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# A new partial wave analysis and the nucleon-nucleon interaction*

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## Abstract

We present some recent results of the Nijmegen partial-wave analysis of all $NN$ scattering data below $T_{\rm lab} = 350$ MeV. We compare the predictions of various $NN$ potential models with the $NN$ scattering data and with the phase parameters at 50 MeV.

21.30.+y, 13.75.Cs, 11.80.Et, 12.40.Qq



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## I. INTRODUCTION

For many years, the Nijmegen group has been investigating the baryon-baryon interaction. After the construction of several hard-core nucleon-nucleon and hyperon-nucleon potentials, this culminated in 1978 in the construction of the soft-core $NN$ potential [1]. This was, and still is, one of the best $NN$ potentials presently available. There is a coordinate-space version [1] and a momentum-space version [2], which are totally equivalent; they produce exactly the same phase parameters at all energies. In order to improve this potential, we needed to get both a better understanding of the underlying theory and a better knowledge of the experimental data. For that purpose we started to develop theoretical and computational tools to perform phase-shift (or rather partial-wave) analyses of the $NN$ scattering data. The first major step, the analysis of the $pp$ scattering data, has been published several years ago [3,4].

At present we have obtained also fits (still preliminary) with the $np$ scattering data and with the combined $pp$ and $np$ scattering data. In this talk we discuss some of the features of our way of analyzing the data. In order to present the results of our analyses, we will discuss some special topics. In Sec. 3 we will discuss our determination of the $NN\pi$ coupling constants. We consider this determination one of the nicest results of our partial-wave analyses. Because we have obtained a good knowledge of the experimental data and because we can compute the experimental observables properly, in Sec. 4 we make comparisons of various $NN$ potential models with the $NN$ data. This is repeated in Sec. 5, where we now compare the potential predictions with the phase parameters at 50 MeV. Our important conclusion is that our value for the mixing parameter $\varepsilon_1$ is in perfect agreement with modern potential predictions.

## II. PARTIAL-WAVE ANALYSIS

The first main problem in doing partial-wave analyses is: How to calculate the scattering amplitudes in terms of phase shifts (and mixing parameters)? We want to stress that this is a non-trivial problem, because the proper treatment of the electromagnetic part of the interaction and its effect on the phase shifts and the scattering amplitude is very important and at the same time rather complicated. For example, in the case of $pp$ scattering the one-pion-exchange (OPE) amplitude has to be calculated properly, including its distortion by the Coulomb interaction. This means that the analytical expression for the OPE scattering amplitude cannot be used. Here also lie some of the differences between the various partial-wave analyses that have appeared in the literature: In the Nijmegen [4], VPI&SU [5], and Saclay [6] analyses, the higher partial waves and Coulomb distortion effects are treated differently.

The second main problem is: What is a good parametrization for the energy dependence of the phase parameters? This is important when doing multi-energy (m.e.) partial-wave analyses. The basis of any partial-wave analysis (PWA) is a good m.e. analysis. In such a m.e. PWA all experimental data at all energies are used. Such an analysis should be compared with several single-energy (s.e.) PWAs, in which the data in some energy interval are used in determining the phase parameters at one single energy. This energy usually corresponds to the middle of that energy interval; however, this is not strictly necessary. In s.e. PWAs at several energies we determine the phase parameters and their errors. These s.e. values for the phase parameters should scatter statistically around the curve representing the phase parameters as determined in the m.e. PWA.

We think that our analyses, and especially our energy dependence of the phase parameters, is of such quality that the situation sketched above is correct. Therefore, we believe that for the Nijmegen analyses the 'best' value for a particular phase parameter is the value as obtained in the m.e. analysis, rather than the value as obtained in the s.e. analysis.

