THEF-NIJM 92.06

Comparison of potential models with the pp scattering data below 350 MeV*

Vincent Stoks and J.J. de Swart
Institute for Theoretical Physics, University of Nijmegen, Nijmegen, The Netherlands

Abstract

We calculate the χ^2 of various NN potential models with respect to the pp scattering data. We find that only the potential models which were explicitly fitted to the pp data give a reasonable description of these data. Most models give a pretty large χ^2 on the very low-energy pp data, due to incorrect $^1S_0$ phase shifts.

13.75.Cs, 12.40.Qq, 21.30.+y

*Published in Phys. Rev. C **47**, 512 (1993).

I. INTRODUCTION

The NN interaction has been the subject of investigation for more than half a century, which is reflected in the numerous potential models that have appeared in the literature. When discussing potential models it has been convenient to divide the range of the interaction into three regions [1]: a short-range, an intermediate-range, and a long-range region. It soon became clear that in the long-range region ($r \gtrsim$ fm) the NN potential is given by one-pion exchange (OPE). The short-range region ($r \lesssim 1$ fm) is generally treated phenomenologically, where in some models a form factor is introduced to make the potential regular at the origin, whereas in other models a hard core is used. For the description of the intermediate-range region ($1 \lesssim r \lesssim 2$ fm) the first logical approach was to include the contributions from two-pion exchange. Examples of the two-pion-exchange (TPE) potentials of the early 1950s are the Taketani-Machida-Ohnuma [2] and the Brueckner-Watson [3] models. However, these TPE models did not give a satisfactory description of the NN scattering data, mainly due to a lack of sufficient spin-orbit force. The necessity of a spin-orbit force was hinted at by Gammel, Christian, and Thaler [4] when they failed to find a phenomenological velocity-independent potential model consisting of central and tensor parts which was able to fit all of the data available at that time.

The breakthrough came in 1957 with the simultaneous construction of the purely phenomenological Gammel-Thaler potential [5] and the semiphenomenological Signell-Marshak potential [6], both models introducing phenomenological spin-orbit potentials. The Gammel-Thaler model gave a good fit to the scattering data up to 310 MeV. The Signell-Marshak model, consisting of the TPE Gartenhaus potential [7] together with a phenomenological spin-orbit force, was successful up to 150 MeV. These potential models were soon improved upon, where we mention the hard-core Hamada-Johnston [8] and Yale [9] models, and the various hard-core and soft-core models constructed by Reid [10].

With the discovery of the heavy mesons in the early 1960s a different approach was launched. In this approach the NN interaction is written as a sum over one-boson-exchange (OBE) potentials. The OBE models are very successful in that the vector-meson and scalar-meson exchanges supply the required spin-orbit forces. However, the short-range part still has to be described phenomenologically. Examples of some of the early OBE models are given in Refs. [11–13], whereas some of the more recent models will be discussed in Sec. III.

Most of the early models gave a for that time reasonable description of the NN scattering data. However, over the years the concept over what is accepted as reasonable has changed. At present, a model which describes the pp scattering data will only be called reasonably good if it has $\chi^2/N_{\rm data} \lesssim 2$, where $N_{\rm data}$ denotes the number of pp scattering data. In Sec. III we show that only few of the models that have appeared in the literature still live up to this criterion of having $\chi^2/N_{\rm data} \lesssim 2$. On the other hand, with the fast computers and the accurate partial-wave analyses presently available, it has now become rather easy to construct phenomenological potential models which have the excellent quality of $\chi^2/N_{\rm data} \approx 1$. Such a potential model for the NN interaction with $\chi^2/N_{\rm data} \approx 1$ is the best one can hope to achieve. The quality of the fit of such a potential model can compete with the fit of a multienergy partial-wave analysis, and in a sense provides another form of partial-wave analysis. Several of such high-quality potential models will be presented in a forthcoming paper [14].

In this paper we will use the Nijmegen representation of the χ^2 hypersurface of the *pp* scattering data to calculate the χ^2 with respect to the *pp* scattering data for a number of different *NN* potential models. The Nijmegen representation is obtained from the single-energy analyses of the Nijmegen *pp* partial-wave analysis [15]. It provides an adequate representation of the scattering data.

