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Determination of the $NN\pi$ coupling constants in NN partial-wave analyses*

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Abstract

The $NN\pi$ coupling constants are extracted in NN partial-wave analyses. The database contains all pp and np scattering data below $T_{\rm lab} = 350$ MeV. Introducing different coupling constants at the different $NN\pi$ vertices, at the pion pole we find for the $pp\pi^0$ coupling $f_p^2 = 0.0751(6)$, for the $nn\pi^0$ coupling $f_n^2 = 0.075(2)$, and for the charged-pion coupling $f_c^2 = 0.0741(5)$. These results allow only small charge-independence-breaking effects in the $NN\pi$ coupling constants. If we assume charge independence, we find $f^2 = 0.0749(4)$. The value found for f_c^2 is in agreement with the determination of the πN scattering data form a recent analysis at Virginia Polytechnic Institute and State University (VPI&SU), and with its determination in an analysis of the charge-exchange reaction $\overline{p}p \rightarrow \overline{n}n$.

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In the construction of the Nijmegen soft-core NN potential [1], the $NN\pi$ coupling constant was determined by fitting to the NN data of 1969 using the Livermore-X phase-shift analysis [2]. Nagels *et al.* found $f^2 = 0.077$ for the value at the pion pole. A few years later, in a phase-shift analysis of the low-energy pp scattering data, the tensor combination of the triplet P waves indicated that the $pp\pi^0$ coupling constant should be small. At that time, a value of $f_p^2 \approx 0.075$ was suggested by us [3]. Again, some years later, in a preliminary partial-wave analysis of the pp scattering data below $T_{\rm lab} = 350$ MeV [4], we found $f_p^2 = 0.0725(6)$. This preliminary version did not contain the magnetic-moment interaction [5] and it used a much smaller database than presently available. The newer, updated value [6] is $f_p^2 = 0.0749(7)$. These values are significantly smaller than the previously accepted value for the charged-pion coupling constant $f_c^2 = 0.079(1)$, as determined from πN scattering [7,8].

In 1987 it was clear to us that there was a large discrepancy between the value for the $pp\pi^0$ coupling constant as determined from the pp scattering data [4] and the value for the charged-pion coupling constant as determined from the πN scattering data. Because there was no obvious reason to doubt either one of these determinations, it was concluded [4] that there apparently is a large breaking of charge independence in the coupling constants. However, subsequent theoretical model calculations have not been able to explain such a large breaking. The differences were always found to be rather small and in most models the charged-pion coupling was found to be smaller than the $pp\pi^0$ coupling (see, e.g., Refs. [9,10]). If we are to believe the theoretical model calculations which rule out a large charge-independence breaking, we can only come to the conclusion that the determination of at least one of these two coupling constants should be incorrect.

We are confident of our value for f_p^2 extracted from the pp scattering data. We therefore believe that the previously accepted high value for f_c^2 as determined in πN scattering can no longer be taken for granted. This means that we are in need of other, independent determinations of this coupling constant. Such determinations have been done recently. A determination of the coupling constant in the analysis of the πN scattering data by Arndt and co-workers [11] at Virginia Polytechnic Institute and State University (VPI&SU) resulted in $f_c^2 = 0.0735(15)$. This coupling constant could also be extracted in an analysis of $\overline{N}N$ scattering data by the Nijmegen group. Analysis of the data on the charge-exchange reaction $\overline{p}p \to \overline{n}n$ below $p_{\rm lab} = 950$ MeV/c resulted in [12] $f_c^2 = 0.0751(17)$. Both results are within one standard deviation from the value for f_p^2 determined in the Nijmegen pp analysis [6], and large charge-independence-breaking effects need no longer be invoked.

In this Rapid Communication we will present another independent determination of f_c^2, using the same technique as in our determination of f_p^2. In a partial-wave analysis (PWA) of all np scattering data below $T_{\rm lab} = 350$ MeV, and in a combined PWA including also the pp scattering data, we have been able to extract the neutral- and charged-pion coupling constants simultaneously. The results are in agreement with the low values obtained in the aforementioned determinations and again provide a strong support for an (approximate) charge independence of the $NN\pi$ coupling constants.

The analysis of the np scattering data is a continuation of our analysis of the experimental NN scattering data. The analysis of the pp scattering data below $T_{\rm lab} = 350$ MeV has already been published [6]. The details of our method are extensively discussed in our earlier publications [13,6], so here we will only briefly outline some of its features. In our analyses we use an energy-dependent boundary-condition model to parametrize the short-range interaction, where the boundary-condition radius is chosen to be $b = 1.4$ fm. The long-range interaction is described by a potential tail. The boundary-condition parametrization is used for the lower partial waves with total angular momentum $J \leq 4$. For the intermediate partial waves ($5 \leq J \leq 8$) we use the phase parameters of the one-pion-exchange (OPE) potential plus the heavy-boson-exchange contributions of the Nijmegen soft-core NN potential [1]. All higher partial waves are given by the OPE phase parameters. In the pp analysis as well as in the np analysis, electromagnetic effects (Coulomb, vacuum polarization, magnetic moments) are accounted for when necessary.