As an example, consider the ${}^3S_1$-${}^3D_1$ mixing parameter $\varepsilon_1$ in $np$ scattering around 100 MeV. In a s.e. analysis at 100 MeV, $\varepsilon_1$ is not accurately known due to the absence of spin-correlation data. This is shown in Fig. 1, where our s.e. results are represented by the black dots. At the adjoining energies at 50 and 150 MeV the available spin-correlation data provide for an accurate determination of $\varepsilon_1$. These data, and the data at lower and higher energies make that $\varepsilon_1$ is fixed rather well in the m.e. analysis, represented by the solid line. This means that also at 100 MeV, $\varepsilon_1$ is in fact much more accurately determined by the data than the s.e. result would suggest. The example demonstrates that a s.e. analysis without an accompanying m.e. analysis is not very useful for making statements with regard to the accuracy with which the phase parameters are determined.

The energy dependence of the lower partial waves in our analyses is determined as follows. In our analysis we divide the interaction into two parts: a long-range part $V_L$ which is well known and essentially model independent, and a short-range part $V_S$ which is treated phenomenologically. The long-range part $V_L = V_{EM} + V_{NUC}$ consists of the complete electromagnetic interaction $V_{EM}$ and the tail of the nuclear potential $V_{NUC}$. The electromagnetic potential contains the non-static Coulomb potential [7] (including relativistic corrections and two-photon-exchange corrections), the magnetic-moment interaction [8], and the vacuum polarization interaction [9]. The tail of the nuclear potential is dominated by the one-pion-exchange (OPE) potential, but contains also contributions of shorter range due to multi-pion exchange or the exchange of heavier bosons (like $\rho, \omega, \eta$). For these shorter-range contributions we use the Nijmegen potential [1]: Nijm78. However, because we only need the tail of the potential (outside $r = 1.4$ fm), any decent potential would have sufficed here.

Using this long-range potential $V_L$, the radial Schrödinger equation

$$(\Delta + k^2)\psi = 2M_{\rm red}V_L\psi \tag{1}$$

is solved for $r > b = 1.4$ fm. Relativistic effects are taken into account via the potential and by using the relativistic expression for the c.m. energy

$$E = \sqrt{m_1^2 + k^2} + \sqrt{m_2^2 + k^2} - (m_1 + m_2)\ . \tag{2}$$

The presence of the centrifugal barrier makes that the Schrödinger equation need only be solved for a small number of lower partial waves. Still, the equation has to be solved for all energies at which experimental data have been measured. The phenomenology, necessary to describe the short-range interaction is represented by a boundary condition at $r = b$. The boundary condition for each of the lower partial waves is parametrized by a square well potential of range $r = b$. The depth $V_S$ of this potential is independent of $r$, but is allowed to be energy dependent and different in the different partial waves. It can be shown that

$V_S$ is an analytic function of the energy. Inclusion of a certain interaction in the long-range potential tail $V_L$ implies that the corresponding left-hand singularities in $V_S$ are removed. In our analyses the nearest left-hand singularity in $V_S$ is a cut starting at $T_{\rm lab} = -40$ MeV and is due to two-pion exchange. The nearest right-hand cut lies at $T_{\rm lab} = 280$ MeV and is due to inelasticities of the pion production threshold. Fortunately, at energies below 400 MeV inelasticities are still very small, so in our analyses we extend the energy range to be considered to 350 MeV. Because $V_S$ is an analytic function of $k^2$, regular in the cut plane between $T_{\rm lab} = -40$ MeV and $T_{\rm lab} = 280$ MeV, we can parametrize it conveniently as a power series in $k^2$. This analytic parametrization guarantees a pretty good energy dependence of our phase parameters.

