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Determination of the charged-pion coupling constant from data on the charge-exchange reaction $\overline{p}p \rightarrow \overline{n}n$*

R.G.E. Timmermans, Th.A. Rijken, and J.J. de Swart
Institute for Theoretical Physics, University of Nijmegen, Nijmegen, the Netherlands

Abstract

The coupling constant of the charged pions to nucleons is extracted from a partial-wave analysis of antinucleon-nucleon scattering data below $p_{\mathrm{lab}} = 950$ MeV/c. For the value at the pion pole we find

$$f_{\mathrm{c}}^2 = 0.0751 \pm 0.0017 \qquad \text{or equivalently} \qquad g_{\mathrm{c}}^2 = 13.6 \pm 0.3 \quad .$$

This result is in agreement with the value found in the recent VPI&SU analysis of πN scattering data. Comparing with the neutral-pion coupling constant as determined in the Nijmegen phase-shift analysis of proton-proton scattering data, we see no evidence for a charge-dependence of the pion-nucleon coupling constants.

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The most accurate determinations of the coupling constant of the charged pions to nucleons came from analyses of πN scattering data. A generally accepted value [1,2] was

$$f_{\rm c}^2 = (79 \pm 1)10^{-3} \quad \text{or} \quad g_{\rm c}^2 = 14.3 \pm 0.2 \ . \tag{1}$$

In the recent Nijmegen phase-shift analyses [3,4] of all pp scattering data below $T_{\rm lab} = 350$ MeV the coupling constant of the neutral pion to protons was determined [3,4] at the pion pole and found to be[1]

$$f_{\rm p}^2 = (74.9 \pm 0.7)10^{-3} \quad \text{or} \quad g_{\rm p}^2 = 13.55 \pm 0.13 \ , \tag{2}$$

where the errors are purely statistical. Since there was no obvious reason to doubt either of these two values, it was concluded [3] that they seemed to indicate possible evidence for an unexpected large breaking of charge-independence of the pion-nucleon coupling constants.

Theoretically, one had a hard time finding an explanation for such a large breaking of charge-independence [5]. Three possible culprits that come to mind turned out to be only small offenders. Electromagnetic radiative corrections to pion-nucleon coupling constants are of the order of 0.5%, or less [6]. Deviations of charge-independence due to the mass difference between up and down quarks are at most 2% [7,8]. Quantummechanical mixing [9] of π^0 and η also cannot do the job. It should be noted that not only the exact size, but also the sign of a possible splitting of $NN\pi$ coupling constants is uncertain. Moreover, if SU(2)-isospin symmetry of pion-nucleon coupling constants is broken at a 5 or 10% level, one expects still larger breakings of SU(3)-flavor symmetry of meson-baryon coupling constants. But this latter assumption seems to be quite reasonable. For instance, we extracted [10] the ΛpK coupling constant from high-quality data on the strangeness-exchange reaction $\overline{p}p \to \overline{\Lambda}\Lambda$. The result, in case of pseudovector coupling, is consistent with the prediction from SU(3), leaving only room for small SU(3) breakings. The case for approximate charge-independence of the strong interaction, and in particular of $NN\pi$ coupling constants, thus appears to be rather strong as far as theory is concerned.

Very recently, in a new analysis [11] of πN scattering data Arndt *et al.* determined the charged-pion coupling constant. They found

$$f_{\rm c}^2 = (73.5 \pm 1.5)10^{-3} \quad \text{or} \quad g_{\rm c}^2 = 13.31 \pm 0.27 \ , \tag{3}$$

at variance with (1), yet by way of charge-independence consistent with (2). According to Arndt *et al.* the main reason for the difference between their value (3) and the old value (1) is that now a much larger and qualitatively much better data set is available. In view of these findings, other determinations of the charged-pion–nucleon coupling constant are most welcome. We want to show here that data on the charge-exchange reaction $\overline{p}p \to \overline{n}n$ can provide valuable independent information on the charged-pion coupling constant [12]. The result is in support of the value (3).

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[1] In Ref. [3] only a preliminary version of the pp phase-shift analysis was presented which did not contain the important magnetic-moment interaction and also the data set was not as complete. Therefore only the value for $f_{\rm p}^2$ given in Ref. [4] should be quoted.

The results presented here are part of a much larger program to perform partial-wave analyses (PWA) of antinucleon-nucleon scattering data below $p_{\rm lab} = 950$ MeV/c. Some preliminary results have already been presented [13] and a detailed account will be published elsewhere [14]. In this communication we concentrate on the determination of the charged-pion coupling constant.

