate significantly from the value $f_c^2 = (79 \pm 1) \times 10^{-3}$ or $g_c^2 = 14.3 \pm 0.2$ for the charged-pion coupling. So, a large SU(2)-symmetry breaking has been indicated by the *pp* phase-shift analysis discussed above.

In the introduction we have described the new features of the most recent *pp* analysis of the Nijmegen group. The most important being the inclusion of the magnetic-moment interaction. In this new phase-shift analysis of all *pp* scattering data below $T_{\text{lab}} = 350$ MeV [1] we found

$$f_0^2 = (74.83 \pm 0.77) \times 10^{-3} \quad \text{or} \quad g_0^2 = 13.53 \pm 0.14 \ . \tag{7}$$

The recommended value for the charged-pion coupling, on the other hand, is [8]

$$f_c^2 = (79 \pm 1) \times 10^{-3} \quad \text{or} \quad g_c^2 = 14.3 \pm 0.2 \ , \tag{8}$$

where $g_c^2 = [(M_p + M_n)/m_c]^2 f_c^2$. Consequently, one observes here a CIB in the pion-nucleon coupling constants $\Delta g^2 \equiv g_c^2 - g_0^2 = 0.8 \pm 0.2$, which is smaller than that found in the earlier analysis [4] which implied $\Delta g^2 = 1.2 \pm 0.2$. In general, the errors on the parameters in the 0–350 MeV energy range are smaller than for the more limited 0–30 MeV energy range. However, one cannot point to a well-defined subset, or type of data, as being in particular responsible for the low value of g_0^2. It rather appears that the data as a whole require a low pion-nucleon coupling constant.

V. NEUTRON-PROTON ANALYSIS 0–30 MEV DATA

Additional information on CIB can be expected from a phase-shift analysis of the *np* data, since here both g_0^2 and g_c^2 occur in the OPE potential. The effect of a large breaking of charge independence for *np* scattering can best be understood as follows. The OPE potential for *pp* scattering can be written as

$$V_{OPE}(pp) = g_0^2 V_\pi(m_0) \ , \tag{9}$$

where m_0 denotes the neutral-pion mass. The OPE potential for *np* scattering in the $I = 1$ states can then be written as

$$V_{OPE}(np) = -g_0^2 V_\pi(m_0) + 2 g_c^2 V_\pi(m_c) \ . \tag{10}$$

(Note that we assume that charge symmetry in the pion-nucleon coupling is still valid, which implies that $g_{pp\pi^0} = -g_{nn\pi^0} = g_0$.)

Neglecting for a moment all effects due to mass differences, we can also write

$$V_{OPE}(np) = V_{OPE}(pp) + 2\Delta g^2 V_\pi \ . \tag{11}$$

With the aforementioned values of g_0^2 and g_c^2, $V_{OPE}(np)$ in the $I=1$ states is found to be stronger than $V_{OPE}(pp)$.

We have performed an ME phase-shift analysis of all np scattering data below $T_{\rm lab}=30$ MeV. The starting point here is a new 0–30 MeV pp phase-shift analysis derived from the 0–350 MeV analysis. This analysis differs from the previously published 0–30 MeV analysis [2] in the following respects. There are added some data which were found to be missing in the previously used dataset and, more importantly, the magnetic-moment interaction is included for consistency reasons (the latter interaction makes no difference for the reached χ^2 minimum). The $I=1$ lower partial waves (^3P_J and $^1D_2$) from this analysis were then used in our np analysis, after correcting them not only for electromagnetic and mass-difference effects, but we also allow for a difference between g_0^2 and g_c^2. These phase shifts are therefore essentially parametrized by the amount of CIB in the coupling constants through Eq. (11). The $I=0$ partial waves up to $J=2$ and the $^1S_0$ phase shift (in order to arrive at the correct np scattering lengths) are parametrized with an energy-dependent P matrix. All higher partial waves ($J>2$) for both $I=0$ and $I=1$ are taken to be pure OPE, including the explicit $\Delta g^2 = g_c^2 - g_0^2$ and mass-difference effects.