A major problem in doing *np* analyses is that the *np* data base is not rich and accurate enough to determine both $I = 0$ and $I = 1$ partial waves. Therefore, in our analysis the $I = 0$ lower partial waves are searched for, whereas the *np* $I = 1$ partial waves (except the $^1S_0$ *np* phase shift) are obtained from the corresponding *pp* partial waves, after correcting them for Coulomb distortion and mass difference effects, and charge-independence breaking of the pion-nucleon coupling constants [10]. At present (June 1991) our data base contains 4208 *NN* scattering data (1766 *pp* scattering data and 2442 *np* scattering data). A large number of data have been removed from the data base because they are of poor quality (more than 3 standard deviations off). We need 51 parameters to parametrize the lower partial waves up to $J = 4$ and we reach $\chi^2_{\rm min} = 4186.3$ which is less than 1 per data point.

From the fact that we need about 50 parameters to obtain a satisfactory description of the scattering data, we tentatively conclude that any potential will need about the same number of parameters for a good description of the *NN* scattering data. This explains why recent 'good' potential models (such as Nijm78 [1], Paris80 [11], Bonn87 [12]) only arrive at a $\chi^2/N_{\rm data}$ in the order of 2: they use only about 13 parameters. In this context it is interesting to remark that the latest Nijmegen Reidlike *pp* potential (see Sec. 4), which fits the *pp* data as well as (and even slightly better than) our m.e. PWA, has exactly the same number of parameters as this PWA.

## III. DETERMINATION OF THE $NN\pi$ COUPLING CONSTANTS

One of the important results of our new partial-wave analysis is the accurate determinations of the various $NN\pi$ coupling constants at the pion pole. Let us sketch the present situation.

In the construction of the various Nijmegen *NN* potentials [13,1], the $NN\pi$ coupling constant was determined by fitting to the *NN* data of pre-1969 using the Livermore phase-shift analyses [14]. In 1975 the rather successful hard-core potential model D used $f^2 = 0.074$, while the soft-core potential Nijm78 used the value $f^2 = 0.077$ at the pion pole. A few years later, in a phase-shift analysis of the low-energy *pp* scattering data, the tensor combination of the triplet $P$ waves indicated that the $pp\pi^0$ coupling constant should be small. At that time, a value of $f_p^2 \approx 0.075$ was suggested by us [15]. Again, some years later, in a preliminary PWA of the *pp* scattering data below $T_{\rm lab} = 350$ MeV [16], we found $f_p^2 = 0.0725(6)$. This preliminary version did not contain the magnetic-moment interaction [8] and it used a much smaller data base than presently available. The newer, updated value [4] is $f_p^2 = 0.0749(7)$. These values are significantly smaller than the at that time accepted value for the charged-pion coupling constant $f_c^2 = 0.079(1)$, as determined from $\pi N$ scattering [17].

In 1987 it was clear to us that there was a large discrepancy between the value for the $pp\pi^0$ coupling constant as determined from the $pp$ scattering data [16] and the value for the charged-pion coupling constant as determined from the $\pi N$ scattering data. Because there was no obvious reason to doubt either one of these determinations, it was concluded [16] that there apparently is a large breaking of charge independence in the coupling constants. However, subsequent theoretical model calculations have not been able to explain such a large breaking. The differences were always found to be rather small and in most models the charged-pion coupling was found to be smaller than the $pp\pi^0$ coupling (see, e.g., Refs. [18,19]). If we are to believe the theoretical model calculations which rule out a large charge-independence breaking, we can only come to the conclusion that the determination of at least one of these two coupling constants should be incorrect.

We are confident of our value for $f_p^2$ extracted from the $pp$ scattering data. We therefore believe that the previously accepted high value for $f_c^2$ as determined in $\pi N$ scattering can no longer be taken for granted. Recent determinations of this coupling constant in the VPI&SU analyses of the $\pi N$ scattering data by Arndt and co-workers [20] resulted in $f_c^2 = 0.0735(15)$. The value for $f_c^2$ could also be determined in an analysis of the data on the charge-exchange reaction $\overline{p}p \to \overline{n}n$ below $p_{\rm lab} = 950$ MeV/$c$, which resulted in [21] $f_c^2 = 0.0751(17)$. Both results are within one standard deviation from the value for $f_p^2$ determined in the Nijmegen $pp$ analysis [4], so large charge-independence-breaking effects need no longer be invoked.