The np database is not rich and accurate enough to determine both the $I = 0$ and the $I = 1$ partial waves. Therefore, in most analyses the $I = 0$ lower partial waves are searched for, whereas the np $I = 1$ partial waves are obtained from the pp $I = 1$ partial waves, after correcting them for Coulomb distortion effects. The only exception is the ${}^1S_0$ np partial wave, which is usually parametrized independently of the pp data. This choice of parametrization is also used in our analyses: The $I = 0$ lower partial waves and the ${}^1S_0$ phase shift are parametrized with an energy-dependent boundary condition, whereas the $I = 1$ partial-wave phase parameters (except the ${}^1S_0$) are obtained from the corresponding pp partial-wave phase parameters by correcting them not only for Coulomb distortion effects, but also for mass difference effects. Moreover, we allow for possible differences between the neutral- and charged-pion coupling constants.

These pion and Coulomb corrections are obtained as follows. First pp phase shifts are calculated with some realistic NN potential V_{NUC} in the presence of the Coulomb potential $V_{\text{C}} = \alpha'/r$. In order to obtain the corresponding np phase shifts, the pp OPE part of the nuclear potential is replaced by the np OPE potential. So the corresponding potentials are given by

$$
\begin{aligned}
pp: &\quad V = V_{\text{NUC}} + V_{\text{C}} \\
np: &\quad V = [V_{\text{NUC}} - V_{\text{OPE}}(pp)] + V_{\text{OPE}}(np) \ .
\end{aligned} \tag{1}
$$

The $np - pp$ phase-shift differences are calculated for various values of the $NN\pi$ coupling constants. These pion and Coulomb differences are then added to the pp phase shifts as obtained in the pp PWA. (A similar procedure is used in a combined PWA by Bohannon, Burt, and Signell [14], but there only the Coulomb potential is taken into account.)

Let us next discuss the OPE potential a little more in detail. In NN scattering we encounter four different pseudovector coupling constants at the vertices: $f_{pp\pi^0}, f_{nn\pi^0}, f_{np\pi^-}$, and $f_{pn\pi^+}$. For the combinations that actually occur in the OPE potential, we use the following definitions

$$
\begin{aligned}
f_p^2 &\equiv f_{pp\pi^0} f_{pp\pi^0} \ , \ \text{for } pp \rightarrow pp \ , \\
f_0^2 &\equiv -f_{nn\pi^0} f_{pp\pi^0} \ , \ \text{for } np \rightarrow np \ , \\
2f_c^2 &\equiv f_{np\pi^-} f_{pn\pi^+} \ , \ \text{for } np \rightarrow pn \ .
\end{aligned} \tag{2}
$$

In case of charge symmetry, one has $f_p^2 = f_0^2$, whereas in case of charge independence, one has $f_p^2 = f_0^2 = f_c^2$. For pp scattering the OPE potential can be written as

$$
V_{\text{OPE}}(pp) = f_p^2 \, V(m_{\pi^0}) \ , \tag{3}
$$

whereas for np scattering it reads

$$V_{\rm OPE}(np) = -f_0^2\, V(m_{\pi^0}) + 2(-)^{I+1} f_c^2\, V(m_{\pi^+}) \ . \tag{4}$$

Here we introduced $V(m)$, which for large values of r is given by

$$\begin{aligned} V(m) &= \frac{1}{3}\left(\frac{m}{m_s}\right)^2 \frac{M}{E}\frac{e^{-mr}}{r} \\ &\times \left[(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + S_{12}\left[1+\frac{3}{(mr)}+\frac{3}{(mr)^2}\right]\right] \ . \end{aligned} \tag{5}$$

The scaling mass m_s is introduced such as to make the pseudovector coupling constant f dimensionless. It is conventionally chosen to be the charged-pion mass. Leaving out the energy-dependent factor M/E results in a small rise in $\chi^2_{\rm min}$ and decreases the values found for the coupling constants. To be explicit, in our pp analysis the rise is $\Delta\chi^2 = 13$ and f_p^2 changes from 0.075 to 0.074. We believe it is better to include the M/E factor.

Due to the spatial extension of the nucleons and pions the coupling constants actually are modified by a form factor. We use an exponential form factor [1] $F(\mathbf{k}^2) = \exp[-(\mathbf{k}^2 + m_\pi^2)/\Lambda^2)]$ with $\mathbf{k}^2$ the momentum transfer squared and Λ a cut-off mass. The normalization is chosen such that at the pion pole $F(-m_\pi^2) = 1$. Because in our analyses we determine the strength of the Yukawa tail (5), this choice of normalization ensures that the results obtained for the coupling constants refer to their value at the pion pole. Obviously, it also ensures that our determination is independent of the value of the cut-off mass Λ that is used.

