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Analysis of the pp analyzing-power data at 50.04 MeV*

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Abstract

The recent, very accurate proton-proton analyzing power measurements by the Zürich group at $T_{\rm lab} = 50.04$ MeV are analyzed. We show that in order to arrive at a proper description of these data, the magnetic-moment interaction has to be included in all partial waves. It is also shown that some of the approximations made in the literature, when trying to incorporate this magnetic-moment interaction, are inadequate. From these beautiful data one can now determine quite accurately the phase shifts at 50 MeV. We compare our results with the phase shifts found in other phase-shift analyses. Some of our phase shifts differ by 6–9 standard deviations from these analyses. Finally, we compare the predictions of some well-known nucleon-nucleon potential models with these high-precision data.

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I. INTRODUCTION

Various nucleon-nucleon phase-shift analyses [1–3] have been able to determine rather accurately the proton-proton phase shifts (and mixing parameters). Still, these analyses sometimes disagree in their phase shifts by more than a few standard deviations. As an example, look at the pp phase shifts at $T_{\rm lab} = 50$ MeV. The BLL analysis of Bystricky, Lechanoine-Leluc, and Lehar [2] (hereafter referred to as BLL) gives $\delta(^3P_2) = 5.92^\circ \pm 0.05^\circ$, whereas the AHR analysis of Arndt, Hyslop, and Roper [3] gives $\delta(^3P_2) = 5.70^\circ \pm 0.04^\circ$, which is a difference of almost 3.5 standard deviations (s.d.). Similarly, the AHR analysis gives $\varepsilon_2 = -1.52^\circ \pm 0.01^\circ$, whereas the Nijmegen analysis [1] gives $\varepsilon_2 = -1.72^\circ \pm 0.02^\circ$, a difference of as much as 9 s.d. These differences are due to a number of reasons. Firstly, the electromagnetic interaction is treated differently in the different analyses. Secondly, the parametrizations of the phase shifts as a function of the energy differ. Finally, there are differences in the treatment of the higher partial waves, and in the data bases that are used in the various analyses. To clear up the situation at 50 MeV one needed some high-precision experiments from which the phase shifts can be determined with much higher accuracy. Perhaps it will then be possible to pinpoint the origin of these differences between the phase shifts.

Another problem with the pp phase shifts is that for certain purposes some of the phase shifts are still not determined accurately enough. For example, the spin-orbit combination Δ_{LS} of the triplet P waves at lower energies can be determined rather accurately in np scattering. A sufficiently accurate determination of Δ_{LS} in pp scattering at the same energies could possibly shed some light on the problem of charge (in)dependence of the nuclear force [4].

In a recently completed experiment [5] at the Paul Scherrer Institute (PSI), the analyzing power A_y in pp scattering for 10 scattering angles at $T_{\rm lab} = 50.04$ MeV has been measured with the high accuracy of $\Delta A_y = 1.5 \times 10^{-4}$, up to an overall normalization error of about 2%. A new method for measuring absolute analyzing powers has been proposed by the same group [6], which would lead to calibrations of polarized proton beams that are up to an order of magnitude better. An experiment using this method is in progress at PSI [7], and a more precise determination of the normalization uncertainty will probably increase the impact these data have on phase-shift analyses and nucleon-nucleon potential models.

Recently, the Nijmegen phase-shift analysis (PSA) of the pp scattering data below $T_{\rm lab} =$ 350 MeV has been finished [1]. The new accurate 50.04 MeV A_y data were not yet included in our data base because at the time we were not aware of their imminent publication. About one and a half years ago, however, we did analyze these data using a preliminary version of our PSA, the results of which are discussed in the paper by the Zürich group [5]. At that time the magnetic-moment (m.m.) interaction was not yet included in our analysis.

