THEF-NIJM 92.05

Pion-nucleon coupling constant*

Vincent Stoks, Rob Timmermans, and J.J. de Swart
Institute for Theoretical Physics, University of Nijmegen, Nijmegen, The Netherlands

Abstract

In view of the persisting misunderstandings about the determination of the pion-nucleon coupling constants in the Nijmegen multienergy partial-wave analyses of pp, np, and $\overline{p}p$ scattering data, we present additional information which may clarify several points of discussion. We comment on several recent papers addressing the issue of the pion-nucleon coupling constant and criticizing the Nijmegen analyses.

PACS numbers: 21.30.+y, 11.80.Et, 13.75.Cs

Typeset using REVTEX

*Published in Phys. Rev. C **47**, 512–520 (1993)

I. INTRODUCTION

There appear to exist some misunderstandings concerning the determination by our group of the pion-nucleon coupling constants in multienergy partial-wave analyses of pp [1], combined pp and np [2], and $\overline{p}p$ [3] scattering data. We have summarized the recent determinations of the pion-nucleon coupling constants in Table I.

In their latest analysis of $\pi^{\pm}p$ scattering data the VPI&SU group has obtained a value for the pion-nucleon coupling constant consistent with our values [4]. Since the values $f^2_{NN\pi} \simeq 0.075$ found by us (see also Refs. [5–9]) are consistently lower than the value $f^2_{NN\pi} = 0.079$ found in the Karlsruhe-Helsinki analyses of $\pi^{\pm}p$ scattering data [10–12], a number of papers [13–19] have been published commenting on our results. In several of these papers one attempts to find an explanation for the difference by trying to point out alleged shortcomings and so-called systematic errors pertaining to our method of analysis. In view of the existing confusion, we have looked again into these and other matters and carefully reexamined possible sources of systematic errors. We felt it would be useful to supply some additional information to further clarify these issues and to address a few points where some of these papers criticizing us are in error.

A very important point that has not been recognized and appreciated enough is that the basis of our accurate determination of the pion-nucleon coupling constants and of the nucleon-nucleon phase shifts is our *multienergy* partial-wave analysis of all nucleon-nucleon scattering data below $T_{\text{lab}} = 350$ MeV. This multienergy partial-wave analysis is much more sophisticated and computer intensive than other similar analyses of nucleon-nucleon scattering data. The energy dependence of the phase shifts in our analysis is described in a much better way. As a consequence, we end up with a multienergy partial-wave solution which gives an excellent fit to all nucleon-nucleon scattering data below $T_{\text{lab}} = 350$ MeV. The phase shifts and pion-nucleon coupling constants as determined by this multienergy partial-wave analysis are the best possible values for these quantities that can be obtained from these data. We strongly feel that statements about the pion-nucleon coupling constant made with the help of single-energy analyses or, even worse, analyses of individual experiments with the help of potential models cannot, in any way, be compared in quality with the results of these multienergy analyses. The reason is that in such analyses at one particular energy the information about the energy dependence of the phase shifts due to one-pion exchange cannot be incorporated. In a multienergy analysis the complete database is used and energy-dependent constraints are properly accounted for. This results in a much better and more precise value for the coupling constant, with a minimal model dependence. For a proper determination the whole database is needed and arguments based on studies of specific experiments are not so reliable.

In our discussion, we take mostly the Nijmegen multienergy analysis of all pp scattering data below $T_{\text{lab}} = 350$ MeV [1] as specific example. The value for the $pp\pi^0$ coupling constant obtained in this analysis is undoubtedly the most compelling evidence for a low pion-nucleon coupling constant, since in this case the required theoretical input is rather small and practically model independent. Therefore, the possible systematic error in this case is very small. Actually, the statistical error on the $np\pi^{\pm}$ coupling constant obtained in the np analysis is smaller than the statistical error on the $pp\pi^0$ coupling constant, although the np data are less accurate and less varied than the pp data. The reason, we think, is that in np scattering pion exchange can be probed more easily, since in pp scattering one always has to deal with the infinite-range repulsive Coulomb potential and one first has to strip the pp data from this long-range Coulomb contribution in order to get a handle on pion exchange. Nevertheless, in the pp analysis we feel almost certain that the systematic error on the $pp\pi^0$ coupling constant is small, since in this case we explicitly investigated all thinkable sources of systematic errors and found no significant effects. We stress that our emphasis on the $pp\pi^0$ coupling constant does not mean that we have doubts about the correctness of the determination of the pion-nucleon coupling constants in the np and $\overline{p}p$ partial-wave analyses. It simply is more difficult to do a similar thorough study for the np and $\overline{p}p$ cases, where the amount of theoretical input into the analyses is larger. For instance, in these cases it is necessary to make from the start some theoretical assumptions about the validity of charge independence.

In the following sections of this paper we will report on our search for systematic errors in the value for the $pp\pi^0$ coupling constant in the pp analysis. Part of the reason we have been able to do this comprehensive investigation is that recently a lot of computing power has become available to us. We begin the next section by discussing some statistics relevant to the analyses. Then we will subsequently discuss the influence of form-factor effects, the sensitivity of the different types of observables to the pion-nucleon coupling constant, and its determination from individual partial waves and from data in different energy ranges. Next, we will investigate more closely some particular pp and np scattering experiments that are brought up in connection with the determination of the coupling constant. Finally, we spend a few words on the Nijmegen analysis of $\overline{p}p$ scattering data.