This result is also supported by the preliminary result of our $NN$ partial-wave analysis, which now also includes the $np$ scattering data. Introducing the three relevant coupling constants

$$\begin{array}{rcrcl} f_p^2 & \equiv & f_{pp\pi^0} f_{pp\pi^0} & \text{for} & pp \to pp \ , \\ f_0^2 & \equiv & -f_{nn\pi^0} f_{pp\pi^0} & \text{for} & np \to np \ , \\ 2f_c^2 & \equiv & f_{np\pi^-} f_{pn\pi^+} & \text{for} & np \to pn \ . \end{array} \tag{3}$$

we find at the pion pole

$$f_p^2 = 0.0751(6) \ , \quad f_0^2 = 0.0752(8) \ , \quad f_c^2 = 0.0741(5) \ , \tag{4}$$

which implies a value for the $nn\pi^0$ coupling constant of $f_n^2 = 0.075(2)$. Assuming that charge independence between the coupling constants holds, we have also performed a combined analysis where we use one coupling constant only, i.e., $f^2 \equiv f_p^2 = f_n^2 = f_c^2$. We then find

$$f^2 = 0.0749(4) \ , \tag{5}$$

and $\chi^2_{\rm min}$ rises with 6.8. Comparing with the result (4) shows that there apparently is no significant charge-independence breaking in the $NN\pi$ coupling constants. This corroborates the results of various theoretical model calculations [18,19] which find that charge-independence-breaking effects are small.

There are several ways for demonstrating that what we determine is indeed the strength of the OPE potential, where we here mention the possibility to extract the corresponding pion mass. We find $m_{\pi^0} = 135.6 \pm 1.3$ MeV, to be compared with the experimental value [22] of $m_{\pi^0} = 134.9739(6)$ MeV, and $m_{\pi^+} = 139.4 \pm 1.0$ MeV, also in excellent agreement with the experimental value [22] $m_{\pi^+} = 139.5675(4)$ MeV.

## IV. QUALITY OF *NN* POTENTIALS

Another result of a complete PWA is that a good knowledge of the data is available. This can then be used to see how well the various potential models fit the experimental data. The s.e. analyses at a number of different energy bins provide us with error matrices for the lower partial-wave phase parameters. Because our combined analysis of the *pp* and *np* data is not in a final stage yet, we here focus mainly on the *pp* analysis. The s.e. analyses provide us with ten error matrices $E_n$. The error matrix is the inverse of half the second derivative matrix of the $\chi^2$ hypersurface with respect to the phase parameters. Denoting the deviation of some model phase parameters from the s.e. phase parameters in each energy bin by $\mathbf{d}$, the $\chi^2$ of the model can now be written as a sum of the s.e. contributions $\chi^2_{\mathrm{se},n}$ and the contributions from the representation matrices $\chi^2_{\mathrm{rep},n}$, i.e.,

$$\chi^2(\mathrm{mod}) = \sum_n \left(\chi^2_{\mathrm{se},n} + \chi^2_{\mathrm{rep},n}\right) = \chi^2_{\mathrm{se}} + \sum_n \mathbf{d}^T E_n^{-1} \mathbf{d} \ . \tag{6}$$

Although the error matrices $E_n$ give a pretty good representation of the $\chi^2$-surface within a certain energy bin, one should note that it is not an exact representation for several reasons. First of all, the higher partial-wave phase parameters and the normalization constants are fixed at their s.e. values. Furthermore, the data have been clustered at some central energy within an energy bin using the results of the m.e. fit, and next to that we have used the approximation that the $\chi^2$ hypersurface is quadratic in the neighborhood of the minimum. The advantage of using this approximate method is that the phase parameters of a potential model to be tested need only be calculated at a small number (10) of energies. This saves a lot of computer time, while the results are more than sufficient for their purpose.