In this paper we present the results of our analysis of these excellent 50.04 MeV A_y data using the results of our recently finished PSA [1] that now does include the m.m. interaction, and compare the model predictions of some realistic nucleon-nucleon potential models with these data. The very high accuracy of these data makes that the way the m.m. interaction is included in the PSA is of importance, so we will first discuss how it is included in our PSA. The long-range part of the m.m. potential is given by

$$V_{\mathrm{m.m.}}(r) \sim -\frac{\alpha}{4M_p^2 r^3}\left[\mu_p^2 S_{12} + (6+8\kappa_p)\mathbf{L}\cdot\mathbf{S}\right] \ , \tag{1}$$

where S_{12} is the tensor operator and $\kappa_p = \mu_p - 1 = 1.792847$ is the proton anomalous magnetic moment. The short-range part of the m.m. potential is more complicated. In the point-particle approximation, i.e., point electric charges and point magnetic moments, there are additional contact terms. In the Nijmegen PSA we have included the proton electric and magnetic form factors as given by [8]

$$G_E(t) = (1 - t/0.71)^{-2} \ , \quad G_M(t) = \mu_p G_E(t) \ , \tag{2}$$

where t denotes the Mandelstam momentum transfer squared. Using these form factors the contact terms in the short-range m.m. potential now become Yukawa-like potentials.

The m.m. partial-wave scattering amplitudes are now obtained [9] by calculating the contribution of the m.m. potential in Coulomb distorted-wave Born approximation (CD-WBA). In the point-particle approximation the contact terms only contribute for $J = 0$ and 1, whereas the effect of the form factors given by Eq. (2) is only of importance in the lowest partial waves ($L \lesssim 4$). The m.m. partial-wave scattering amplitudes are then summed with Legendre polynomials up to $L = 1000$.

We do not find any significant difference in our results when we replace the form factors of Eq. (2) by their point-particle approximation. This is not surprising in view of the short-range effect of the contact terms and the fact that the lower partial waves up to $J = 4$ are parametrized.

Because also the Coulomb amplitude and the vacuum polarization amplitude are added separately in our PSA [1,10], this means that now the electromagnetic amplitude (consisting of the Coulomb, magnetic moment, and vacuum polarization amplitudes) is clearly separated from the nuclear amplitude. Because the phase shifts which parametrize the nuclear amplitude are now phase shifts with respect to electromagnetic (e.m.) wave functions [9], the partial-wave S matrix $S_{\mathrm{e.m.}+N}$ appears in the scattering amplitude as (see also Ref. [10] for definitions and notations)

$$(S_{\mathrm{e.m.}+N} - 1) = (S_{\mathrm{e.m.}} - 1) + S_{\mathrm{e.m.}}^{1/2}(S_{\mathrm{e.m.}+N}^{\mathrm{e.m.}} - 1)S_{\mathrm{e.m.}}^{1/2} \ , \tag{3}$$

where $(S_{\mathrm{e.m.}} - 1)$ represents the contribution of the partial-wave e.m. scattering amplitude and $(S_{\mathrm{e.m.}+N}^{\mathrm{e.m.}} - 1)$ represents the partial-wave scattering amplitude of the nuclear plus e.m. interaction with respect to e.m. wave functions, and the latter part is usually parametrized in a PSA. The adjustment for the fact that the partial-wave e.m. scattering amplitude is calculated separately, is accounted for by multiplying the partial-wave nuclear scattering amplitude to the left and right with $S_{\mathrm{e.m.}}^{1/2}$.

It has been customary to include the m.m. interaction in phase-shift analyses only approximately. These approximations will be discussed next. Although the influence of the m.m. interaction on the scattering amplitude is largest in the lower partial waves, one can argue [11] that it is not necessary to include these effects explicitly, since the lower partial-wave phase shifts are parametrized anyway. In this approximation the m.m. interaction is not included in the $S_{\mathrm{e.m.}}^{1/2}$ factor, while it is included in the electromagnetic scattering amplitude for the higher partial waves only. This approximation is used in the BLL analysis [2], where they calculate the higher partial-wave m.m. scattering amplitudes in plane-wave Born approximation (BA). Moreover, they only take account of the spin-orbit part of the m.m. interaction, neglecting the tensor part. In the AHR analysis [3], the total m.m. scattering amplitude is calculated in BA and is only approximately corrected for Coulomb distortion effects. However, the nuclear scattering amplitude is not adjusted for the fact that the m.m. scattering amplitude is added in all partial waves, i.e., in this analysis the m.m. interaction is not included in the $S^{1/2}_{\mathrm{e.m.}}$ factor either.

We will show that for a proper description of the new A_y data both approximations are no longer suitable. The accuracy of these data requires a more thorough treatment of the m.m. interaction. Before the publication of this experiment, the improvement of including the m.m. interaction in a PSA in this way was found to be small and could only be seen in a data set that contained all data in a sufficiently large energy range (see Ref. [12], which discusses a similar treatment for including the m.m. interaction, but only approximately corrects for Coulomb effects). The high accuracy of this excellent experiment is such that the necessity for including the m.m. interaction in all partial waves, properly adjusting the partial-wave nuclear amplitudes, can already be seen clearly on these data alone. The influence of such an improved treatment is likely to become even more important when more high-precision measurements will become available.