The method of analysis used is essentially the same as used in the *pp* phase-shift analysis (PSA), but there are some additional complications in that both isospin 0 and 1 contribute and there is a large amount of annihilation into mesonic channels. Let us repeat the argument [15] that in a single-energy proton-proton PSA one needs for each angular momentum J on the average 2.5 real parameters, but in antinucleon-nucleon scattering 20 real parameters are required for $J \neq 0$ and 8 for $J = 0$. This means that in order to perform a partial-wave analysis a lot of theoretical input is necessary. As a consequence, the same degree of uniqueness as in a nucleon-nucleon PSA cannot be reached.

For each partial wave the relativistic Schrödinger equation [16] is solved for the coupled $\overline{p}p$ and $\overline{n}n$ channels, starting with a boundary condition at $r = b = 1.25$ fm. The relativistic Schrödinger equation is a differential form of the relativistic Lippmann-Schwinger integral equation, which in its turn is equivalent to three-dimensional relativistic integral equations like the Blankenbecler-Sugar equation [17]. The boundary condition, called the P matrix, is the logarithmic derivative of the wave-function matrix. In the outer region $r > b$ the C-parity transformed Nijmegen soft-core OBE potential [18] is used as intermediate- and long-range interaction. This potential gives an excellent description of the rich and accurate data on nucleon-nucleon scattering. It is also one of the main reasons for the success of our PWA. The poorly known short-range interaction at $r < b$ is treated phenomenologically by parametrizing the P matrix as a function of energy. The mesonic annihilation is taken care of by using a complex P matrix, leading to a non-unitary S matrix for the coupled $\overline{p}p$ and $\overline{n}n$ channels. Actually the P-matrix parametrization chosen can be translated into a simple local short-range optical potential. The mass differences between p and n and between $\pi^{\pm}$ and π^0 and the Coulomb interaction are taken exactly into account (we work on the physical particle basis).

The data set on antinucleon-nucleon scattering below $p_{\rm lab} = 950$ MeV/c is extensively described and analyzed in [14]. For the purpose of this study we restrict ourselves to the data between $p_{\rm lab} = 400$ and 900 MeV/c, which is where the accurate data on charge-exchange differential cross sections [19,20] were taken at KEK and LEAR. The final set also contains recent high-quality data on elastic asymmetry [21,22], and the very recent charge-exchange analyzing-power data [23]. The excellent data from the pre-LEAR era and from KEK on the elastic differential cross section [24–26] are included as well. We have a total of 884 data-points. To achieve a good fit to these data we need 23 parameters, of which only 3 are for the annihilation. With this parameter set we reach an excellent $\chi^2_{\rm min}/{\rm data} = 1.15$.

In $\overline{N}N$ scattering one encounters three $NN\pi$ coupling constants, namely $f^2_{\rm p}$, $f^2_{\rm c}$, and $f^2_{\rm n} = f^2_{{\rm nn}\pi^0}$. Once charge-dependence of the $NN\pi$ coupling constants is accepted, one naturally expects that $f^2_{\rm c} \neq f^2_{\rm p} \neq f^2_{\rm n} \neq f^2_{\rm c}$. Since it is not possible to determine all three coupling constants from the data, one would like to have some theoretical input about the way charge-independence is broken. However, as stated above, there is no unambiguous prescription available. We tried a few alternatives, but it luckily turned out that the results are rather insensitive to $f_{\rm n}^2$, so we used $f_{\rm p}^2 = f_{\rm n}^2$.

For the coupling between pions and nucleons we use the pseudovector interaction lagrangian

$$\mathcal{L}_{\rm PV} \;=\; \frac{f}{m_{\rm S}}\sqrt{4\pi}\;(\overline{\psi} i\gamma_\mu\gamma_5\psi)\partial^\mu\phi \;\;, \tag{4}$$

because pseudovector coupling is favored over pseudoscalar coupling [10]. Here $m_{\rm S}$ is a scaling mass in order to make the pseudovector coupling constant f dimensionless. It is conventionally chosen to be equal to the charged-pion mass $m_{\rm S} = m_{\pi^+}$.

As in Ref. [3], a simple one-pion-exchange (OPE) potential without a form factor is used for $r > b$

$$V_{\scriptscriptstyle OPE}(r) = f^2\left(\frac{m}{m_{\rm S}}\right)^2\frac{1}{3}\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + S_{12}\left(1+\frac{3}{(mr)}+\frac{3}{(mr)^2}\right)\right]\frac{e^{-mr}}{r} \;\;, \tag{5}$$

where

$$\begin{array}{lll} f^2 = f_{\rm p}^2\,, & m = m_{\pi^0} & \text{for}\;\; \overline{p}p\rightarrow\overline{p}p, \\ f^2 = f_{\rm n}^2\,, & m = m_{\pi^0} & \text{for}\;\; \overline{n}n\rightarrow\overline{n}n, \\ f^2 = 2f_{\rm c}^2, & m = m_{\pi^+} & \text{for}\;\; \overline{p}p\leftrightarrow\overline{n}n. \end{array} \tag{6}$$

By using only the tail of the OPE potential, we determine the coupling constant at the pion pole. As stated above, the heavy-boson-exchange part of the Nijmegen potential is used.

