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The charge dependence of the pion-nucleon coupling constants*

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Abstract

The recent work in Nijmegen on the determination of the NNπ-coupling constants is discussed. No evidence for any charge dependence of these coupling constants is found. The best value for the charge independent coupling constant is

$$f^2/4\pi = 0.075 \ ,$$

which is definitely a lot smaller than the commonly accepted value $f^2/4\pi = 0.079$.

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INTRODUCTION

In this talk I will restrict myself mainly to the recent work in Nijmegen on the determination of the meson-nucleon coupling constants [1]- [4]. This work is part of a much larger project of studying the baryon-baryon (BB) and the antibaryon-baryon ($\overline{\text{B}}$B) interactions. To put this work in the right perspective, we have to go back to the seventies, when we tried to construct good BB-potentials [5]. This culminated in 1978 in the construction of a soft-core, one-boson-exchange (OBE) NN-potential: Nijm78 [6], which was and still is, one of the best NN-potentials available. We realized then, that if one wants to improve significantly upon this potential, then one needs to have either a much better understanding of the underlying theory or one must have a much better knowledge of the experimental data. Of course, improvements on both sides are also possible and welcome. We decided to spend a lot of time doing phase-shift analyses (PSA) or partial-wave analyses (PWA) of the experimental NN scattering data in order to improve our knowledge of these data.

To understand another reason, why we look so closely at the experimental data, one must again go back to the end of the seventies, when many new multiquark states, like the Q^6-dibaryon states [7] and the $Q^2\overline{Q}^2$-baryonium states [8] were predicted and studied. Around that time it became quite clear that quarks were not objects to be studied only by high-energy physicists, but that quarks are, or at least should be, also important in nuclear physics [9]. However, one of the amazing things was and still is:

> 'Where are these quark effects in the NN-interaction?'

In the hope to find possible evidence for such quark effects we decided to study the experimental data very carefully.

We started therefore with partial-wave analyses of the NN scattering data [10,11]. At present, about 10 years later, we have finally finished more or less the analyses [2] of the proton-proton scattering data below $T_{\text{lab}} = 350$ MeV and we are now working on the analyses of neutron-proton scattering data in the same energy range.

In our partial-wave analyses of the pp-data [1,2,12] with $T_{\text{lab}} < 350$ MeV we could determine very accurately the ppπ^0-coupling constant **at the pion pole**. We found

$$f^2_{pp\pi^0}/4\pi = 0.0750(7) \quad \text{or} \quad g^2_{pp\pi^0}/4\pi = 13.55(13) \; . \tag{1}$$

This value of the ppπ^0-coupling constant must be compared with the value of the NNπ-coupling constant as determined in the analyses of the πN-scattering data. In such analyses one determines the charged-pion-nucleon coupling constant at the pole. One found [13]

$$f^2_c/4\pi = 0.079(1) \quad \text{or} \quad g^2_c/4\pi = 14.3(2) \; . \tag{2}$$

When we compared the results in (1) and (2) we came to the conclusion that either there is a large breaking of charge independence, or the result (2) is not correct.

We were at that time confident enough about our result (1), that this value was not seriously questioned anymore by us. But we also had no reason to doubt seriously the value (2), so we concluded that there must be a large breaking of charge independence for these coupling constants. At that same moment we expected that quark model calculations would be able to explain such a surprisingly large breaking of charge independence in terms of up-down quark mass differences and/or other effects [12]. In the meantime several quark model calculations have been performed, all indicating that it is not so simple to obtain large charge independence breaking effects. Therefore it has been slowly dawning on us that perhaps the value (2) obtained from the πN-data is not correct. Several new determinations of the charged-pion-nucleon coupling constant are now (almost) available [3,4,14]. These new determinations are all consistent with $f_c^2/4\pi = 0.075$. There is therefore no evidence at present for a breaking of charge independence in the values of the NNπ-coupling constants.

THE NIJMEGEN PARTIAL-WAVE ANALYSES

The experimental NN-scattering data are at each energy described by an **infinite** number of phase-shifts, which are functions of the energy. Obviously there exists only a **finite** amount of experimental data. This implies that partial-wave analyses are only possible with a sufficient theoretical input.

Because the long-range interaction $V_{\rm L}$ between nucleons is theoretically well-known one can calculate reliably the higher ℓ phases. The unknown short-range interaction is in the higher ℓ phases sufficiently screened by the large centrifugal barrier. This all means, that in a partial-wave analysis one has to determine only a finite number of phases with low ℓ.