In the analysis of the np scattering data the boundary conditions of the lower partial waves are parametrized with 21 parameters. The two $NN\pi$ coupling constants f_0^2 and f_c^2 are also included as free parameters. The np data do not determine the $pp\pi^0$ coupling f_p^2. However, the np $I=1$ lower partial-wave phase parameters are obtained by adding the pion and Coulomb corrections to the corresponding pp partial-wave phase parameters, which in turn do contain the coupling constant f_p^2 in their parametrization. In the np analysis we take for the $pp\pi^0$ coupling constant the result as determined in the pp analysis, i.e., $f_p^2 = 0.075$. Our np database consists of 2302 scattering observables or, including the normalization data, 2442 scattering data. Taking into account the 23 model parameters and the floated normalizations which are to be fitted, we are left with 2264 degrees of freedom. We obtain $\chi^2_{\rm min}(np) = 2429.6$, or $\chi^2_{\rm min}/N_{\rm DF} = 1.07$, where $N_{\rm DF}$ denotes the number of degrees of freedom.

For the two $NN\pi$ coupling constants at the pion pole with the $pp\pi^0$ coupling fixed at $f_p^2 = 0.075$, we find

$$f_0^2 = 0.0753(8) \ , \qquad f_c^2 = 0.0740(5) \ . \tag{6}$$

Our result for f_c^2 is in good agreement with the recent result of the VPI&SU πN analysis [11] and with the result of the analysis of the charge-exchange reaction in $\overline{N}N$ scattering [12], but disagrees with the value $f_c^2 = 0.079(1)$ as found in an earlier πN analysis [7,8].

In the np PWA the $I=1$ phase parameters are obtained from the pp PWA using the pion and Coulomb corrections. These phase parameters are therefore totally determined by the pp scattering data. In a combined PWA, all NN scattering data are analyzed simultaneously, so in such an analysis the $I = 1$ lower partial-wave phase parameters are not only determined by the pp data, but *also* by the np data. The pp database contributes 1636 pp scattering observables or, including the normalization data, 1766 scattering data. Next to the $pp\pi^0$ coupling constant f_p^2 we need 27 parameters to parametrize the boundary conditions of the pp lower partial waves. The total NN database thus comprises 4208 scattering data. Taking into account the 48 boundary-condition parameters, the three $NN\pi$ coupling constants, and the floated normalizations which are to be fitted, the total number of degrees of freedom in the combined analysis is $N_{\rm DF} = 3850$. We reach $\chi^2_{\rm min} = 4186.3$, consisting of $\chi^2_{\rm min}(pp) = 1771.8$ and $\chi^2_{\rm min}(np) = 2414.5$. Comparing with our pp analysis, $\chi^2(pp)$ has risen with 6.3, in order to allow for a simultaneous drop in $\chi^2(np)$ of 15.1.

For the three $NN\pi$ coupling constants at the pion pole we find

$$f_p^2 = 0.0751(6)\ , \quad f_0^2 = 0.0752(8)\ , \quad f_c^2 = 0.0741(5)\ , \tag{7}$$

which implies a value for the $nn\pi^0$ coupling constant of $f_n^2 = 0.075(2)$. The inclusion of the np scattering data has no influence on the result for the $pp\pi^0$ coupling constant. Again the difference between f_p^2 and f_0^2 is only very small. Also the value for the charged-pion coupling constant remains lower than the value for the $pp\pi^0$ coupling constant.

Assuming that charge independence between the coupling constants holds, we have also performed a combined analysis where we use one coupling constant only, i.e., $f^2 \equiv f_p^2 = f_n^2 = f_c^2$. We then find

$$f^2 = 0.0749(4)\ , \tag{8}$$

and $\chi^2_{\rm min}$ rises with 6.8. Comparing with the result (7) shows that there apparently is no significant charge-independence breaking in the $NN\pi$ coupling constants. This corroborates the results of various theoretical model calculations [9,10] which find that charge-independence-breaking effects are small. On the other hand, it refutes our earlier observation of a large charge-independence breaking in our analysis of the NN scattering data below $T_{\rm lab} = 30$ MeV. There [15] we found a breaking of $\Delta f^2 \equiv f_c^2 - f_p^2 = 0.0088(15)$. The reason for this discrepancy will be explained below.

At the time of our 0–30 MeV np analysis [15], the 0–350 MeV pp analysis was not yet finished, and we used the pp results of our 0–30 MeV analysis [13]. In that way we are able to do a combined analysis of the 0–30 MeV data consistently. One of the shortcomings of these low-energy analyses was that they did not contain the magnetic-moment interaction, which has some influence on the results for the values of the coupling constants.