The importance of the 50.04 MeV measurement is therefore twofold. On the one hand, the statistical error is very small which allows for a better, and especially more accurate determination of the phase shifts at 50 MeV. For example, the error on the $^3P_0$ phase shift is now reduced by a factor of two. On the other hand, the data require an improvement in the theoretical framework used in a PSA for describing the scattering observables. The data are significantly better described when the scattering amplitude of the m.m. interaction is included in *all* partial waves, adjusting the partial-wave nuclear amplitudes accordingly. These data are the first where this improvement can be seen explicitly.

Of course, if one properly wants to compare some nucleon-nucleon (*NN*) potential model prediction with these data, the effects of the m.m. interaction have to be taken into account also. We find that, including the m.m. interaction properly, the older Nijmegen [13] and Paris [14] *NN* potential models are still in very good agreement with the new A_y data, while the prediction of the much more recent Bonn *pp* potential [15] is much worse. This is because the tensor potential of the Bonn potential is too strong and its spin-orbit potential is too weak.

II. ANALYSIS OF THE DATA

A. Multienergy analysis

We start with the recently finished multienergy (m.e.) PSA by the Nijmegen group [1] of the *pp* scattering data below $T_{\mathrm{lab}} = 350$ MeV. The data set contains the *pp* scattering data published in a regular physics journal as of 1955 and is updated up to August 1989. In our PSA a number of data were discarded from the data base as a consequence of our statistical criterion that data should not be off more than three s.d., which left us with 1626 scattering observables or, including the 129 normalization data, 1755 scattering data. We will hereafter refer to this data base as PP89. To parametrize the lower partial waves up to $J = 4$ we use 28 parameters and we find $\chi^2_{\min} = 1760.6$. The effect of including the m.m. interaction in all partial waves in our PSA was small, but certainly not negligible, since it caused a drop of 28.6 in $\chi^2_{\min}$.

The effect of including the m.m. interaction in our PSA is even more strikingly seen when we use this PSA to predict the χ^2 on the new 50 MeV A_y data. The results are shown in Table I, where we give the χ^2 values on these A_y data (i) using the model parameters from our original m.e. PSA [1] and (ii) after refitting the model parameters. Both analyses were done once including the m.m. interaction and once leaving it out. The PSA predictions on the A_y data before refitting of the model parameters are given in parentheses. We also give the $\chi^2_{\min}$ on the 1755 data of our original data base PP89 (i.e., the data base of our PSA as published in Ref. [1]), and the $\chi^2/N_{\rm dat}$ on the 1766 data of the total data base, i.e., including the $10+1$ A_y data. In Ref. [1] we showed that the effect of including the m.m. interaction in our PSA was a 5 s.d. effect (a drop of 28.6 in $\chi^2_{\min}$). When we now add these 11 A_y data to our data base and redo the same analysis, then the effect of including the m.m. interaction is a drop of 106.3 in $\chi^2_{\min}$. So in the new data base the inclusion of the m.m. interaction is a 10 s.d. effect.

Let us now first discuss in a little more detail the analysis where we omit the m.m. interaction. The PSA prediction then gives a χ^2 on the A_y data which is very large, the main contribution coming from the forward angle data. Because of the high accuracy of these data their weight as compared to the other data is relatively high, so the χ^2 can be lowered considerably by refitting the model parameters. In that case the drop in χ^2 on these data can be much larger than the rise in χ^2 on the other data. But now one of the forward-angle A_y data is still not described well enough and is off by more than three standard deviations. Moreover, the $\chi^2_{\min}$ on the 1755 data of the original data base PP89 rises with as much as 52.7. Adding the 11 A_y data to our data base results in a rise of 82.1 in $\chi^2_{\min}$. In view of this large rise, one should consider two possibilities. The first one is that the data contain large unknown systematic errors. In such a case one should ask oneself the question if these data should not be discarded. The second possibility is that the model used to describe the scattering data is not totally correct. In that case, these new, much more accurate data require improvements in the model description, which up to now could be neglected or approximated.