We want to stress that such a comparison between potential models can only be done fairly when for *all* potential models the χ^2 with respect to the data is calculated correctly and in the same way. This in order to avoid any ambiguities regarding whether or not any specific electromagnetic corrections are accounted for. For that purpose we calculate the phase shifts for each potential model by solving the radial Schrödinger equation in the presence of the Coulomb interaction. For the $^1S_0$ phase shift the vacuum polarization potential and the two-photon-exchange modification to the Coulomb potential are also included (for details see, e.g., Ref. [15]). We make one exception, however: For the momentum-space Bonn potential [16] we use the phase shifts as provided by one of the authors. The reason is that a proper treatment of the electromagnetic interaction is very important if the *pp* scattering data are to be described properly (see, e.g., Ref. [17]), and we do not have the software programs for handling electromagnetic effects in momentum-space calculations. For $T_{\rm lab} < 30$ MeV, the $^1S_0$ phase shifts of the Bonn potential were adjusted by us such as to account for vacuum polarization and modified Coulomb effects. The proper way of how to make these adjustments is described in Ref. [18].

Another important aspect for a fair comparison between various potential models is that they should all be compared with the same database. We use the database of the Nijmegen partial-wave analysis [15]. This database is as complete as possible and has been scrutinized very carefully where all bad data or groups of bad data have been removed.

In Sec. II we explain how the Nijmegen representation of the χ^2 hypersurface of the *pp* scattering data can be used to calculate χ^2 with respect to these data for any particular potential model. The advantage of using the Nijmegen representation is that the phase shifts of a model that is to be investigated need only be calculated at a small number of energies, namely at the 10 energies of the single-energy analyses of the Nijmegen partial-wave analysis [15]. This small number (10) should be compared with the much larger number of energies at which experimental data have been measured (about 200). Therefore, using the Nijmegen representation of the *pp* data rather than the data themselves saves a lot of computer time, while the results are almost always sufficient for their purpose.

In Sec. III we present our results for some of the more well-known potential models that have appeared in the literature, most of which are still commonly used in other calculations involving the *NN* interaction. Examples of such calculations are *pp* bremsstrahlung, three-nucleon elastic scattering, few-nucleon bound-state calculations, and nuclear matter calculations. The list of potential models that we discuss is not complete. For example, momentum-space potentials will not be considered here, due to reasons already discussed above. The only exception is the momentum-space Bonn potential, since it is widely used and a specific *pp* version has been published [16]. Four of the more recent potential models which give a good description of the *pp* data are then studied in more detail.

II. REPRESENTATION OF χ^2 HYPERSURFACE

The details of the Nijmegen way of analyzing the *pp* scattering data are extensively discussed elsewhere [18,15], and will not be repeated here. Here we want to stress that the Nijmegen partial-wave analysis is not particularly important for the present calculations, because we compare various potential models with the experimental data. What is important is that we can use the same computer programs as used in the Nijmegen partial-wave analyses to compute properly the *pp* scattering amplitudes. In doing the comparison we make use of a representation of the χ^2 hypersurface of the *pp* scattering data, which was produced by the Nijmegen single-energy partial-wave analyses of the *pp* scattering data. This χ^2 hypersurface is somewhat dependent on the Nijmegen multienergy (m.e.) analysis, but the crucial point is that it provides an excellent representation of the *pp* data, as will be demonstrated below.

The Nijmegen representation of the χ^2 hypersurface is obtained as follows. In the single-energy (s.e.) analyses the 1787 *pp* scattering data below $T_{\rm lab} = 350$ MeV are clustered at 10 energies from 382.54 keV (the interference minimum) up to 320 MeV. The total $\chi^2_{\rm se}$ of all 10 s.e. analyses amounts to $\chi^2_{\rm se} = 1676.3$. These s.e. analyses provide us with 10 error matrices E_n. The error matrix is the inverse of half the second-derivative matrix of the χ^2 hypersurface with respect to the phase shifts within a particular energy bin. The Nijmegen representation of the χ^2 hypersurface of the *pp* scattering data consists of the number $\chi^2_{\rm se} = 1676.3$ and the 10 error matrices E_n at the 10 different energies. It provides a good representation of the scattering data within each energy bin. However, this representation is not exact. First of all, the higher partial-wave phase shifts are fixed at their m.e. values. Furthermore, the data have been clustered at some central energy within an energy bin using the results of the m.e. fit, and next to that we have used the approximation that the χ^2 hypersurface is quadratic in the neighborhood of the minimum.

The representation of the data can be used as follows. The phase shifts of a model are calculated at the 10 central energies of the s.e. analyses. Denoting by $\mathbf{d}_n$ the deviation of the phase-shift predictions of the model from the s.e. phase shifts in the nth energy bin, the χ^2 of the model can be written as a sum of the s.e. contributions $\chi^2_{{\rm se},n}$ and the contributions from the inverse error matrices $\chi^2_{{\rm rep},n}$, i.e.,

$$\begin{aligned} \chi^2(\text{model}) &= \sum_n \left(\chi^2_{{\rm se},n} + \chi^2_{{\rm rep},n}\right) \\ &= \chi^2_{\rm se} + \sum_n \mathbf{d}_n^T E_n^{-1} \mathbf{d}_n \ . \end{aligned} \tag{1}$$

In using Eq. (1), we account for the correlations between the different phase shifts, because this information is stored in the error matrices.