If we take $\Delta g^2=0$ with $g_0^2=g_c^2=14.3$, we obtain $\chi^2=454.1$ for $N_{\rm dat}=478$ ($N_{\rm df}=425$). However, if we have Δg^2 as a free parameter, χ^2 drops to $\chi^2=439.2$ and we find

$$\Delta g^2 = 0.61 \pm 0.14 \ , \tag{12}$$

which is in reasonable agreement with $g_c^2 = 14.3\pm0.2$ and $g_0^2=13.53\pm0.14$ of the new 0–350 MeV analysis. This is smaller than the previous result [4] of $\Delta g^2 = 1.28\pm0.12$, which was based on pp and np analyses without the magnetic-moment interaction. The drop in χ^2 is mainly due to a better description of the polarization data. The polarization in this energy range can be approximated by

$$\sigma(\theta)A_y(\theta) = \frac{3}{k^2}\sin^2\delta(^3S_1)\Delta^p_{LS}\sin\theta \ . \tag{13}$$

The reason for the drop in χ^2 is primarily due to the fact that a better description of the spin-orbit combination Δ^p_{LS} has been attained. The OPE potential does not contain a spin-orbit interaction and hence the improvement is due to a second-order effect in the pion-nucleon coupling constants. (The different central and tensor phase shifts, which are first order in Δg^2 give only minor improvements in the description of the data.)

Whereas the magnetic-moment interaction for the $T_{\rm lab}=0-30$ MeV pp analysis is not visible in χ^2, its inclusion is necessary for a proper description of the low-energy np data. Especially the description of the very accurate analyzing-power measurements of Holslin *et al.* [29] (10.03 MeV), Tornow *et al.* [30] (16.9 MeV), and Sromicki *et al.* [31] (25.0 MeV), is improved significantly, as shown in Table III.

VI. DISCUSSION

Theoretically, the large CIB has not found an explanation. It is generally believed that the charge-independence breaking (CIB) is rather small (see, e.g., [9]). For example, the electromagnetic breaking by radiative corrections is rather small. Typically $|f_0 - f_c|/f_c \approx$

0.5% [32]. Moreover, this correction is in the "wrong" direction. Recently, Henley and Zhang have estimated the effects of the mass difference between the u and the d quark [33]. They use a nonrelativistic quark model (masses $\approx$ 300 MeV) with $m_d - m_u = 6$ MeV. The pion-nucleon coupling in their model originates from a bremsstrahlung gluon, which produces a virtual quark-antiquark pair that, after a rearrangement of quarks, leads to an amplitude for the formation of a pion (hereafter called $^3S_1$ model). Their results show a breaking of the pion-nucleon coupling constant of $0.1 - 1.0\%$ and again into the wrong direction. (In Ref. [5] it was prematurely concluded from [34] that the $^3S_1$ model supported the large CIB.) The CIB due to $\pi^0 - \eta$ mixing is small and again in the "wrong" direction [35]. Similar results are obtained in [36] using a Nambu and Jona-Lasinio model [37] for calculation of the pion and nucleon states. Also, $\rho^0 - \omega$ mixing gives a small CIB. In a cloudy bag model calculation [38], one obtained that $g_{nn\pi^0}$ is about 0.4% bigger than $g_{pp\pi^0}$, which is again much smaller than one expects from the Nijmegen phase-shift analysis. However, another bag model calculation [39], which used the MIT bag model [40] wave functions, reported a small CIB into the other direction.

Finally, we will discuss CIB in the context of the $^3P_0$ model [41–43]. The results of this model are rather similar to those of [33]. However, in contrast to the gluon model, the interaction Lagrangian, which is akin to mass terms, breaks chiral symmetry. Since the $^3P_0$ model links the hadron couplings to the physical vacuum, it suggests the possibility to relate the CIB in the pion-nucleon coupling to the CIB of the vacuum. However, we noticed that the pion-nucleon coupling in the $^3P_0$ model is rather sensitive to the radii of the pions and the nucleons. (This was claimed not to be so for the $^3S_1$ model, but we found this rather to depend on the gluon propagator used [44].) Ignoring this dependence for a moment, we make an estimation of the CIB in the $^3P_0$ model pair-creation constant γ, which would be needed if the CIB was entirely to be explained this way.

The creation of the quark-antiquark pair is described by the interaction Lagrangian density

$$\mathcal{L}(^3P_0) = [\bar{q}_i(x)(O_{\bar{q}q})_{i,j}q_j(x)]_{N.V.} \ , \tag{14}$$

where the subscript $N.V.$ means that the operator does not work on the valence quarks. So, this interaction either creates a $\bar{q}q$ pair from the vacuum or destroys such a pair. The matrix O is a matrix in quark-flavor space and it is here supposed not to give quark mixing. However, we will allow for SU(3)-symmetry breaking and hence, we give it the form $(O_{\bar{q}q})_{i,j} = \gamma_i \delta_{ij}$ $(i, j = u, d, s)$. In this paper we will be concerned with the up and down quark only, and so we can restrict ourselves to the upper 2×2 matrix. Introducing the $I = 0$ and $I = 1$ components of the $^3P_0$ pair by the combination