This latter possibility is demonstrated dramatically by the result of the second analysis, where the improvement in the theoretical description of the data by including the m.m. interaction in our way has a drastic influence on the quality of the description. In this case the PSA prediction [case (i)] is already in excellent agreement with the A_y data. Refitting of the model parameters [case (ii)] is not really necessary. Including the 11 A_y data to our data base gives in this case a rise of 4.4 in $\chi^2_{\min}$. This is a clear indication for the necessity of including the m.m. interaction if one wants to describe these data correctly.

The new data are also accurate enough as to question the validity of approximations made in the literature in treating the m.m. interaction. When we use the BLL way of including the m.m. interaction in our PSA including the 50 MeV data, we arrive at $\chi^2(A_y) = 26$ which is rather high for 11 data. But what is worse is that, when we then compare the χ^2 on the remaining data base with the results of the corresponding PSA without these data, we observe a rise of $\Delta\chi^2 \approx 32$, so the inclusion of the 11 A_y data causes a total rise in χ^2 of

58, which is pretty high. When we use the AHR way of including the m.m. interaction, we find $\chi^2(A_y) = 5.5$ and a rise of $\Delta\chi^2 = 3.5$ on the other data, which is already much better. For the Nijmegen way of including the m.m. interaction, we obtain $\chi^2(A_y) = 4.3$, where the $\chi^2_{\rm min}$ on the other data rises with only 0.1.

In view of these results, it is therefore obvious that the present accuracy of analyzing power data requires the improved treatment for including the effects of the m.m. interaction in a PSA. Before the measurements of the Zürich group, the effect of including the m.m. interaction was found to be small only, and could be conveniently approximated by including the spin-orbit part of the interaction in the higher partial waves only. With these new 50 MeV data, however, this treatment is no longer found to be acceptable. The m.m. scattering amplitude has to be included in *all* partial waves and one has to adjust the partial-wave nuclear scattering amplitudes accordingly.

B. Single-energy analysis

Next, we redid the single-energy (s.e.) analysis as presented in Ref. [1] where the m.m. interaction has been included. Before the inclusion of the new 50 MeV data we arrived at $\chi_{\rm s.e.} = 207.0$ for 218 scattering observables in the energy range 35–75 MeV. With the new data we now obtain $\chi_{\rm s.e.} = 212.0$ for 228 scattering observables. The χ^2 values on the 50 MeV A_y data are also given in Table I, where again the χ^2 prediction of the s.e. analysis without these data is parenthesized. In Fig. 1 we show the difference between the s.e. result including the new A_y data and the experimental results. The curve denotes the difference between the analysis including these data and the prediction of our original s.e. analysis [1] without these data. This figure is similar to Fig. 8 of the Zürich paper [5], which contains the results of our preliminary analysis without the m.m. interaction of these data. Comparing both figures demonstrates dramatically the effect of the m.m. interaction.

The phase shifts for $J \leq 2$ as determined in the s.e. analysis are presented in Table II and are compared with the results of the BLL and AHR analyses. These latter results were taken from Table 3 of Ref. [5]. The high accuracy of the new 50 MeV A_y data allows for a more accurate determination of the phase shifts when compared with previous analyses where these data were not included. Our very accurate determination of the $^1D_2$ phase shift is mostly due to the accurate $\sigma(\theta)$ data of Berdoz *et al.* [16] at 50.06 MeV. These data were not included in the AHR and BLL analyses as far as we know, perhaps because they were not available at that time. The errors in the triplet P phase shifts are strongly correlated, and it is more convenient to express these phase shifts in terms of central, tensor, and spin-orbit combinations, rather than in terms of the phase shifts themselves. The central combination Δ_C is largely determined by the differential cross sections $\sigma(\theta)$, whereas the tensor and spin-orbit combinations Δ_T and Δ_{LS} are more directly related to the analyzing power. These phase-shift combinations are also given in Table II. Because of the correlations between the P waves it is impossible for us to give errors on the corresponding phase-shift combinations of the BLL and AHR analyses. Our results for Δ_T and Δ_{LS} (accurately determined because of the accuracy of the analyzing power data) are in reasonable agreement with the results of the AHR analysis. However, the three different analyses still disagree with one another and for some phase shifts the differences amount to more than six standard deviations. We think these differences can be explained as follows.