II. STATISTICAL CONSIDERATIONS

The Nijmegen database of pp scattering data below $T_{\rm lab} = 350$ MeV contains at present $N_{\rm obs} = 1656$ scattering observables. Since there are 119 experimental groups with a finite overall normalization error as well as 12 additional normalization parameters (from angle-dependent normalizations) we have a total of $N_{\rm dat} = 1787$ pp data. In the Nijmegen partial-wave analysis these data are fitted with $N_{\rm par} = 22$ model parameters, which includes the $pp\pi^0$ coupling constant. Because there are also 22 groups with a floated normalization the number of degrees of freedom is $N_{\rm df} = 1612$. If the database is a correct statistical ensemble and if the theoretical model is correct, one expects $\langle\chi^2_{\rm min}\rangle = N_{\rm df} \pm \sqrt{2N_{\rm df}} = 1612 \pm 57$. In our latest analysis we reach $\chi^2_{\rm min} = 1786$, which is only 174, or 3 standard deviations, higher than the expectation value. This difference is at least partially due to small theoretical shortcomings in our model. This implies, therefore, that there is still some room for theoretical improvements in the pp partial-wave analysis. We have investigated the statistical quality of the final pp data set by calculating the momenta of the theoretically expected χ^2 distribution and comparing them to the ones actually found in the analysis. The results are presented in Table II. For details about the statistical tools used in the Nijmegen partial-wave analyses we refer to Ref. [20]. The second and higher central moments found in the analysis are in excellent agreement with their expectation values. This shows that the statistical quality of the pp data set used is very good. It shows that our analysis with its $\chi^2_{\rm min}/N_{\rm df} = 1.108$ is already quite good. Nevertheless, we hope that a significant drop in

$\chi^2_{\rm min}$ can still be obtained giving better agreement between $N_{\rm df}$ and $\chi^2_{\rm min}$. In this way our partial-wave analysis becomes a tool, because it can decide on the quality of the proposed improvements in theory.

We have demonstrated that our multienergy partial-wave solution is essentially correct statistically. This has rather strong consequences: it means that the values for the phase shifts and coupling constants as well as the statistical errors on these quantities as determined in the multienergy analyses are essentially correct. Here the statistical error on a particular quantity (e.g., a phase shift) is the error as obtained in the standard way via the χ^2-rise-by-one rule, as discussed for example in Sec. V A 2 of Ref. [20]. Including new experiments in the multienergy analysis will change the phase shifts and coupling constants not more than 1 or 2 multienergy standard deviations. The same is not necessarily true for quantities determined in a single-energy analysis. For example, in a single-energy *np* analysis around 100 MeV there are no *np* spin-correlation data to pin down the ε_1 mixing parameter, whereas in the multienergy analysis the presence of spin-correlation data at the adjoining energies near 50 and 150 MeV also allows for an accurate determination of the ε_1 mixing parameter at 100 MeV. Therefore, the multienergy values and errors for the phase shifts and coupling constants are much more realistic than the single-energy determinations.

In our latest *pp* analysis [9] we used a parametrization for the ${}^1S_0$ partial wave different from the one used in previous analyses [1,8]. This parametrization has less parameters but gives a somewhat higher $\chi^2_{\rm min}$. We feel, however, that it is a better parametrization, since the ${}^1S_0$ wave was probably overparametrized in the older analyses. This means that results presented for this analysis are not necessarily also true for the older analyses, especially with respect to statements about the ${}^1S_0$ wave. In spite of a large amount of effort on our side, the description of this partial wave is still not as good as we would like.

In our *pp* analysis we determine the $pp\pi^0$ coupling constant at the pion pole. We find now $f^2_{pp\pi^0} = 0.0750(5)$, where the error is purely statistical. If we fix this coupling at the old value 0.079 the result is $\chi^2_{\rm min} = 1842$, which is $\Delta\chi^2_{\rm min} = 56$, or 7.5 standard deviations, higher than the minimum. We stress again that no particular data set is responsible for the specific value of the coupling constant, but that all data when fitted in a multienergy partial-wave analysis contribute to this value. From these numbers one can get a feeling about how unreliable it must be to make statements on the pion-nucleon coupling constant with the help of potential models which fit the data with $\chi^2_{\rm min}/N_{\rm dat} \simeq 2$, instead of drawing conclusions based on multienergy analyses with $\chi^2_{\rm min}/N_{\rm dat} \simeq 1$. Nevertheless, in some recent papers [17,19] conclusions about the pion-nucleon coupling constant are drawn with the help of potential models that have $\chi^2_{\rm min}/N_{\rm dat} \simeq 2$ or $\chi^2_{\rm min} \simeq 3600$ which is about 2000, or 35 standard deviations, and not just 174, or 3 standard deviations, higher than the expectation value $\langle\chi^2_{\rm min}\rangle = 1612$. The point we want to make here is that these potential models can still be improved in so many different places in so many different ways, that it is very presumptuous to try to make any conclusions about a $\Delta\chi^2_{\rm min} = 56$ effect. All such conclusions are inevitably very model dependent, which seems to be generally overlooked by the authors criticizing us.

III. FORM FACTORS

Especially persistent is the suggestion of systematic errors due to form-factor effects [13–16,18,19], although we have repeatedly [2,3,8,9] stressed that we determine the pion-nucleon coupling constant at the pion pole, and that therefore form factors are irrelevant to the issue. We did explicitly check this by adding an exponential form factor to the pion-exchange potential in our analysis. We used

$$F(\mathbf{k}^2) = \exp[-(\mathbf{k}^2 + m_\pi^2)/\Lambda_0^2] \ , \tag{1}$$

normalized such that at the pion pole $F(-m_\pi^2) = 1$. For values of the cutoff mass Λ_0 as low as 500 MeV we have found no significant changes. This can be seen in Table III, where the results for the $pp\pi^0$ coupling constant and the corresponding values for $\chi^2_{\rm min}$ are presented as a function of the cutoff mass Λ_0 in MeV. Evidently there are no significant changes for realistic values of the cutoff mass. In spite of our statements, however, it seems to have been suggested recently in Ref. [18] that the value of the coupling constant may depend critically on the shape of the form factor and that we, like other groups, should use a form factor of the Feynman type

$$F(\mathbf{k}^2) = \left[\frac{\Lambda_2^2 - m_\pi^2}{\Lambda_2^2 + \mathbf{k}^2}\right]^2 \ , \tag{2}$$

again normalized such that at the pion pole $F(-m_\pi^2) = 1$. The square appears because one takes a monopole form factor at each vertex. Let us start by saying that we strongly feel that an exponential form factor is more physical than a Feynman-type form factor. The exponential form factor follows quite naturally from Regge-pole theory and from constituent-quark models with harmonic-oscillator wave functions. A Feynman-type form factor, on the other hand, which is essentially a phenomenological regulator, has an annoying singularity at $\mathbf{k}^2 = -\Lambda_2^2$ in the unphysical region. This Feynman-type form factor is chosen mainly for reasons of convenience.