In order to investigate the quality of the Nijmegen representation of the χ^2 hypersurface, we tested it in several ways. First, we used our m.e. phase shifts as model phase shifts and calculated the corresponding χ^2 contribution. These $\chi^2_{{\rm rep},n}$ contributions of the m.e. phase shifts are listed in the last column of Table I. They should be added to the $\chi^2_{{\rm se},n}$ of the s.e. analyses given in the next to last column of Table I to give the total χ^2 within each energy bin. For all 10 energy bins we find that the agreement of $\chi^2_{{\rm se},n} + \chi^2_{{\rm rep},n}$ with the corresponding $\chi^2_{{\rm me},n}$ of the m.e. analysis is very good. The difference between the total $\chi^2(\text{model}) = 1786.8$ given by Eq. (1) and the $\chi^2_{\rm me} = 1786.4$ reached in our m.e. analysis is only 0.4. This means that the χ^2 as calculated directly on the data and the χ^2 calculated via Eq. (1) only differ by 0.02%. It shows that the approximation, that the χ^2 hypersurface of the s.e. analyses is quadratic up to the minimum $\chi^2_{\rm me}$ of the m.e. analysis, is actually very good. For completeness, we have also listed in Table I the number of scattering data $N_{\rm data}$ within each energy bin, which is the number of scattering observables plus the number of normalizations with an experimental error. The information presented in Table I forms the basis for our test of the quality of various potential models to be discussed in the sections below.

As a second test for the quality of the Nijmegen representation of the χ^2 hypersurface, we used the Nijmegen soft-core potential (Nijm78) [19] to compare the χ^2(model) obtained using Eq. (1) with the χ^2(data) obtained from a direct comparison with the data. We are now farther away from the minimum χ^2, so we expect that the χ^2 hypersurface will no longer be quadratic. As a consequence, the result for χ^2(model) using Eq. (1) will be less accurate. For the Nijm78 model we find $\chi^2(\text{data}) = 3387.5$ and $\chi^2(\text{model}) = 3462.8$. The difference of 75.3 is now about 2%, which is sufficiently small. When we are farther away from the minimum χ^2, this difference will be even larger, but Eq. (1) is still correct within the order of magnitude. This allows us to use Eq. (1) to make statements regarding the quality of some potential model.

The difference between χ^2(model) and χ^2(data) can be understood as follows. For the calculation of the χ^2 via Eq. (1) only the lower partial waves of the potential model up to $J = 4$ are used. All higher partial waves are taken from the m.e. analysis. Also all normalization constants are fixed at the values as obtained in the s.e. analyses. On the other hand, in the direct comparison with the data for the calculation of χ^2(data) all partial waves of the potential model up to $J = 8$ are used. Furthermore, all normalization constants are automatically adjusted such as to give the best agreement with the data. This means that χ^2(data) is smaller than χ^2(model).

III. COMPARISON OF *NN* POTENTIAL MODELS

The Nijmegen representation of the χ^2 hypersurface of the *pp* scattering data can be used to test the quality of an *NN* potential model. For that purpose we calculate the χ^2(model) using Eq. (1). The results for a number of models are shown in Table II, where we present the $\chi^2_{{\rm rep},n}$ contributions. To obtain the total χ^2 within a particular energy bin, one should add the $\chi^2_{{\rm se},n}$ as given in Table I. As discussed in the previous section, the larger entries ($\gtrsim 1000$) in Table II are inaccurate, but they still correctly represent a large $\chi^2_{{\rm rep},n}$. Before discussing each of the models in more detail we first note some general features.

For some models the 50.0 MeV energy bin gives a relatively large contribution to χ^2. This is partially due to a recently published very accurate analyzing power experiment at 50.04 MeV [20]. The accuracy of this 50.04 MeV experiment means that especially the triplet P phase shifts at 50 MeV are now very accurately known. Because the triplet P phase shifts of the various potential models are not always in too good an agreement with these new accurate values, they will produce a large contribution to χ^2.

Many models give a relatively poor or even very bad description of the low-energy data.