In the BLL analysis the energy range is much larger and hence contains many more data (10–225 MeV, 1164 data) than the AHR or Nijmegen analyses. However, our data base in the same energy range contains almost as many data. The small difference is because of the fact that we do not include data that were published before 1955 or that have only been reported in conference proceedings. We also do not include total cross-section data (σ_{tot}, $\Delta\sigma_T$, $\Delta\sigma_L$), because we do not know how to calculate them model independently. Because of their larger data base, the influence of the 11 A_y data in the BLL analysis cannot be that large since the phase shifts have to describe the more than thousand other data as well. The AHR and Nijmegen analyses are less constrained. It is also not clear to us what kind of phase shifts they use. It seems they do not include the Coulomb correction factor as given by Breit [17], which means that their phase shifts cannot be directly compared with our phase shifts without correcting for this.

In the AHR analysis the energy range is slightly smaller (36–54 MeV) and contains 162 *pp* observables as well as 307 *np* observables, whereas we have 228 *pp* observables and no *np* observables in a slightly larger energy range. The Nijmegen analysis gives *pp* phase shifts, whereas the AHR analysis gives isospin $I = 1$ phase shifts (*pp* as well as *np*). Arndt and co-workers [3] assume that they know how to obtain *pp* phase shifts from *np* phase shifts, and vice versa. They assume, therefore, that they know the charge-independence breaking effects and can correct for them. We would say that in the AHR analysis the *pp* phase shifts are "contaminated" by the *np* data, which explains the differences between our results. On the other hand, the $^1S_0$ phase shift of the AHR and Nijmegen analyses agree fantastically. But then the *pp* $^1S_0$ phase shift in the AHR analysis is determined by the *pp* data alone. The Δ_T phase shift also agrees rather well. The reason is that the *np* data do not determine the Δ_T phase shift very accurately. This phase-shift combination, therefore, is solely determined by the *pp* data. Similarly, the Δ_{LS} phase shift of the AHR analysis is in reasonable agreement with our result, which is because the accurate 50 MeV *pp* measurement does not allow for the results for this phase shift to differ too much. The two s.d. difference is probably due to the bulk of *np* analyzing power data around 50 MeV which are likely to influence the *pp* Δ_{LS} phase shift in the AHR analysis.

Let us summarize the impact the new data have on phase-shift analyses. The data are in excellent agreement with our earlier analysis where these new data were not included. The accuracy of these A_y data allows for a more accurate determination of the phase shifts at 50.0 MeV. Before the inclusion of these data the different analyses sometimes disagreed by a few standard deviations in their phase shifts. Because of the higher accuracy, however, some of our phase shifts now differ by more than 6 s.d. when compared with the results of other analyses, so this experiment is very important in that these differences are now much more pronounced. For example, before the inclusion of the A_y data the difference between our result for the $^3P_0$ phase shift and the result of the BLL analysis was within 2 s.d., whereas with these new data included this phase shift can now be determined twice as accurately, resulting in a 6 s.d. difference. Similarly, the difference between our determination of the $^3P_2$ phase shift and the result of the AHR analysis was within 3 s.d., whereas now there is a 7 s.d. difference. We do think that our phase shifts are more in accordance with the data, because we have included the m.m. interaction in a more accurate way and we obtain a χ^2 per degree of freedom of 1.12, which is considerably lower than the other analyses that obtain about 1.3.

III. POTENTIAL MODEL RESULTS

Finally, we would like to compare some nucleon-nucleon potential model predictions with these A_y data. In order to make a proper comparison possible, we now also have to include the effects of the m.m. interaction. To demonstrate the importance of the m.m. interaction, we will first discuss its effects on the potential model comparison using the four different treatments for including the m.m. interaction as discussed earlier. In our discussion of these four different treatments, we will restrict ourselves to the Nijmegen soft-core *NN* potential [13] (Nijm78). Other potential models give similar results.