In our mind, it is hard to understand how the *type* of form factor can matter once it has been demonstrated, using one particular type, that the value of the coupling constant is determined at the pole. When comparing form factors, it should always be kept in mind that the form factor is related to the size of the nucleon. An exponential form factor and a Feynman-type form factor give approximately the same nucleon size when $\Lambda_0 = \Lambda_2/\sqrt{2}$. Reliable values of the cutoff mass are hard to determine in *NN* scattering since these depend, for instance, on which heavy mesons are included in the model [19]. But, in general, if a cutoff mass is used which affects the pion-exchange potential drastically at distances where the nucleons do not overlap anymore, the obvious conclusion should be that this is an unrealistic low value for the cutoff mass. As an example, look at Fig. 4 in Ref. [18], where one is willing to accept a 20% reduction of the tensor force due to pion exchange at a distance of 2.5 fm. In the Nijmegen soft-core potential [21] the exponential cutoff mass is 965 MeV and in the Bonn potential [22] the Feynman-type cutoff mass for the pion-exchange potential is 1300 MeV. In fact, it has, until very recently [19], always been claimed by the Bonn group that a satisfactory fit to the *NN* scattering data is impossible with a lower cutoff mass. If one looks at potential models, a value of 500 MeV for a cutoff mass in a form factor is quite low.

To meet all criticism, however, and to avoid new misconceptions, we have again explicitly checked our conjectures by repeating the pp analysis using a pion-exchange potential with Feynman-type form factor. As we expected, our findings were entirely similar to those with an exponential form factor. We conclude therefore that neither the shape of the form factor nor the value of its cutoff mass (as long as it is not unreasonably low) has a significant influence on our determination of the pion-nucleon coupling constant. The obvious reason for this nice feature is to be found in the specific method of analysis which allows the extraction of the coupling constant from the asymptotic behavior of the one-pion-exchange potential in configuration space and its determination is not sensitive to short-range modifications.

IV. DETERMINATION FROM DIFFERENT OBSERVABLES, PARTIAL WAVES, AND ENERGY RANGES

It is an interesting exercise to investigate which particular types of observables are the most sensitive to variations in the coupling constant. We have repeated the pp analysis for four different values of the $pp\pi^0$ coupling constant and the np analysis, with the $NN\pi^0$ coupling constants fixed at 0.075, for four values of the $np\pi^{\pm}$ coupling constant. The resulting values of $\chi^2_{\rm min}$ are tabulated in Table IV for the different types of pp scattering observables and in Table V for the np observables. The value found for the charged-pion coupling constant in the np analysis is $f_c^2 = 0.0748(3)$. It can be seen that in the np analysis the difference between $f_c^2 = 0.075$ and 0.079 is $\Delta\chi^2_{\rm min} = 255$, so this is a difference of 16 standard deviations. We did already mention above that the corresponding difference in the pp analysis, for the $pp\pi^0$ coupling constant, amounts to $\Delta\chi^2_{\rm min} = 56$, or 7.5 standard deviations. Furthermore, it can be seen from these tables that no particular type of observable is solely responsible for the low value of the coupling constant, but that essentially all types of pp as well as np scattering data favor a low pion-nucleon coupling constant.

In order to investigate in what way the different partial waves are sensitive to the $pp\pi^0$ coupling constant, we introduced first of all two different couplings, one for the singlet waves and one for the triplet waves. We then find $\chi^2_{\rm min} = 1786$ for 1611 degrees of freedom. The resulting coupling constants are 0.0753(7) for the singlet waves and 0.0750(6) for the triplet waves. This shows that both singlet and triplet waves favor a low coupling constant.

Next, we introduced in turn a different $pp\pi^0$ coupling constant for a specific partial wave and one for all other partial waves. This exercise was done for all parametrized waves in the analysis: ${}^1S_0$, ${}^1D_2$, ${}^1G_4$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$-${}^3F_2$, ${}^3F_3$, and ${}^3F_4$-${}^3H_4$. The results for the coupling constants are shown in Table VI. For all cases excepting the ${}^1S_0$ wave the results favor a low coupling constant. In the case that a separate $pp\pi^0$ coupling was introduced in the ${}^1S_0$ channel an improvement of $\Delta\chi^2_{\rm min} = 6.6$ was found for a high coupling constant. This is presumably a consequence of our new parametrization of the ${}^1S_0$ partial wave which is evidently not perfect. When the coupling constant in the ${}^1S_0$ wave is fixed at 0.075 the results for all remaining partial waves are nicely consistent with a $pp\pi^0$ coupling constant somewhat lower than 0.075. Apparently the ${}^1S_0$ wave enhances this to $f^2_{pp\pi^0} = 0.0750(5)$. This is probably also the reason that, of the different types of observables, only the differential cross sections favor a coupling somewhat larger than 0.075 (see Table IV), since differential cross sections are more sensitive to this wave than spin-dependent observables. There is thus some indication for a small systematic error pertaining to the ${}^1S_0$ partial wave in the pp analysis. Further study is required to find out to what extent this is to be attributed to friction between the data or to a flaw in our theoretical treatment of the ${}^1S_0$ channel. In view of the above findings, it may be better to fix the coupling constant in the ${}^1S_0$ channel at a value of 0.075 and determine $f^2_{pp\pi^0}$ from the remaining partial waves. We then find for 1612 degrees of freedom $\chi^2_{\rm min} = 1787$ and the coupling constant becomes $f^2_{pp\pi^0} = 0.0746(6)$. This is within one standard deviation from the value quoted above $f^2_{pp\pi^0} = 0.0750(5)$ determined from all partial waves including the ${}^1S_0$.