The quality of the representation of our $\chi^2$-surface was tested in two ways. First, we used our m.e. phase parameters as model phase parameters and calculated the corresponding $\chi^2$-contribution. The difference between $\chi^2(\mathrm{mod})$ given by Eq. (6) and the $\chi^2_{\mathrm{min}}$ reached in our m.e. analysis is only 0.35. This means that the $\chi^2$ as calculated directly on the data and the $\chi^2$ calculated via Eq. (6) only differ by 0.02%. This shows that the approximation that the $\chi^2$ hypersurface of the s.e. analyses is quadratic up to the $\chi^2_{\mathrm{min}}$ of the m.e. analysis, is actually very good. As a second test we used the Nijm78 soft-core potential [1] to compare the $\chi^2$-value obtained using Eq. (6) with the $\chi^2$-value obtained from a direct comparison with the data. We are now farther away from the minimum $\chi^2$. The difference is now about 2%, which is satisfactory. It allows us to use Eq (6) to make statements with regard to the quality of some potential model.

We have compared a number of different *NN* potential models which have appeared in the literature with the *pp* scattering data. However, in order to make a fair comparison possible, we only consider the 3–350 MeV region. The reason is that the $^1S_0$ phase shift values at the interference minimum (0.38254 MeV) and at 1.0 MeV are very accurately known. So if the $^1S_0$ of a potential model is a little bit off, the $\chi^2$ contribution can be enormous. For example, the Paris potential [11] gives a $\chi^2$ contribution of more than 4500 on these two energy bins alone, whereas at the other energy bins between 3 and 350 MeV the quality of the potential is very satisfactory.

Still, even in the 3–350 MeV region the quality of most potential models is very poor. This is partly due to the following. In our analysis (and also in the VPI&SU analysis [5]), there is a difference between the $pp$ $^1S_0$ and the $np$ $^1S_0$ phase shifts of about 2°. At low energies the difference is even larger (about 5° at 10 MeV). Such a difference cannot be obtained using a nuclear potential model where one only includes the electromagnetic interaction. Indeed, adding the electromagnetic interaction to the Argonne $v_{14}$ potential [23] which was fitted to the $np$ data, we arrive at a $\chi^2/N_{\rm data}$ of more than 7 for the $pp$ data. This large value is for a part due to an incorrect $pp$ $^1S_0$ phase shift. When we give the Argonne $v_{14}$ potential perfect $pp$ $^1S_0$ phase shifts, the $\chi^2/N_{\rm data}$ on the $pp$ data drops to about 4.3. For the Nijm78 [1] and Paris80 [11] potentials such a situation does not apply, since these models were explicitly fitted to the $np$ as well as the $pp$ scattering data.

Therefore, in the following we will focus on 4 potential models which were constructed to explicitly fit the $pp$ data: the Nijm78 soft-core potential [1], the parametrized Paris80 potential [11], an update of the Bonn potential especially fitted to the $pp$ scattering data [24] (denoted by Bonn89), and a Reidlike Nijmegen potential [10] NijmRdl 91, where each partial wave is fitted separately to the $pp$ scattering data. The Reidlike potential is constructed in order to have a phenomenological potential model which reproduces the phase parameters of our partial-wave analysis. It provides a very good representation of this analysis. The results are shown in Table I. We note that the old Nijm78 and Paris80 potentials, and the new Bonn89 potential are roughly of the same quality: $\chi^2/N_{\rm data} \approx 2$. The new NijmRdl 91 potential fits the data much better and is as good as (and even a little bit better than) the m.e. analysis: $\chi^2/N_{\rm data} = 1.0$.