The differences between the Nijm78 predictions using these four approximations for including the effects of the m.m. interaction and the results of the s.e. analysis are shown in Fig. 2. There we have also plotted the difference between the experimental data and the s.e. analysis result. In the first two cases, i.e., no m.m. interaction at all (dotted curve), and the BLL way for including the m.m. interaction (dashed curve), the differences are found to be very large, due to an incorrect description of the forward-angle data. The corresponding χ^2 values are $\chi^2(A_y) = 596$ and $\chi^2(A_y) = 843$, respectively, indicating pretty bad fits. For the third case (dash-dotted curve), i.e., the AHR way for including the m.m. interaction, we obtain $\chi^2(A_y) = 24$, which is already rather good. For the last treatment (solid curve), which is the treatment of the m.m. interaction as used in the Nijmegen PSA, we find $\chi^2(A_y) = 10.5$, where the difference is due to a better description of the forward-angle data as can be seen in Fig. 2. From these results we conclude that the m.m. interaction should be included in *all* partial waves. Furthermore, the additional adjustment of the partial-wave nuclear amplitudes as applied in the Nijmegen treatment gives a significant improvement in the description of the data, indicating the importance of this adjustment. Using this latter treatment for including the m.m. interaction we can now properly confront some well-known *NN* potential models (Nijmegen, Paris, and Bonn) with these A_y data.

The soft-core Nijmegen *NN* potential [13] (Nijm78) as well as the parametrized Paris *NN* potential [14] (Paris80) were fitted mainly to the *NN* data of 1969 using the Livermore-X PSA [18]. In Table III we give the χ^2 values of these potentials, where we have restricted ourselves to the 1569 data in the 3–350 MeV energy range, which includes the new A_y data. This is because the $^1S_0$ phase shift of the Paris80 *pp* potential at low energies ($T_{\rm lab} \lesssim 1$ MeV) is in error [10]. Therefore, the low-energy data (0–3 MeV) give a relatively high contribution to χ^2, and inclusion of these data would result in a value of $\chi^2/N_{\rm dat} = 4.5$.

Using the AHR analysis [3] and the computer code SAID (Ref. [19]), the Bonn *NN* potential [20] was fitted to the world *NN* data of 1987. A proper confrontation with the *pp* data of the full Bonn *NN* potential is impossible for us, because the Coulomb interaction can only be treated approximately in this case. The only way we can confront the Bonn potential with the *pp* data is to use the coordinate space version OBEPR (Bonn87) of Ref. [20]. This potential does not fit the *pp* data very well as is shown in Table III. Because the Bonn87 $^1S_0$ potential is fitted to the low-energy *np* data, inclusion of the 0–3 MeV data would again give rise to a relatively high contribution to χ^2.

Recently, two newer versions of this OBEPR potential have been constructed [21], but they still do not fit the *pp* data very well. We obtain $\chi^2/N_{\rm dat} = 10$ and 9, respectively, on the 3–350 MeV *pp* data.

In their quest for better potentials the Nijmegen group [22] as well as the Bonn group [15] constructed new *pp* potentials. The new Bonn *pp* potential [15] (Bonn89) is an adaption to the *pp* data of the older full Bonn potential of Ref. [20]. The new Nijmegen *pp* potential [22] (Nijm89) is a Reid-like potential based on the original Nijm78 potential. In Table III we present the χ^2 values for both these potentials, where we again have restricted ourselves to the 3–350 MeV *pp* data in order to make a fair comparison with the other potential models possible. However, the Nijm89 as well as the Bonn89 *pp* potentials give an excellent description of the 0–3 MeV *pp* data. We note that if we omit the low-energy data, the older Nijm78 and Paris80 potentials and the newest Bonn89 potential are all found to be of about the same quality. The Nijm89 potential, on the other hand, has a much lower χ^2 on the *total* data base ($\chi^2/N_{\rm dat} = 1.09$) and is almost as good as our m.e. PSA.

In Table III we also give the χ^2 comparison of the five potential models with the new accurate 50 MeV A_y data alone. The Nijm78 and Paris80 potentials give a reasonably good description of these data. The prediction of the Bonn89 potential model is much worse, which is mainly due to its triplet P phase-shift combinations Δ_T and Δ_{LS}, which are 8 and 10 s.d. off, respectively, the tensor combination being too strong and the spin-orbit combination being too weak. This is shown in Table II where we included the phase shifts of the potential models. (The results of the Bonn87 potential are even worse and will no longer be considered here.) The differences in the Δ_T and Δ_{LS} phase shifts lead to an incorrect forward-angle analyzing power when compared with the experimental data, which can be seen in Fig. 1. There we present the differences between the model predictions of the four *NN* potential models (the results of Bonn87 are not shown) and the experimental data. The new Nijm89 potential is in excellent agreement with the data, but then its model parameters were adjusted using the s.e. results of our PSA [1] which already gives a very good description of these data.