$$\gamma_0 = \tfrac{1}{2}(\gamma_u + \gamma_d) \ , \quad \text{and} \quad \gamma_1 = -\tfrac{1}{2}(\gamma_u - \gamma_d) \ , \tag{15}$$

it is now straightforward to calculate the different πN coupling constants by working out the matrix elements $\langle N_2\pi|H(^3P_0)|N_1\rangle$. Here we include into the constants γ_i $(i = u, d)$ the overlap integrals, assuming these to be independent of the particular pion and nucleons. We then find the following results

$$\begin{array}{rlrl} g_{pp\pi^0} = & g_0 & g_{pn\pi^+}/\sqrt{2} = & g_0 + \tfrac{3}{2}\Delta g \\ -g_{nn\pi^0} = & g_0 + 2\Delta g & g_{np\pi^-}/\sqrt{2} = & g_0 + \tfrac{1}{2}\Delta g \end{array} \ , \tag{16}$$

which implies that $g_c - g_0 \approx \Delta g = \frac{4}{9\sqrt{3}}\gamma_1$. Using the result from [5], we get $\gamma_1/\gamma_0 = 4.6\%$. On the other hand, for the new analysis [1], with the magnetic-moment interaction included, we get $\gamma_1/\gamma_0 = 3.0\%$.

Coming back to the dependence on the pion radius, one would expect that $R_{\pi^\pm} > R_{\pi^0}$, due to the Coulomb interaction between the quarks. It appears in the (naive) ${}^3P_0$ model that $R_{\pi^\pm} - R_{\pi^0} \approx 0.05$ fm would be sufficient to explain the CIB we found. However, it must be stressed that we do not know much about the difference between the pion radii. Even the definition of the radius that would be operative here, is rather unclear. Therefore, how much of the CIB is due to the internal structure of, e.g., the pions and how much there has to be explained for example as an effect of a genuine CIB of the vacuum, is at present obscure.

To summarize, we have discussed briefly various theoretical possibilities for the origin of CIB. However, for most of them it has already been demonstrated in the literature that they are unable to produce sizable effects. Therefore, we conclude that more effort is needed, both experimentally and theoretically, to settle the size of the CIB. We expect to gain further information from the 0–350 MeV np phase-shift analysis.

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TABLES

TABLE I. Determionation of the neutral-pion coupling constant $10^3 \times f_0^2$. (a) and (b) indicate different data sets (from [5]).

Bugg	[24]	75.2±3.9	
MacGregor, Arndt, Wright	[25]	81.4±4.6	
Breit *et al.*	[26]	73.1–81.8	
Bugg *et al.*	[15]	77.8±3.6	
Kroll	[27]	80.3±2.2	
Bergevoet *et al.*	[2]	80.2±6.6	(a)
		74.1±5.5	(b)

TABLE II. Results *pp* phase-shift analysis for f_0^2 for different potential tails (from [5]).

V_{NUC}	b[fm]	χ^2_{min}	$10^3 \times f_0^2$
V_{OPE}	1.8	1288.9	71.9±0.8
$V_{OPE} + V_{HBE}^N$	1.4	1266.7	72.6±0.6
$V_{OPE}^s + V_{HBE}^N$	1.4	1265.9	72.5±0.6
$V_{OPE} + V_{HBE}^P$	1.4	1273.3	74.6±0.6

TABLE III. The change in χ^2 due to the inclusion of the magnetic-moment interaction for the recent low-energy polarization data.

Holslin *et al.*	[29]	10.03 MeV	12 data	$\chi^2 = 17.1 \rightarrow 10.3$
Tornow *et al.*	[30]	16.9 MeV	15 data	$\chi^2 = 19.6 \rightarrow 16.4$
Sromicki *et al.*	[31]	25.0 MeV	16 data	$\chi^2 = 29.2 \rightarrow 18.1$

FIGURES

FIG. 1. Ellipses of constant χ^2 in the (m, f^2) plane, for optimal adjustment of the P-matrix parameters. Solid ellipse: 69% CL region ($\Delta\chi^2 = 2.4$). Dashed ellipse: 95.5% CL region ($\Delta\chi^2 = 6.2$). Filled circle with vertical bar: value and error for m with free f_0^2. Open circle with horizontal bar: value and error for f_c^2 from πN scattering (from [8]).