At this point it is perhaps good to clarify a question which was raised at the Elba Conference. There somebody tried to imply that there is something wrong with the Nijmegen data base. We have compared our data base and our predictions with the VPI&SU analyses using the SAID 1989 solution. This concerns $NN$ data in the 8–325 MeV energy range. The data bases contain

$$\begin{aligned} 1113\ pp \text{ data and } 2265\ np \text{ data } &= 3378\ NN \text{ data in SAID 89,} \\ 1382\ pp \text{ data and } 2274\ np \text{ data } &= 3656\ NN \text{ data in Nijmegen PWA.} \end{aligned}$$

From these numbers it appears definitely not true that we reject more data than VPI&SU. We have compared the Paris80 potential directly (not using Eq. (6)) with these two data sets. The $\chi^2/N_{\rm data}$ of the Paris80 potential on the interval 8–325 MeV yields

$$\begin{aligned} pp \text{ data} : \chi^2/N_{\rm data} &= 2.27 \text{ in SAID 89} \\ &= 2.15 \text{ in Nijmegen PWA;} \\ np \text{ data} : \chi^2/N_{\rm data} &= 3.31 \text{ in SAID 89} \\ &= 3.28 \text{ in Nijmegen PWA;} \\ NN \text{ data} : \chi^2/N_{\rm data} &= 2.96 \text{ in SAID 89} \\ &= 2.85 \text{ in Nijmegen PWA.} \end{aligned}$$

The differences between these two comparisons with the data are: SAID 89 contains phase parameters slightly different from the phase parameters calculated by us using the Paris80 potential, the inclusion of the Coulomb interaction in the $pp$ analyses is definitely a source of difference, different $NN\pi$ coupling constants are perhaps used, etc. We make the observation that for $np$ the analyses agree very well, while also for $pp$ the agreement is reasonable. We would like to stress here that in the Paris80 potential it is possible to calculate the *pp* phase parameters as well as the *np* phase parameters. When one has only *np* phase parameyters available and one tries to compare with the *pp* data, one should not complain when the results are shown to be incorrect.

It is also very instructive to see how the different partial waves contribute to the total $\chi^2$. For that purpose we start with the m.e. phase shifts and substitute the $^1S_0$ phase shift of the different potential models. We then calculate the difference $\Delta\chi^2$ between this $\chi^2$ and the $\chi^2$ of the m.e. analysis. This is also done for the $^1D_2$, the triplet $P$ and the triplet $F$ phase shifts. These four separate contributions can then be summed and compared with the $\chi^2$ as obtained when we take all the potential phase shifts together. The results are presented in Table II.

The agreement between the sum of the $\Delta\chi^2$-contributions substituting the potential model phase shifts one at a time, and the $\Delta\chi^2$-contribution using all potential model phase shifts simultaneously is satisfactory for the old Nijm78 potential and the new NijmRdl 91 potential. For the Nijm78 potential the $\Delta\chi^2$-contributions are about the same for each of the separate contributions. On the other hand, for the Paris80 potential the $^1S_0$ phase shift is rather good, whereas the $^3P$ (and less the $^1D_2$) phase shifts are not. Similarly, for the Bonn89 potential both the $^1S_0$ and the $^3P$ phase shifts are not very good. Moreover, for both the Paris80 and Bonn89 potentials, the last column in Table II (the sum of the separate contributions) is substantially higher than the second column (using all potential phase shifts together). This means that the correlation between the various phase shifts in these models is very important. This is not a rather nice feature.

## V. ANALYSES AT 50 MEV

Recently, a very accurate *pp* analyzing power experiment at 50.04 MeV [25] and a measurement of the spin-correlation parameter $A_{zz}$ in *np* scattering at 67.5 MeV [26] were reported. These experiments, together with the other *pp* and *np* scattering data already present in the 50 MeV region, provide us with a fairly complete set of *NN* scattering data around 50 MeV. This makes that the phase parameters at 50 MeV can now be determined rather accurately. It will be interesting to see how the results of the various potential models compare with these phase parameters.