IV. CONCLUSIONS

We have analyzed the accurate analyzing power data that have been measured recently at PSI. Because of these data, the phase shifts at 50 MeV can now be determined much more accurately. We show that the accuracy of these data requires a more thorough treatment of the m.m. interaction in a phase-shift analysis. These measurements are the first where the effects of such an improved treatment in the description of the data can be seen clearly and explicitly. We show that these improvements should also be taken into account when one wants to compare an *NN* potential model prediction with such accurate experimental results. The A_y predictions of the older Nijm78 and Paris80 potentials are found to be in rather good agreement with the data, whereas the recent Bonn89 potential is doing much worse. This is because the tensor potential of the Bonn89 potential is too strong and its spin-orbit potential is too weak. This feature of the Bonn potential was already pointed out [23] at the 1983 Karlsruhe conference.

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TABLES

TABLE I. χ^2 values on the 50.04 MeV A_y data and on the total data base (PP89) of our PSA. Values are given (i) before and (ii) after the inclusion of the new 50 MeV data and in the first case predicted values are parenthesized. Results are given for the multienergy (m.e.) analyses with or without the m.m. interaction included and for the single-energy (s.e.) analysis with the m.m. interaction.

		50.04 MeV	PP89	χ^2/N_{dat}
Number of data:		11 A_y	1755	1766
m.e. analyses				
no m.m.	(i)	(268.5)	1789.2	1.17
	(ii)	29.4	1841.9	1.06
with m.m.	(i)	(4.5)	1760.6	1.00
	(ii)	4.3	1760.7	1.00
s.e. analyses				
	(i)	(7.9)		
	(ii)	4.0		

TABLE II. Phase shifts in degrees at 50.0 MeV of the s.e. BLL analysis [2], the s.e. AHR analysis [3], and of the present Nijmegen analysis including the new 50.04 A_y data. Also shown are the central, tensor, and spin-orbit combinations of the triplet P phase shifts and the corresponding phase shifts of four potential models (see text).

	s.e. analyses			NN potential models			
	BLL	AHR	Nijmegen	Nijm78	Paris80	Bonn89	Nijm89
$^1S_0$	39.24(8)	39.05(10)	39.05(9)	39.58	38.75	38.25	38.82
$^1D_2$	1.73(4)	1.59(4)	1.72(1)	1.63	1.80	1.68	1.70
$^3P_0$	10.37(14)	11.54(13)	11.43(12)	11.80	11.81	12.66	11.43
$^3P_1$	–8.41(8)	–8.39(5)	–8.29(4)	–8.36	–8.41	–8.34	–8.28
$^3P_2$	5.97(5)	5.65(2)	5.85(2)	5.78	5.72	5.56	5.80
ε_2	–1.68(4)	–1.53(1)	–1.73(2)	–1.77	–1.78	–1.75	–1.73
Δ_C	1.67	1.62	1.76(1)	1.73	1.69	1.72	1.73
Δ_T	–3.61	–3.74	–3.72(2)	–3.78	–3.79	–3.88	–3.72
Δ_{LS}	2.86	2.53	2.58(3)	2.53	2.52	2.29	2.58

TABLE III. χ^2 values on the new 50 MeV A_y data and on the 3–350 MeV data base including these data for five potential models (see text). The third column contains the $\chi^2/N_{\rm dat}$ for this data base.

	3–350 MeV	$\chi^2/N_{\rm dat}$	50.04 MeV
Number of data	1569	1569	11
Nijm78	3153	2.01	10.5
Paris80	3145	2.00	17.5
Bonn87	21356	13.60	283.8
Bonn89	3074	1.96	184.8
Nijm89	1750	1.12	3.7

FIGURES

FIG. 1. Difference $\Delta A_y(\text{mod})$ between the four potential model predictions A_y^{mod} and the experimental data A_y^{exp}, and the difference $\Delta A_y(\text{fit})$ between the best fit of the data A_y^{fit} and the experimental data A_y^{exp}. The curve denotes the difference between the final fit including the A_y data and the prediction of the PSA without these data.

FIG. 2. Difference between the A_y predictions of the Nijmegen NN potential [13] using four different treatments for including the m.m. interaction and the s.e. analysis result. The difference between the s.e. result and the experimental data points is also shown. Dotted curve: no m.m. interaction; dashed curve: BA in higher partial waves; dash-dotted curve: no adjustment of nuclear phase shifts; solid curve: full treatment. Details are given in the text.