We can also bypass the apparent friction in the ${}^1S_0$ partial wave by removing from the database the data taken at very low energies below 1 MeV, namely, the Los Alamos cross sections around the interference minimum measured by Brolley *et al.* [23] and the Zürich cross sections from Thomann *et al.* [24]. The reason, of course, is that at these low energies the ${}^1S_0$ phase shift is very accurately known and gives a very strong constraint on the parametrization of the ${}^1S_0$ phase shift. The results from this 3–350 MeV partial-wave analysis are presented in Table VII. As expected, we find a somewhat smaller coupling constant $f^2_{pp\pi^0} = 0.0743(6)$ instead of 0.0750(5). If we fix the coupling constant in the ${}^1S_0$ channel at 0.075, we find in the 3–350 MeV analysis $f^2_{pp\pi^0} = 0.0744(6)$. So there are strong indications that the value for the neutral-pion coupling constant as determined in the pp partial-wave analysis is somewhat smaller than 0.075. In the last line of Table I we quote $f^2_{pp\pi^0} = 0.0745(6)$.

In order to demonstrate that it is not only the data at low energies that pin down the coupling constant, Table VII also contains the results for the $pp\pi^0$ coupling constant obtained from a number of analyses in different energy ranges. It can be seen from this Table that the data at energies higher than 10 or 30 MeV favor a low coupling constant as well. Similar results are found if we restrict the energy range at the high end, by doing an analysis of the data up to, say, 280 MeV. These findings once more underline our claim that the database as a whole contributes to a low value for the pion-nucleon coupling constant, and not some particular experiment(s) or the data in a restricted energy bin.

V. pp ANALYZING-POWER DATA AROUND 10 MEV

Let us next turn to our investigations of specific experiments that are discussed in connection with the pion-nucleon coupling constant. In Ref. [19], for instance, it is stated that the prime reason for a low $pp\pi^0$ coupling constant was our analysis of pp analyzing-power data around 10 MeV. In Ref. [18], the same statement can be found in a different form, where it is said that the ${}^3P$ phase shifts around 10 MeV are very important in the determination. However, these arguments only reflect our statements based on a preliminary pp analysis by our group [6]. Apparently, these critics have overlooked our amendment to these statements as discussed in our paper on the completed 0–350 MeV pp analysis [1], where we incorporated many theoretical improvements and included much more experimental data.

In this latter paper it is explicitly stated that the ^3P waves are not especially important in the determination of $f^2_{pp\pi^0}$ and that it is not possible to pinpoint some specific type of observables as particularly constraining. Concerning the analyzing-power data around 10 MeV, the 15 Wisconsin data points at 9.85 MeV [25] have in our latest multienergy *pp* analysis $\chi^2_{\rm min} = 16$. If we fix the $pp\pi^0$ coupling constant at 0.079 and refit, $\chi^2_{\rm min}$ on these data increases to 31. If we leave out this group (lowering the number of degrees of freedom with 15 to 1598), $\chi^2_{\rm min}$ drops about 16 from 1786 to 1770 and the value for the coupling constant becomes $f^2_{pp\pi^0} = 0.0751(6)$. This clearly shows that these data are not alone responsible for the low value of the coupling constant, although this group clearly favors a low coupling constant. We stress once more, however, that this latter conclusion is only justified when reached in a multienergy partial-wave analysis using the data as a whole. We did already demonstrate that the data above 30 MeV give for the coupling constant $f^2_{pp\pi^0} = 0.0743(6)$, so the analyzing-power data around 10 MeV are absolutely not crucial to a low value for $f^2_{pp\pi^0}$.

A criticism of our *pp* analysis in this context is the fact that we do not include in our database another group of analyzing-power data around 10 MeV (and 25 MeV) taken by the Erlangen group of Kretschmer *et al.* [26,27]. We do not do this because it is our policy not to include data that have not been published in a regular physics journal. Moreover, in this specific case we have committed ourselves to not publishing any analysis of these specific data prior to their publication by the Erlangen group. Of course we are well aware of the existence of these data and we did analyze them. We can state that we find no reason whatsoever to modify any of our conclusions regarding the $pp\pi^0$ coupling constant. It is definitely not true that it is crucial which one of these two data sets (the Wisconsin or the Erlangen set) around 10 MeV is included, as is concluded in Ref. [19] from a study with meson-exchange potential models that have $\chi^2_{\rm min}/N_{\rm dat} \gtrsim 2$.

VI. np BACKWARD DIFFERENTIAL CROSS SECTIONS

A point of discussion regarding the *np* analysis is the normalizations of the *np* differential cross sections and their relation to our determination of the $np\pi^\pm$ coupling constant (see, for instance, Ref. [18]). It is common folklore that the $np\pi^\pm$ coupling constant is determined mainly by the peak present in backward *np* differential cross sections. It was suggested by Chew [28] as early as 1958 that this is a good place to extract the pion-nucleon coupling constant. If this is true, then a very important group of data should be the Los Alamos set of backward cross sections measured by Bonner *et al.* [29]. However, we have seen already from Table V that all observables, and not the differential cross sections in particular, favor a low coupling constant. Our results and conclusions regarding the data from Bonner *et al.* are the following. The way we handle the normalization of a group of data and its uncertainty is explained in detail in Ref. [20]. One group of 42 cross sections at 194.5 MeV is rejected completely, as well as 1 data point at 344.3 MeV. For the remaining 607 backward *np* cross sections at 10 different energies we find $\chi^2_{\rm min} = 630$ for $f^2_c = 0.075$ and $\chi^2_{\rm min} = 655$ for $f^2_c = 0.079$, so $\Delta\chi^2_{\rm min} = 25$. Of these ten groups, seven groups have a floated normalization and three groups have a finite normalization error of 4%, where the sensitivity of these last three groups (242 data points) to f_c^2 is rather small. So we see that it is mainly the *shape* of the cross section and not the normalization that makes these Los Alamos data favor a low coupling constant. In Table VIII we give the results obtained for the charged-pion coupling constants for the ten individual groups by fitting a parabola through four values of $\chi^2_{\rm min}$ for four different coupling constants. We see that most groups favor a low coupling constant, but the errors are rather large.