In principle, we could try to determine both $f_{\rm p}^2$ and $f_{\rm c}^2$ by adding them both as parameters. This would give us both coupling constants with a certain statistical error and a correlation between them. However, our main goal is to investigate the possible charge-dependence of the $NN\pi$ coupling constants. In view of the values (1) and (3) it is the value of the charged-pion coupling constant $f_{\rm c}^2$ that is controversial. Moreover, one cannot hope to extract $f_{\rm p}^2$ in $\overline{p}p$ scattering more accurately than in pp scattering. We therefore fixed $f_{\rm p}^2$ at the value (2) found in the Nijmegen pp PSA and leave the more comprehensive study for the future. It is also worth pointing out that since $f_{\rm p}^2$ is determined only by the data on elastic $\overline{p}p\rightarrow\overline{p}p$ scattering, and $f_{\rm c}^2$ by the data on charge-exchange $\overline{p}p\rightarrow\overline{n}n$ scattering, one expects that the correlation between these two parameters will not be very large. $f_{\rm c}^2$ and the P-matrix parameters are fitted to the data. The value found for $f_{\rm c}^2$ at the pole is

$$f_{\rm c}^2 = (75.1 \pm 1.7)10^{-3} \quad \text{or} \quad g_{\rm c}^2 = 13.6 \pm 0.3 \;\;. \tag{7}$$

Again the error is statistical only. We thus confirm the value (3) for $f_{\rm c}^2$ determined by Arndt *et al.*. Comparing with the value (2) for $f_{\rm p}^2$ we find no evidence for a charge-dependence of the $NN\pi$ coupling constants.

Because the essentially unknown short-range interaction is parametrized phenomenologically, possible large systematic errors due to the model dependence are eliminated. Systematic errors may also come from the tail of the potential. We checked explicitly that adding a form factor to the OPE potential has no influence on the final results. We used a Gaussian form factor $F(k^2) = \exp[-(k^2 + m_\pi^2)/\Lambda^2]$, normalized such that $F(-m_\pi^2) = 1$. Varying Λ

between ∞ and 600 MeV, we found no significant change in the value for the charged-pion coupling constant.

In charge-exchange scattering only isovector mesons can be exchanged, the most important, next to the π, being the vector $\rho(770)$. The scalar $a_0(980)$ and the 'diffractive' piece of the tensor $a_2(1320)$ potentials are also included, but these contribute only very little to the tail of the OBE potential. Their inclusion does not affect the results. The tensor potentials of the $\pi^\pm$ and $\rho^\pm$ add up in $\overline{p}p \rightarrow \overline{n}n$ and their spin-spin potentials have opposite signs. To investigate a possible systematic error due to the tail of the ρ-exchange potential, we scaled this potential with a scale parameter γ which we add as another parameter. Refitting the parameter set we find $\gamma = 0.8 \pm 0.4$, and the same value and error for f_c^2 as in (7). In view of this, we think that our calculation is free of substantial systematic errors.

Another consistency test is to determine in an analogous way the mass of the charged pion, which also appears in the expression for the tail of the OPE potential. Adding m_{π^+} as a parameter, we find $m_{\pi^+} = 145 \pm 5$ MeV, in nice agreement with the experimental value $m_{\pi^+} = 139.57$ MeV. A large correlation between f_c^2 and m_{π^+} is seen. Because the mass found is consistent with the experimental value, this correlation strengthens our belief in the correctness of the determination of the coupling constant.

To summarize our findings, we confirm the low value for the charged-pion coupling constant found in the recent VPI&SU analysis of πN scattering data [11]. Comparing these values with the neutral-pion coupling constant as determined in the Nijmegen pp PSA [3,4], one sees that there is no evidence for a breaking of charge-independence of $NN\pi$ coupling constants. A recommended [27] value for the charge-independent $NN\pi$ coupling constant at the pion pole is

$$f^2(-m_\pi^2) = 0.075 \quad \text{or} \quad g^2(-m_\pi^2) = 13.55 \ . \tag{8}$$

Using a Gaussian form factor with $\Lambda = 779$ MeV the value at $k^2 = 0$ is found to be

$$f^2(0) = 0.0738 \quad \text{or} \quad g^2(0) = 13.34 \ . \tag{9}$$

This is the value obtained when one uses naively the Goldberger-Treiman relation with $f_\pi = 92.4 \pm 0.2$ MeV [28] and $|g_\mathrm{A}/g_\mathrm{V}| = 1.2650 \pm 0.0016$ [29].

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