In Table III we present the *pp* phase shifts at 50 MeV. The first column gives the phase shifts as determined in the Nijmegen partial-wave analysis. In the following columns we give the *pp* phase shifts of the different potential models where in square brackets we give the difference with the PWA in standard deviations. For example, the $^3P_0$ phase shift in the PWA is $11.36 \pm 0.12°$. The Nijm78 potential predicts 11.80° for this phase shift. The difference with the PWA value is 0.44° which amounts to 4 standard deviations. Important to note is that the new Bonn89 potential has a $^3P_0$ phase shift which is 11 standard deviations too large and a $^3P_2$ phase shift which is 14 standard deviations too low. These are indications for the fact that this new Bonn89 *pp* potential has a tensor potential which is too strong, whereas its spin-orbit potential is too weak. We would like to point to the very good performance of the NijmRdl 91 potential for these $I = 1$ phase parameters.

In Table IV we present some of the $I = 0$ phase parameters at 50 MeV. The numbers between square brackets again denote the difference with the PWA in standard deviations. For the Bonn potential we have now given the results of the full Bonn87 potential [12]. From Table IV we can draw the conclusion that the NijmRdl 91 potential again gives an excellent fit, while the other potentials give roughly similar predictions. Important is our conclusion that the $\varepsilon_1$ mixing parameter at 50 MeV is $\varepsilon_1 = 2.16 \pm 0.48°$. This is substantially lower than the result of the analysis by the Basel group [26] of $\varepsilon_1 = 2.9 \pm 0.3°$, shown as an open circle in Fig. 1. We want to stress here again that the data base for both analyses are similar. Only the conclusions differ. Our value for $\varepsilon_1$ is in perfect agreement with the modern $NN$ potentials.

## VI. CONCLUSIONS

The Nijmegen partial-wave analyses of all $NN$ scattering data below $T_{\rm lab} = 350$ MeV has a $\chi^2/N_{\rm data} = 1.00$. This is a very satisfactory result. Because we have $N_{\rm df} = 3850$ degrees of freedom we would expect $\langle\chi^2_{\rm min}\rangle = 3850 \pm 90$. We find $\chi^2_{\rm min} = 4171$. This means that there is still room for improvement of our analysis of $\Delta\chi^2 \approx 320$, provided that the data form a **statistical** data set. This possibility for improvement can be used as a powerful tool to check theoretical improvements.

The analysis can also be used to test the quality of potential models. We can calculate the potential phase shifts and make a direct comparison with the data. But we can also include the tail of a potential model in the long-range part $V_L$ which enters the analysis via the Schrödinger equation (1). Performing a m.e. analysis using this potential tail then provides information on its quality.

In our analysis we use the heavier-boson exchanges of some potential model (in the present analysis we use the Nijmegen soft-core potential [1]) to give a non-OPE contribution to the intermediate partial waves with $5 \leq J \leq 8$. This also provides us with a test for the quality of the non-OPE part of a particular potential model.

Our way of analyzing the data by using a potential tail allows us to determine the $NN\pi$ coupling constants. Perhaps in a later stage we will also be able to determine, e.g., the $\omega$ and $\rho$ coupling constants.

Finally, we can study charge-independence-breaking effects between the $pp$ and $np$ $^1S_0$ phase shifts and the triplet $P$ waves.

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## TABLES

TABLE I. $\chi^2$ values at the 8 single energies between 3 and 350 MeV of the four potential models mentioned in the text and the *pp* partial-wave analysis. For the total $\chi^2$ one has to add the $\chi^2_{\rm se} = 1510.5$ of the s.e. analyses as in Eq. (6).