We have also tabulated the norm of these groups as determined in the multienergy analysis, once again following the χ^2-rise-by-one rule using the full error matrix. It can be seen that our multienergy solution pins down these normalizations with very small errors, of the order of 0.5%, which is much smaller than the 4% error quoted by the experimentalists. There is essentially no difference between the groups with a floated normalization and the groups with a finite normalization error.

Our conclusion is that the relevance of the backward cross sections for determining the charged-pion coupling constant is more limited than is generally assumed. There are other experiments that are much more constraining for the coupling constant. Here we mention the 12 analyzing-power data at 10.03 MeV taken by Holslin *et al.* [30], the 16 spin correlations measured by Bandyopadhyay *et al.* [31] at 220 MeV, and the 19 spin correlations taken by the same group at 325 MeV. Again, these statements apply to an analysis of the groups within multienergy partial-wave analyses of the complete database, and do not follow from studies of the individual experiments. However, we want to stress once more that our low value of the charged-pion coupling constant is not only due to these accurate analyzing-power and spin-correlation data. The analysis without these data still yields $f_c^2 = 0.0750(4)$, demonstrating that also the other data favor a low, but slightly less accurate, value.

VII. DETERMINATION FROM CHARGE-EXCHANGE DATA

In this section, we add some remarks about our coupled-channels partial-wave analysis [3] of antiproton scattering data. In this case a neutral pion can be exchanged in elastic $\overline{p}p \to \overline{p}p$ scattering and a charged pion in charge-exchange $\overline{p}p \to \overline{n}n$ scattering. Until recently it was believed by probably everybody (including ourselves) that a partial-wave analysis of these reactions was out of the question. We find it gratifying that the methods used in the partial-wave analyses of pp and np scattering data could be extended to the case of the antiproton elastic and charge-exchange scattering. The value for the $np\pi^{\pm}$ coupling constant $f_c^2 = 0.0751(17)$ found in our analyses of charge-exchange data is in nice agreement with the values found in the analyses of NN data. In fact, if it is possible to measure the differential cross section for $\overline{p}p \to \overline{n}n$ with the accuracy stated by Bradamante (private communication, see also Ref. [32]), the charge-exchange reaction will be an even more competitive place to study the isovector-meson coupling constants.

We want to stress that the still popular (but now rather outdated) few-parameter optical-potential models can in no way be compared to a sophisticated partial-wave analysis. In the first approach at best a crude qualitative description of a limited number of data is possible. No $\chi^2_{\rm min}$ is ever presented. It is easy for us to construct a similar optical-potential model, by supplementing the C-parity-transformed Nijmegen potential [21] by an imaginary potential containing two free parameters. We then find at best $\chi^2_{\rm min} \sim 10^9$ for a database of 3309 observables. This should be compared to $\chi^2_{\rm min} = 3592.5$ reached in our multienergy partial-wave analysis on the same set of data. Of course, our two-parameter model is as bad or as good as any other few-parameter optical-potential model. For instance, for the 1968 prototype Bryan-Phillips model [33] we find a $\chi^2_{\rm min}$ which is even much larger. One can never hope to describe all $\overline{p}p$ scattering data with just 2 or 3 free parameters, when one needs already about 20 free parameters to fit the pp data. In a single-energy $\overline{p}p$ analysis in principle eight times as many phase-shift parameters are required compared to a pp analysis. One should keep in mind that in the past these optical-potential models were never intended for a quantitative comparison to the data. The 1980 Dover-Richard model [34], for instance, served an excellent purpose in examining what could be expected qualitatively when the CERN Low Energy Antiproton Ring (LEAR) would come into operation in 1983. At present, however, this approach seems hardly justified anymore. In our opinion, these naive models that do not fit the presently available data at all are completely inadequate to address in a reliable manner issues like the value of the pion-nucleon coupling constant, as was attempted very recently in Ref. [19]. This can be seen, for instance, from the bad fit in Ref. [19] to very recent accurate charge-exchange analyzing-power data from LEAR [35]. After almost 10 years of data-taking at LEAR, it unfortunately still is a common practice to compare the data to the "predictions" of these museum models and then draw strong conclusions about the physics behind these models from such a comparison.

VIII. CONCLUSIONS

To summarize, we firmly believe that the value of the pion-nucleon constant found in the Nijmegen partial-wave analyses of pp, np, and $\overline{p}p$ scattering data is essentially correct and free of significant systematic errors. An excellent $\chi^2_{\rm min}$ is reached in all cases, reflecting both the statistical consistency of the data sets and the quality of the analyses. The specific method of analysis allows the extraction of the coupling constant at the pion pole from the asymptotic pion-exchange potential and ensures a clean separation from short-range form-factor effects and heavy- or multi-meson-exchange forces. We stress that in all cases we have also determined the mass of the exchanged pion and always found agreement with the experimental values. For instance, in the combined analysis of pp and np data [2] it was found that $m_{\pi^0} = 135.6(1.3)$ MeV and $m_{\pi^\pm} = 139.4(1.0)$ MeV. This success is a very strong argument against the presence of significant systematic errors, such as form-factor effects. Furthermore, there is consistency in the results from different analyses as well as agreement with the value $f_c^2 = 0.0735(15)$ found by Arndt and co-workers in their latest VPI&SU analysis of $\pi^\pm p$ scattering data [4,36,37].

We pointed out before [3] that the Goldberger-Treiman relation [38] also favors a low pion-nucleon coupling constant. Our present results indicate a need for reconsideration of calculations on the so-called Goldberger-Treiman discrepancy (see, e.g., Ref. [16]). Recently, Workman, Arndt, and Pavan [39] showed that the Goldberger-Miyazawa-Oehme sum rule [40] provides rather model-independent evidence for a low coupling constant as well.