Another very important ingredient in a multi-energy partial-wave analysis is the energy dependence of these low ℓ phases. The quality of the parametrization of this energy dependence will to a large extent determine the quality of the phase-shift analyses.

In our PWA we solve for each partial-wave and each energy the relativistic wave equation [9]

$$(\Delta + p^2)\psi = 2mV\ \psi\ , \tag{3}$$

where p^2 is the cm-momentum, which is related to the cm-energy E by

$$E = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} - (m_1 + m_2)\ .$$

In order to obtain a solution of the radial wave equation in each partial-wave we assume an energy-dependent boundary condition at $r = b = 1.4$ fm. At that point we specify the P-matrix, which is the logarithmic derivative of the radial wave function [15]. For $r > b$ we take the potential equal to the theoretically well-known long-range interaction $V_{\rm L}$. These ingredients (the energy dependent boundary condition, the well-known long range potential $V_{\rm L}$, and the relativistic wave equation) guarantee us a realistic energy dependence of our phase parameters.

The theoretically well-known long-range interaction $V_{\rm L}$ for $r > b$ we write as

$$V_{\rm L} = V_{\rm EM} + V_{\rm OPE} + V_{\rm HBE}\ , \tag{4}$$

where $V_{\rm EM}$ is the complete electromagnetic interaction [16], $V_{\rm OPE}$ the one-pion-exchange potential, and $V_{\rm HBE}$ is a contribution of the heavier-boson exchanges. For this heavier-boson exchange potential we normally take the Nijmegen potential Nijm78. The electromagnetic interaction $V_{\rm EM}$ we write as

$$V_{\rm EM} = V_{\rm C1} + V_{\rm C2} + V_{\rm MM} + V_{\rm VP} \ . \tag{5}$$

Here $V_{\rm C1} = \alpha'/r$ is the non-static Coulomb potential with

$$\alpha' = \alpha \frac{E_1 E_2 + p^2}{m(E_1 + E_2)} \approx \alpha(1 + 3T_{\rm lab}/4M) \ .$$

$V_{\rm C2}$ is a rather unimportant relativistic correction, which is of order
α^2/Mr^2. The magnetic-moment interaction $V_{\rm MM}$ is of the order α/M^2r^3. The vacuum-polarization potential $V_{\rm VP}$ is of the order $\alpha^2 e^{-2m_e r}/r$.

An important part of the long-range potential (4) is the one-pion-exchange potential $V_{\rm OPE}$. This potential contains the NNπ-coupling constant. This constant can be determined in the phase-shift analyses of the NN-data, because it is contained in the description of **every** phase-shift. Somewhat later we will come back to this $V_{\rm OPE}$.

The importance of the inclusion of the best long-range potential is that its inclusion improves very much the analyticity properties of the P-matrix in the complex energy plane. Our P-matrix will not have any left-hand cuts or singularities due to the Coulomb interaction, the vacuum polarization potential, or the OPEP [17]. The first left-hand singularity in P appears at the start of the two-pion-exchange cut. This large region of analyticity for the P-matrix makes the parametrization of the energy-dependence quite simple. This implies a very realistic energy-dependence for our phase parameters. In our PWA [2] of the pp-data with $T_{\rm lab} \leq 350$ MeV we fitted to $N_{\rm d} = 1766$ scattering data which amounted to $N_{\rm df} = 1586$ degrees of freedom. We reached $chi^2_{\rm min} = 1760.7$, which means $\chi^2/N_{\rm d} = 0.997$.

QUALITY OF PP-POTENTIALS

When one has finished a PWA of all pp-data with $T_{\rm lab} < 350$ MeV, then one has such an understanding of these data, that it becomes rather easy to determine the quality of the various pp-potentials. It is then relatively easy to see how well a certain potential fits the experimental scattering data. The most direct way to answer this question is to compare the predictions of the potential directly with all the experimental data. However, this is an enormous job. At each experimental energy one needs to calculate a sufficient amount of phases, such that the experimental observables (differential cross sections, polarizations) can be calculated with sufficient accuracy. For this one needs fancy programs, that can incorporate the more difficult to calculate electro-magnetic effects, like the magnetic moment interaction and the vacuum polarization. The normalization constants for the different data sets need to be determined in a direct comparison of the potential predictions with the experimental data. To do this for only one potential requires already a lot of effort and computer time.