| energy | Nijm78 | Paris80 | Bonn89 | NijmRdl 91 | PWA91 |
|---|---|---|---|---|---|
| 5 | 52.2 | 18.4 | 8.6 | 15.1 | 17.7 |
| 10 | 76.4 | 33.2 | 46.0 | 13.8 | 18.8 |
| 25 | 66.2 | 12.0 | 20.0 | 3.2 | 0.9 |
| 50 | 531.2 | 347.7 | 333.2 | 9.7 | 16.4 |
| 100 | 131.1 | 42.0 | 56.0 | 23.0 | 20.4 |
| 150 | 206.1 | 419.5 | 306.0 | 20.6 | 21.8 |
| 215 | 198.0 | 170.3 | 312.0 | 11.9 | 15.2 |
| 320 | 425.5 | 572.1 | 518.9 | 7.1 | 3.3 |
| 3–350 | 1686.7 | 1615.2 | 1600.6 | 104.3 | 114.5 |
| $\chi^2/N_{\rm data}$ | 2.0 | 2.0 | 2.0 | 1.0 | 1.0 |

TABLE II. The difference $\Delta\chi^2$ (see text) in the 3–350 MeV energy range of various potential models using all potential phase shifts, or using one particular phase shift only.

| Model | All | Only one phase shift | | | | |
|---|---|---|---|---|---|---|
| | phases | $^1S_0$ | $^3P$ | $^1D_2$ | $^3F$ | Sum |
| Nijm78 | 1572.2 | 302.0 | 462.8 | 403.6 | 442.1 | 1610.5 |
| Paris80 | 1500.7 | 137.2 | 835.0 | 637.0 | 459.6 | 2068.8 |
| Bonn89 | 1486.1 | 630.5 | 743.4 | 341.4 | 193.8 | 1909.1 |
| NijmRdl 91 | –10.2 | 5.6 | 0.0 | –12.0 | 1.1 | –5.3 |

TABLE III. *pp* phase shifts in degrees at 50 MeV of the partial-wave analysis and of various potential models. The numbers between square brackets denote the difference with the partial-wave analysis in standard deviations.

| phase | PWA | Nijm78 | Paris80 | Bonn89 | NijmRdl 91 |
|---|---|---|---|---|---|
| $^1S_0$ | 39.14±0.09 | 39.58[5] | 38.75[4] | 38.25[10] | 38.82[4] |
| $^1D_2$ | 1.70±0.01 | 1.63[7] | 1.80[10] | 1.68[2] | 1.70[0] |
| $^3P_0$ | 11.36±0.12 | 11.80[4] | 11.81[4] | 12.66[11] | 11.43[1] |
| $^3P_1$ | –8.26±0.04 | –8.36[2] | –8.41[4] | –8.34[2] | –8.28[0] |
| $^3P_2$ | 5.84±0.02 | 5.78[3] | 5.72[2] | 5.56[14] | 5.80[2] |

TABLE IV. $np$ $I = 0$ phase parameters in degrees at 50 MeV of the partial-wave analysis and of various potential models. The numbers between square brackets denote the difference with the partial-wave analysis in standard deviations.

| phase | PWA | Nijm78 | Paris80 | Bonn87 | NijmRdl 91 |
|---|---|---|---|---|---|
| $^1P_1$ | –9.83±0.24 | –8.64[5] | –10.94[5] | –10.48[3] | –9.78[0] |
| $^3S_1$ | 62.70±0.50 | 60.40[5] | 62.30[1] | 62.20[1] | 62.50[1] |
| $^3D_1$ | –6.43±0.08 | –6.60[2] | –6.77[4] | –6.98[7] | –6.44[0] |
| $\varepsilon_1$ | 2.16±0.48 | 2.27[0] | 1.89[1] | 2.08[0] | 2.15[0] |

## FIGURES

FIG. 1. Mixing parameter $\varepsilon_1$ in degrees versus $T_{\rm lab}$ in MeV. $\bullet$: s.e. result; $\circ$: result Basel group [26]; solid curve: m.e. result; dash-dotted curve: Nijmegen potential; dotted curve: Paris potential; dashed curve: Bonn potential.