In a recent study of the deuteron properties [17] it was shown that a low value for the pion-nucleon coupling constant implies that the value for $\kappa_\rho \equiv f_{NN\rho}/g_{NN\rho} = 6.6$ as determined in the Karlsruhe-Helsinki analyses of $\pi^{\pm}p$ scattering data [10] must be wrong. It was further shown that the preferred value for κ_ρ is in agreement with the value $\kappa_\rho = 4.2$ as found in the 1978 Nijmegen soft-core nucleon-nucleon potential [21] and with the value $\kappa_\rho = 3.7$ which follows from vector-meson dominance of nucleon electromagnetic form factors.

We feel that especially the value for the $pp\pi^0$ coupling constant as determined in the pp analysis is compelling evidence for a low pion-nucleon coupling constant. With the exception of the ${}^1S_0$ wave, the $pp\pi^0$ coupling constant can be determined from all parametrized partial waves, all values being consistent. Given the $pp\pi^0$ coupling constant, it seems to us that claims for a high $np\pi^{\pm}$ coupling constant are untenable, since in that case not only three independent recent determinations of this coupling constant must be wrong, but one also has to cope with a large breaking of charge independence, which is theoretically very difficult to accommodate [3]. Therefore, we strongly recommend that in future work on nucleon-nucleon scattering the value $f^2_{NN\pi} = 0.0745$ at the pion pole is taken as a starting point from which further consequences can be discussed.

ACKNOWLEDGMENTS

Discussions with Prof. D. Bugg, Dr. Th. Rijken, and M. Rentmeester are gratefully acknowledged. Part of this work was included in the research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM) with financial support from the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

REFERENCES

[1] J.R. Bergervoet, P.C. van Campen, R.A.M. Klomp, J.-L. de Kok, T.A. Rijken, V.G.J. Stoks, and J.J. de Swart, Phys. Rev. C **41**, 1435 (1990).

[2] R.A.M. Klomp, V.G.J. Stoks, and J.J. de Swart, Phys. Rev. C **44**, R1258 (1991); V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, and J.J. de Swart, Phys. Rev. C **48**, 792 (1993).

[3] R.G.E. Timmermans, Th.A. Rijken, and J.J. de Swart, Phys. Rev. Lett. **67**, 1074 (1991), and in preparation.

[4] R.A. Arndt, Z. Li, L.D. Roper, and R.L. Workman, Phys. Rev. Lett. **65**, 157 (1990).

[5] J.J. de Swart, T.A. Rijken, J.R.M. Bergervoet, P.C.M. van Campen, W.M.G. Derks, and W.A.M. van der Sanden, in *Proceedings of the European Workshop on Few-Body Physics*, Rome, Italy, 1986, Few-Body Systems, Suppl. **1** (Springer Verlag, Wien, 1986), p. 1.

[6] J.R. Bergervoet, P.C. van Campen, T.A. Rijken, and J.J. de Swart, Phys. Rev. Lett. **59**, 2255 (1987).

[7] T.A. Rijken, V.G.J. Stoks, R.A.M. Klomp, J.-L. de Kok, and J.J. de Swart, in *Proceedings of the XIIth International Conference on Few-Body Problems in Physics*, Vancouver, Canada, 1989 (North-Holland, Amsterdam, 1990); Nucl. Phys. **508**, 173c (1990).

[8] J.J. de Swart, R. Klomp, R. Timmermans, and Th.A. Rijken, in *Proceedings of the International Workshop on Quark-Gluon Structure of Hadrons and Nuclei*, Shanghai, China, 1990, edited by L.S. Kisslinger and Xijun Qiu, (International Academic Publishers, Beijing, 1990), p. 112.

[9] J.J. de Swart, R.A.M. Klomp, T.A. Rijken, and V.G.J. Stoks, in *Proceedings of the XIIIth European Conference on Few-Body Problems*, Elba, Italy, 1991, Few-Body Systems, Suppl. **6** (Springer Verlag, Berlin, 1992), p. 105.

[10] G. Höhler and E. Pietarinen, Nucl. Phys. **B95**, 210 (1975).

[11] R. Koch and E. Pietarinen, Nucl. Phys. **A336**, 331 (1980).

[12] O. Dumbrajs, R. Koch, H. Pilkuhn, G.C. Oades, H. Behrens, J.J. de Swart, and P. Kroll, Nucl. Phys. **B216**, 277 (1983).

[13] A.W. Thomas and K. Holinde, Phys. Rev. Lett. **63**, 2025 (1989).

[14] F. Gross, J.W. Van Orden, and K. Holinde, Phys. Rev. C **41**, R1909 (1990).

[15] K. Holinde and A.W. Thomas, Phys. Rev. C **42**, R1195 (1990).

[16] S.A. Coon and M.D. Scadron, Phys. Rev. C **42**, 2256 (1990).

[17] R. Machleidt and F. Sammarucca, Phys. Rev. Lett. **66**, 564 (1991).

[18] T.E.O. Ericson, in *Proceedings of the XIIIth Conference on Few-Body Problems in Physics*, Adelaide, Australia, 1992 (North-Holland, Amsterdam, 1992); Nucl. Phys. **A543**, 409c (1992).

[19] J. Haidenbauer, K. Holinde, and A.W. Thomas, Phys. Rev. C **45**, 952 (1992).

[20] J.R. Bergervoet, P.C. van Campen, W.A. van der Sanden, and J.J. de Swart, Phys. Rev. C **38**, 15 (1988).

[21] M.M. Nagels, T.A. Rijken, and J.J. de Swart, Phys. Rev. D **17**, 768 (1978).

[22] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).

[23] J.E. Brolley, Jr., J.D. Seagrave, and J.G. Beery, Phys. Rev. **135**, B1119 (1964); M.L. Gursky and L. Heller, *ibid.* **136**, B1693 (1964).