A more practical way to go about it, is to use single energy phase-shift analyses in which one has determined at several energies the best values for the phases, the corresponding minimal χ^2, the error matrix, and the normalization constants for the various data sets. This way a quadratic χ^2-surface for all experimental data has been constructed. When confronting a potential with the experimental data one uses this χ^2-surface. The normalization constants are given the values as determined by the single energy partial-wave analyses. We have

0 − 350	3 − 350		ref.
		Phase shift analyses	
0.95	0.96	Nijm. single-energy 90	[2]
1.00	1.01	Nijm. multi-energy 90	[2]
		NN-potentials	
10.	6.1	Hamada-Johnston 62	[19]
1.92		Nijmegen 78	[6]
4.4	2.0	Paris 80	[18]
2.3		Reid 68,82	[20]
	7.2	Argonne 84	[21]
	14.	Bonn(r) 87	[22]
	10.	Bonn-A 89	[23]
	9.2	Bonn-B 89	[23]
		pp-potentials	
1.89		Bonn(pp) 89	[24]
1.09		Nijmegen Rdl 90	[25]

TABLE I. The $\chi^2/N_{\rm d}$ with the pp-data for the various potentials. When there are no numbers given in the column $0-350$ MeV then this means that the potential was fitted to the np scattering length and the number in this column is larger than 500.

checked the method for the Nijmegen [6] and for the Paris potential [18], where we calculated the total χ^2 both ways. The agreement between both methods was good.

In Table I we give for the different potentials the $\chi^2/N_{\rm d}$ with the pp-data. Because several potentials do not fit the very low energy pp-data very well, we will for these cases compare with the $N_{\rm d} = 1569$ pp-data from $3-350$ MeV instead of with the $N_{\rm d} = 1766$ pp-data in the $0-350$ MeV range.

From Table I we note that apart from the very low energy region the Nijm78 potential [6] and the Paris80 potential [18] give similar χ^2-values. For the very low energy data, $T_{\rm lab} \approx 1$ MeV, the $^1S_0$-phase of the Paris potential is in error.

Surprising is the poor quality of the recent Argonne84 [21] and the Bonn(r) potentials [22,23] of 1987 and 1989. They fit the pp-data worse than the old Hamada-Johnston potential [19] of 1962. This is what one calls progress.

Recently the Bonn group constructed [24] a special pp-potential Bonn(pp)89. The fit of this newest potential to the pp-data is of the same quality as the fits of the much older Nijmegen and Paris potentials.

Also in Nijmegen we constructed new pp-potentials. We have several of them. The best one at present is the Reidlike potential Nijmegen Rdl90 [25], which fits the pp-data with $\chi^2/N_{\rm d} = 1.09$, which is almost as good as the fit of the Nijmegen multi-energy phase-shift analyses with $\chi^2/N_{\rm d} = 1.00$. A much lower χ^2 than the χ^2 of the Nijmegen multi-energy PSA is not possible.

THE ONE-PION-EXCHANGE POTENTIAL

The NNπ-interaction is described by the phenomenological interaction Lagrangians $\mathcal{L}_{\rm PS}$ or $\mathcal{L}_{\rm PV}$, where

$$\mathcal{L}_{\rm PS} = g(\bar{\psi}_2 i\gamma_5\psi_1)\phi \quad \text{and} \quad \mathcal{L}_{\rm PV} = (f/m_s)(\bar{\psi}_2 i\gamma_\mu\gamma_5\psi_1)\partial^\mu\phi \ .$$

The corresponding potentials $V_{\rm OPE}$ are equivalent when

$$f/m_s = g/(M_1+M_2) \ .$$

The scaling mass m_s is introduced in $\mathcal{L}_{\rm PV}$ in order to make the coupling constant f dimensionless. Different choices for the scaling mass are possible. We feel that the best one is to use $m_s = m_+$ the charged pion mass. Quark model calculations support this choice. In that case charge independence of the PS-coupling constants g implies that for the PV-coupling constants f charge independence is only slightly broken, due to the factor $M_1 + M_2$. However, this breaking is so small, that in this case charge independence for the g's implies charge independence for the f's and vice versa.

Another unfortunate choice sometimes made is to take $m_s = m$, where m is the mass of the exchanged pion. Because of the large mass difference between the π^0 and the π^+ charge independence for the f's will mean a large breaking of charge independence for the g's. In order **not** to introduce an artificial breaking of charge independence it is better **not** to use the choice $m_s = m$, but to take always $m_s = m_+$.