The reason is that the $^1S_0$ phase shifts at 382.54 keV and 1.0 MeV are very accurately known. So when the $^1S_0$ phase shift of a potential model is a little bit off, the χ^2 contribution will already be enormous. However, there still remains the fact that most of these models claim to fit the scattering length and the effective range pretty well, so one would expect that the χ^2 contribution of the low-energy data should not be too large (or as large as it is for some of the models). In order to investigate whether the high χ^2 value for some of the models is only due to an erroneous $^1S_0$ phase shift at these lowest energies, we also compared the various models in a slightly different energy range. In Table II we therefore also give the χ^2 contribution on the 2–350 MeV energy range, which contains 1590 *pp* scattering data. If now the quality of a potential model is bad, it is not totally due to a slightly incorrect $^1S_0$ phase shift at low energies.

In the following we chronologically list some of the better-known *NN* potentials that have appeared in the literature, most of which are still commonly used in other calculations involving the *NN* interaction. The older phenomenological and TPE models of the early 1950s are not included.

HJ62: Hamada-Johnston potential [8]. The energy-independent Hamada-Johnston potential is a hard-core potential. It includes the OPE potential and a phenomenological part consisting of central, tensor, spin-orbit, and quadratic spin-orbit terms. At the time of its presentation it provided a good representation of the *pp* and *np* scattering data below 315 MeV. The 28 model parameters were fitted to the Yale phase shifts [21]. In 1970, Humberston and Wallace [22] introduced an additional parameter to improve the deuteron properties of the model. From Table II we see that the data in the 50 MeV bin and the very low-energy data give a large contribution to χ^2. This is not surprising in view of the much higher accuracy with which these phase shifts are known nowadays. For the description of the remaining 1347 data the old HJ62 model is still surprisingly good, but it is only sparsely used anymore.

Reid68: Reid soft-core potential [10]. In the paper by Reid, a number of different hard-core and soft-core potentials are presented. In these models each partial wave with total angular momentum $J \leq 2$ is parametrized phenomenologically in terms of Yukawa functions of multiples of the pion mass. The OPE part itself is explicitly included. In some partial waves an explicit distinction between central, tensor, and spin-orbit parts is used. A shortcoming of the soft-core versions is that the potentials are not regular in the origin, but still have an r^{-1} singularity. The parameters were fitted to the *pp* and *np* phase shifts of the Yale [21] and early Livermore [23] analyses. In 1981, Day [24] extended the potential for partial waves with $J > 2$. Also for this model the description of the very low-energy data is a bit off. The remaining data are described pretty well.

TRS75: Super-soft-core potential [25]. This $pp + np$ potential contains the π-, ρ-, and ω-exchange contributions where the coupling constants are taken from other sources. The other important intermediate-range contributions to the *NN* force are parametrized phenomenologically through OBE potential functions with 32 free ranges and amplitudes. The potential contributions are regularized at the origin by steplike functions which also serve to construct the short-range phenomenological cores, whence the name super-soft-core potential. The model is an improved version of an earlier super-soft-core model by the same group [26]. The model is very good for the 0.5–35 MeV energy region, whereas for higher energies the description rapidly becomes worse. Also the very low-energy data of the 0.382 54

MeV bin are not described very well, even though the pion and nucleon masses used in the pp and np potentials were especially adjusted so as to account for the difference between the pp and np $^1S_0$ phase shifts.

OBEG75: Funabashi potentials [27]. These potentials are constructed from the π, η, ρ, and ω OBE potentials. Also included are the contributions of two scalar mesons δ and σ, the masses of which were fitted to the scattering data. The potential contains the standard OBE part and a retardation part. The off-energy-shell effects coming from the retardation, albeit of little importance to the two-nucleon system, are expected to play an important role in many-nucleon systems. The potentials were evaluated in coordinate space for the sake of future investigations regarding the influence of off-energy-shell effects in finite nuclei. The various treatments of the inner region in these potentials are a hard core, a Gaussian soft core, and a velocity-dependent core. In each case an attractive spin-orbit core is included to improve the triplet P phase shifts. Furthermore, all potentials are regularized by means of a steplike cutoff function. The results presented here refer to the Gaussian soft-core potential, denoted by OBEG. From Table II we see that the overall behavior of this model is rather bad.