[24] Ch. Thomann, J.E. Benn, and S. Münch, Nucl. Phys. **303**, 457 (1978).
[25] M.D. Barker, P.C. Colby, W. Haeberli, and P. Signell, Phys. Rev. Lett. **48**, 918 (1982); **49**, 1056(E) (1982).
[26] W. Kretschmer, M. Haller, A. Rauscher, R. Schmitt, W. Schuster, W. Grüebler, C. Forstner, V. König, and P.A. Schmelzbach, in Book of Contributions, Eleventh International IUPAP Conference on Few-Body Systems in Particle and Nuclear Physics, Tokyo and Sendai, 1986, edited by T. Sasakawa, K. Nisimura, S. Oryu, and S. Ishikawa (unpublished).
[27] W. Kretschmer, M. Haller, R. Höpfl, S. List, A. Rauscher, W. Schuster, R. Weidmann, W. Grüebler, M. Bittcher, M. Clajus, P. Egun, and P.A. Schmelzbach, in Contributed Papers to the XIIth International Conference on Few-Body Problems in Physics, Vancouver, 1989, edited by B.K. Jennings, TRIUMF Report No. TRI-89-2.
[28] G.F. Chew, Phys. Rev. **112**, 1380 (1958).
[29] B.E. Bonner, J.E. Simmons, C.L. Hollas, C.R. Newsom, P.J. Riley, G. Glass, and Mahavir Jain, Phys. Rev. Lett. **41**, 1200 (1978).
[30] D. Holslin, J. McAninch, P.A. Quin, and W. Haeberli, Phys. Rev. Lett. **61**, 1561 (1988).
[31] D. Bandyopadhyay, R. Abegg, M. Ahmad, J. Birchall, K. Chantziantoniou, C.A. Davis, N.E. Davison, P.P.J. Delheij, P.W. Green, L.G. Greeniaus, D.C. Healey, C. Lapointe, W.J. McDonald, C.A. Miller, G.A. Moss, S.A. Page, W.D. Ramsay, N.L. Rodning, G. Roy, W.T.H. van Oers, G.D. Wait, J.W. Watson, and Y. Ye, Phys. Rev. C **40**, 2684 (1989).
[32] M.P. Macciotta, A. Masoni, G. Puddu, S. Serci, A. Ahmidouch, E. Heer, C. Mascarini, D. Rapin, J. Arvieux, R. Bertini, J.C. Faivre, R.A. Kunne, R. Birsa, F. Bradamante (Spokesman), A. Bressan, S. Dalla Torre-Colautti, M. Giorgi, M. Lamanna, A. Martin, A. Penzo, P. Schiavon, F. Tessarotto, A.M. Zanetti, E. Chiavassa, N. De Marco, A. Musso, and A. Piccotti (The PS199 Collaboration), Proposal to the CERN, Report No. CERN/SPSLC 92-17 (1992).
[33] R.A. Bryan and R.J.N. Phillips, Nucl. Phys. **B5**, 201 (1968); **B7**, 481(E) (1968).
[34] C.B. Dover and J.-M. Richard, Phys. Rev. C **21**, 1466 (1980); **25**, 1952 (1982).
[35] R. Birsa, F. Bradamante, S. Dalla Torre-Colautti, M. Giorgi, M. Lamanna, A. Martin, A. Penzo, P. Schiavon, F. Tessarotto, M.P. Macciotta, A. Masoni, G. Puddu, S. Serci, T. Niinikoski, A. Rijllart, A. Ahmidouch, E. Heer, R. Hess, R.A. Kunne, C. Lechanoine-Le Luc, C. Mascarini, D. Rapin, J. Arvieux, R. Bertini, H. Catz, J.C. Faivre, F. Perrot-Kunne, M. Agnello, F. Iazzi, B. Minetti, T. Bressani, E. Chiavassa, N. De Marco, A. Musso, and A. Piccotti (The PS199 Collaboration), Phys. Lett. B **246**, 267 (1990); **273**, 533 (1991).
[36] R.A. Arndt and R.L. Workman, Phys. Rev. C **43**, 2436 (1991).
[37] R.A. Arndt, Z. Li, L.D. Roper, and R.L. Workman, Phys. Rev. D **44**, 289 (1991).
[38] M.L. Goldberger and S.B. Treiman, Phys. Rev. **110**, 1178 (1958); Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960).
[39] R.L. Workman, R.A. Arndt, and M.M. Pavan, Phys. Rev. Lett. **68**, 1653, (1992); **68**, 2712(E) (1992).
[40] M.L. Goldberger, H. Miyazawa, and R. Oehme, Phys. Rev. **99**, 986 (1955).

TABLES

TABLE I. Recent determinations of the pion-nucleon coupling constants from dispersion-relation (DR) analyses of pion-nucleon scattering data and from partial-wave analyses (PWA) of nucleon-nucleon and antinucleon-nucleon scattering data.

Group	Year	Method	$10^3\, f^2_{pp\pi^0}$	$10^3\, f^2_c$
Karlsruhe-Helsinki [12]	pre-1983	$\pi^{\pm}p$ DR		79(1)
Nijmegen [1]	1987-1990	pp PWA	74.9(0.7)	
VPI&SU [4]	1990	$\pi^{\pm}p$ DR		73.5(1.5)
Nijmegen [2]	1991	combined NN PWA	75.1(0.6)	74.1(0.5)
Nijmegen [3]	1991	$\overline{p}p$ PWA		75.1(1.7)
this work	1992	pp and np PWA	74.5(0.6)	74.8(0.3)

TABLE II. Comparison between the moments of the χ^2-probability distribution expected from theory and those determined in our partial-wave analysis (PWA) of pp data. Tabulated are $\langle\chi^2_{\rm min}\rangle/N_{\rm df}$ and the central moments μ_n for $n = 2, 3, 4$.

	Theory	PWA
$\langle\chi^2_{\rm min}\rangle/N_{\rm df}$	1.000±0.035	1.108
μ_2	1.81±0.12	1.83
μ_3	5.55±0.74	5.40
μ_4	29.8±4.5	27.6

TABLE III. The $pp\pi^0$ coupling constant as a function of the cutoff mass in the exponential form factor. The number of degrees of freedom is 1612.