Due to the spatial extension of the hadrons the coupling constant in momentum space is not really a constant, but is modified by a form factor $F(k^2)$, where k^2 is the momentum transfer. Different functional forms and different normalizations are in use. We write

$$g^2(k^2) = g^2 \ F(k^2) \ .$$

When one normalizes the form factor such that $F(-m_\pi^2) = 1$, then g^2 will be the value of the coupling constant at the pion pole. In order to demonstrate the effect of a form factor we take

$$F(k^2) = (\Lambda^2 - m^2)/(\Lambda^2 + k^2) \ ,$$

which is normalized at the pole. For scalar-meson exchange the potential $V(k^2)$ in momentum space is then

$$V(k^2) = -g^2 \frac{\Lambda^2 - m^2}{\Lambda^2 + k^2} \ \frac{1}{k^2 + m^2} = -g^2 \left\{ \frac{1}{k^2+m^2} - \frac{1}{k^2+\Lambda^2} \right\} \ .$$

In coordinate space we get

$$V(r) = -\frac{g^2}{4\pi} \ \frac{e^{-mr}}{r} \left\{ 1 - e^{-(\Lambda - m)r} \right\} \ .$$

We see clearly that for large values of r the second term inside parenthesis goes to zero and the potential $V(r)$ becomes the ordinary Yukawa-potential. The form factor modifies the potential in the inner region only. In our PWA we only use the OPE-potential outside $r = 1.4$ fm. This means that we clearly determine the coupling constant at the pole. In order to demonstrate very clearly that our value for $f^2/4\pi$ at the pion pole is practically independent of the cutoff Λ we give in Table II the value for the ppπ^0-coupling constant at the pole as determined in our PWA using different values of Λ and gaussian form factors $F(k^2) = \exp(-(k^2+m^2)/\Lambda^2)$.

From our PWA of the pp-data we find

$$f^2_{pp\pi^0}/4\pi = 0.0750(7) \ ,$$

where the quoted error is purely statistical. There could, of course, always be systematic errors of unknown origin, but we feel quite confident about our value. This means that there is either evidence for a large breaking of charge independence or the value of the charged coupling constant as determined in πN scattering is incorrect.

Λ (MeV)	∞	965	550	500
$f^2/4\pi$	0.0750	0.0750	0.0751	0.0753

TABLE II. The value of the ppπ^0-coupling constant at the pole as determined in pp-scattering for different values of Λ. The statistical error is 0.0007.

THE CHARGED-PION-NUCLEON COUPLING CONSTANT

From past experiences in high energy physics and from recent quark model calculations [26] it seems unlikely and also rather unbelievable, that there is such a large breaking of charge independence. This then indicates that one must reconsider the present value for the charged NNπ-coupling constant $f_c^2/4\pi = 0.079(1)$.

This charged coupling constant can be seen in several different reactions. First of all in elastic πN scattering. A recent analysis [14] of the πN-scattering data gives

$$f_c^2/4\pi = 0.0735(15) \ .$$

The cause for this new value is the many new scattering data that have recently become available.

Charged-pion exchange is also possible and important in np-scattering, where this charge exchange gives at intermediate and higher energies rise to a large peak in the backward differential cross section. In a preliminary analysis [27] of the np-data with $T_{\rm lab} < 30$ MeV we found evidence for a sizable difference $\Delta f^2 = f_c^2 - f_0^2$. In a more extensive study we discovered later several sources of systematic errors in the 0–30 MeV np-phase shift analyses. These systematic errors did give rise to large effects. This caused us to abandone (at least temporary) our PWA of the np-data with $T_{\rm lab} < 30$ MeV and to concentrate on the analyses of the data with $T_{\rm lab} < 350$ MeV. This Nijmegen PWA [4] of the np-scattering data with $T_{\rm lab} \leq 350$ MeV is still in a preliminary stage. In this analysis we find for the charged coupling constant

$$f_c^2/4\pi = 0.0747(5) \ .$$

In Nijmegen we are also very busy with a PWA of the $\overline{\mathrm{N}}\mathrm{N}$ scattering data [3]. The charged pion is also exchanged in the charge-exchange reaction

$$\overline{p} + p \rightarrow \overline{n} + n \ .$$

In our analysis of this charge-exchange reaction we find

$$f_c^2/4\pi = 0.0751(17) \ .$$

When we look at all these various determinations of the charged pion-nucleon coupling constant we come to the conclusion that we have at present no evidence for any breaking of charge independence of the NNπ-coupling constants.

As value to use for the charge independent NNπ-coupling constant we recommend

$$f^2(-m_\pi^2)/4\pi = 0.075 \ .$$

This is the value at the pion pole. Using a gaussian form factor with $\Lambda = 615$ MeV gives then at $k^2 = 0$ the value $f^2(0)/4\pi = 0.07124$, which is the value predicted by a naive application of the Goldberger-Treiman relation.

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