A more serious shortcoming was the following. In our 0–350 MeV pp analysis [6], we found that the quality of the data around $T_{\rm lab} = 10$ MeV is dubious. This is reflected in the spin-orbit combination Δ_{LS} of the triplet P waves. A single-energy analysis of the data around $T_{\rm lab} = 10$ MeV yields a Δ_{LS} which differs by almost three standard deviations from the 0–350 MeV multi-energy analysis. The spin-orbit combination in the 0–30 MeV analysis is almost entirely determined by the analyzing-power data at 9.85 MeV [16]. In the 0–350 MeV analysis it is determined by the higher-energy analyzing-power data as well. In view of the results of the latter analysis, we believe that this experiment at 9.85 MeV is more or less in disagreement with the other data in our database. This implies that $\Delta_{LS}(pp)$ which occurs in the 0–30 MeV analysis of Ref. [4] is incorrect. Unfortunately, this had important consequences for our 0–30 MeV np analysis.

In our np analysis, $\Delta_{LS}(np)$ is obtained from $\Delta_{LS}(pp)$ by adding the corresponding pion and Coulomb corrections. In Ref. [17] it is demonstrated that $\Delta_{LS}(np)$ increases when Coulomb distortion and mass difference effects are accounted for, whereas it decreases for increasing values of the coupling constant difference Δf^2. Since the OPE potential does not contain a spin-orbit interaction, its influence on Δ_{LS} can only be of second order in the coupling constants. Therefore, starting with a $\Delta_{LS}(pp)$ which is too high, one requires a large charge-independence breaking Δf^2 in the pion and Coulomb corrections in order to arrive at the value for $\Delta_{LS}(np)$, which is fairly well fixed by the np analyzing-power data. This is precisely the reason why in our earlier 0–30 MeV np analysis [15] we found a large breaking of charge independence. Indeed, a reanalysis of the 0–30 MeV np data where we use the pp phase shifts from the 0–350 MeV pp analysis rather than those from the 0–30 MeV pp analysis, results in a charge-independence breaking which is in agreement with our present determination. In retrospect it is clear that the 0–350 MeV partial-wave analysis is to be preferred over the 0–30 MeV partial-wave analysis.

Finally, there are several ways for showing that what we determine is indeed the strength of the OPE potential tail. One way is to include for the long-range nuclear interaction the OPE potential only. In that case it turns out that the value of $b = 1.4$ fm for the boundary-condition radius is unreasonable. This indicates that heavy-boson-exchange forces are not negligible outside 1.4 fm. We therefore use $b = 1.8$ fm and we also have to increase the number of boundary-condition parameters. Even then the description is not very good: in the combined analysis we now obtain $\chi^2_{\text{min}} = 4287.1$. However, for the coupling constants we find $f_p^2 = 0.0749(9)$ and $f_c^2 = 0.0742(5)$, in excellent agreement with the result (7). A second way for demonstrating that we determine the strength of the OPE potential tail is to extract the corresponding pion mass. In Ref. [4] the neutral-pion mass could be extracted from the pp scattering data and the updated result is now found to be $m_{\pi^0} = 135.6 \pm 1.3$ MeV, in agreement with the experimental value [18] of $m_{\pi^0} = 134.9739(6)$ MeV. In the present analysis we can also extract the charged-pion mass from the np scattering data and we find $m_{\pi^+} = 139.4 \pm 1.0$ MeV, in excellent agreement with the more accurate value $m_{\pi^+} = 139.5675(4)$ MeV. The fact that we find the correct values for the neutral- and charged-pion masses shows that we really look at the OPE potential and it therefore gives more confidence in our determination of the $NN\pi$ couplings.

Summarizing, we confirm the low value for the charged-pion coupling constant f_c^2, as determined in the recent VPI&SU analysis of the πN scattering data and in the analysis of the charge-exchange reaction $\overline{p}p \to \overline{n}n$. The result is slightly lower than the value for the $pp\pi^0$ coupling constant f_p^2, as determined in the Nijmegen analysis of the pp scattering data. It supports the results of several theoretical model calculations where only small charge-independence-breaking effects are predicted. The value for the $nn\pi^0$ coupling constant can be determined much less accurately. If charge independence between the $NN\pi$ coupling constants is to be assumed, we recommend

$$f^2(-m_\pi^2) = 0.075 \ .$$

This result is in good agreement with the value that is obtained when one naively uses the

Goldberger-Treiman relation with $|g_A/g_V| = 1.2650(16)$ [19], $f_\pi = 92.4(2)$ MeV [20], and a cut-off mass for the form factor of about $\Lambda = 770$ MeV.

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