Λ_0 (MeV)	500.0	750.0	1000.0	1250.0	∞
$10^3\, f^2_{pp\pi^0}$	75.2(0.5)	75.0(0.5)	75.0(0.5)	75.0(0.5)	75.0(0.5)
$\chi^2_{\rm min}$	1786.3	1786.4	1786.4	1786.4	1786.4

TABLE IV. χ^2 results for four values of the $pp\pi^0$ coupling constant for the different types of observables in the analysis of pp scattering data. The numbers in the two last columns are obtained by fitting a parabola to the numbers in the four preceding columns. The number of degrees of freedom is 1613 for each value of $f^2_{pp\pi^0}$.

Type	N_{dat}	$10^3\, f^2_{pp\pi^0} = 73$	75	77	79	χ^2(min)	$10^3\, f^2_{pp\pi^0}$(min)
$\mathrm{d}\sigma/\mathrm{d}\Omega$	821	838.7	825.7	823.0	830.8	822.7	76.5(0.9)
A_y	558	585.2	580.0	585.7	602.2	580.0	75.0(0.9)
A_{ii},C_{nn}	66	52.8	55.5	60.0	66.3	51.9	71.0(2.1)
D,D_t	97	105.2	107.5	112.0	118.9	104.9	72.0(1.9)
R,R',A,A'	209	194.5	193.1	194.9	199.8	193.1	74.9(1.6)
rest	36	24.8	24.7	24.6	24.5		
all	1787	1801.2	1786.4	1800.2	1842.4	1786.4	75.0(0.5)

TABLE V. χ^2 results for four values of the $np\pi^\pm$ coupling constant for the different types of observables in the analysis of np scattering data. The numbers in the two last columns are obtained by fitting a parabola to the numbers in the four preceding columns. The $NN\pi^0$ coupling constants are taken to be 0.075. The number of degrees of freedom is 2331 for each value of f^2_c.

Type	N_{dat}	$10^3\, f^2_c = 73$	75	77	79	χ^2(min)	$10^3\, f^2_c$(min)
σ_{tot},$\Delta\sigma_{\mathrm{L}}$,$\Delta\sigma_{\mathrm{T}}$	252	232.9	229.7	232.4	242.4	229.5	75.1(1.1)
$\mathrm{d}\sigma/\mathrm{d}\Omega$	1350	1379.0	1364.2	1367.9	1391.8	1363.2	75.6(0.6)
A_y	738	737.7	720.3	745.9	830.4	717.8	74.8(0.4)
A_{yy},A_{zz}	86	77.2	72.6	91.2	136.0	71.2	74.4(0.6)
D_t	43	42.8	39.8	42.0	51.6	39.5	75.1(1.1)
R_t,R'_t,A_t,A'_t	43	54.6	58.6	68.5	88.4	54.7	73.1(1.0)
all	2512	2524.3	2485.3	2547.9	2740.7	2480.4	74.8(0.3)

TABLE VI. Results for the pion-nucleon coupling constant introduced separately in each parametrized partial wave in the analysis of pp scattering data. For fitting the coupling constant in non-S waves the coupling in the $^1S_0$ wave is fixed at 0.075. The number of degrees of freedom is 1611 in each case.

Partial wave	$10^3\, f^2$(wave)	$10^3\, f^2$(rest)	χ^2_{min}
$^1S_0$	79.7(1.9)	74.5(0.6)	1779.8
$^1D_2$	74.6(0.8)	74.6(0.6)	1786.0
$^1G_4$	74.6(2.1)	74.6(0.6)	1786.0
$^3P_0$	72.7(1.7)	74.8(0.6)	1784.6
$^3P_1$	74.9(0.7)	74.3(0.8)	1785.6
$^3P_2$–$^3F_2$	74.7(0.8)	74.6(0.6)	1786.0
$^3F_3$	73.3(1.3)	74.8(0.6)	1784.8
$^3F_4$–$^3H_4$	75.1(0.9)	74.5(0.6)	1785.6

TABLE VII. Values for the $pp\pi^0$ coupling constant determined in a partial-wave analysis pp scattering data within different energy ranges in MeV.

$T_{\rm lab}$ range	$N_{\rm df}$	$10^3\ f^2_{pp\pi^0}$	$\chi^2_{\rm min}$	$\chi^2_{\rm min}/N_{\rm df}$
0-350	1612	75.0(0.5)	1786.4	1.108
3-350	1435	74.3(0.6)	1596.6	1.113
10-350	1312	73.7(0.7)	1488.8	1.135
30-350	1237	74.2(0.8)	1397.6	1.130
0-280	1243	75.5(0.6)	1389.3	1.118
3-280	1066	74.5(0.7)	1189.7	1.116

TABLE VIII. The charged-pion coupling constant f_c^2 determined from the backward np cross sections of Bonner *et al.* [29] in the np partial-wave analysis. The numbers are obtained by fitting a parabola through the $\chi^2_{\rm min}$ results for four different coupling constants. For $T_{\rm lab} = 265.8$ MeV these four numbers were consistent with a straight line.

$T_{\rm lab}$ (MeV)	$N_{\rm dat}$	χ^2(min)	norm	$10^3\ f_c^2$(min)
162.0	43	60.0	1.092(7)	69.9(3.0)
177.9	44	44.0	1.083(7)	70.2(3.1)
211.5	43	31.0	1.063(7)	72.8(3.3)
229.1	49	62.3	1.058(7)	69.5(3.6)
247.2	53	38.5	1.042(7)	69.7(9.3)
265.8	63	—	1.028(6)	—
284.8	73	79.7	1.052(5)	75.3(3.5)
304.2	80	79.9	1.003(4)	74.6(3.4)
324.1	82	91.7	1.057(5)	78.0(4.3)
344.3	80	74.6	1.035(5)	74.8(3.9)