Nijm78: Nijmegen potential [19]. The mesons which give rise to the meson-exchange forces of the Nijmegen potential are the non-strange mesons of the pseudoscalar, vector, and scalar nonets. They can be identified with the dominant parts of the nine lowest-lying meson trajectories in the complex J plane. The model also includes the dominant $J = 0$ parts of the Pomeron, f, f', A_2 trajectories, which essentially lead to repulsive central Gaussian potentials. The inner region is adjusted with an exponential form factor. The 13 model parameters were fitted to the phase-shift error matrices of the 1969 Livermore analyses [28]. These model parameters can be checked with meson-nucleon coupling constants and cutoffs obtained from other sources. An important feature of this model is that there exist exactly equivalent versions of this potential for use in coordinate space or momentum space. Using the same set of parameters, both the coordinate-space and momentum-space versions produce the same phase shifts at all energies (see also Ref. [29]). The overall description of the pp data is good; only the χ^2 contribution to the 50 MeV bin is a little bit high.

Paris80: Parametrized Paris potential [30]. The original Paris potential [31] was obtained by calculating the TPE contributions to the NN forces from the pion-nucleon phase shifts and from the pion-pion interaction using dispersion relations. The π- and ω-exchanges were then also explicitly included. A balanced fitting to the phase-shift error matrices of the 1969 Livermore analysis [28] and to the pp and np scattering data themselves required a total of 12 parameters. In 1980 a parametrized version [30] consisting of a set of Yukawa functions provided a phenomenological representation of the Paris potential. Except for the very low-energy region, this model gives a good description of the pp scattering data, where also the description of the 50 MeV bin is not too bad.

Urb81: Urbana potential [32]. The Urbana potential is a purely phenomenological v_{14} potential where 14 represents the number of different potential types (central, spin-spin, tensor, spin-orbit, quadratic spin-orbit, centrifugal, centrifugal spin-spin, and an overall isospin dependence), rather than the number of phenomenological parameters. Next to OPE and a 14-parameter representation of TPE, the short-range part is represented by two Woods-Saxon potentials using a total of 20 parameters. All potential types are regularized by means of a cutoff function. The parameters were fitted to the np phase shifts of the

1977 energy-dependent phase-shift analysis by Arndt *et al.* [33]. The 50 MeV bin and the 150 MeV bin give relatively large contributions to χ^2. Also here the description of the very low-energy data is a bit off.

Arg84: Argonne potential [34]. The Argonne potential is similar to the Urbana potential. It was fitted to a 1981 phase-shift analysis of Arndt and Roper (an update of the analysis of Ref. [33]) for the *np* scattering data in the 25–400 MeV energy range. Next to OPE and a 14-parameter representation of TPE, the short-range part of the Argonne potential is represented by a Woods-Saxon potential using 16 parameters. The main reason for constructing this new v_{14} model was to have a phase-equivalent standard of comparison for the v_{28} model, which includes operators which represent all possible processes with $N\Delta\pi$ or $\Delta\Delta\pi$ vertices. The description of the very low-energy data is bad, which is not surprising in view of the fact that the model was fitted to the *np* data with $T_{\rm lab} > 25$ MeV. Also the 50 MeV and 150 MeV bin are not described too well. Still, in the 25–350 MeV region the Argonne model provides an improvement over the Urbana model.

Bonn87: Coordinate-space Bonn potential [35]. The full Bonn potential is an *NN* momentum-space potential. Next to π, ω, and δ exchanges, the model also contains an explicit determination of the TPE contribution, including ρ exchange and virtual isobar excitation. The combined $\pi\rho$-exchange diagrams are included as well. The coordinate-space version is obtained from a simple parametrization of the full model by six OBE terms (three pairs of pseudoscalar, vector, and scalar mesons, respectively). The potentials are regularized at the origin by means of dipole form factor functions. In this paper we use the coordinate-space OBE version in order to be able to include the electromagnetic interaction. We find that the description of the very low-energy data is very poor, while for the higher energies the description is not very good either. It demonstrates that the coordinate-space Bonn potential is only a poor substitute for the full Bonn model, which is claimed to give a reasonably good description of the scattering data [35]. There have also appeared a number of other adjusted OBE coordinate-space versions [36], Bonn A and Bonn B, which also give a very poor description of the *pp* data ($\chi^2/N_{\rm data} > 8$ in the 2–350 MeV energy range).

Bonn89: Updated Bonn potential [16]. This potential model is an adaptation of the full momentum-space Bonn potential [35] to the *pp* scattering data. This was done by including the Coulomb interaction in the momentum-space calculations and making small adjustments to guarantee a reasonable confrontation with the *pp* data. The scalar-meson coupling constants were changed in such a way as to explicitly fit the *pp* $^1S_0$ phase shift below 2 MeV (as suggested in Ref. [18]), while keeping the deuteron properties in the *np* $^3S_1$-$^3D_1$ channel at the values of the original full Bonn model. Indeed, now the overall description of the *pp* data is good.

NijmRdl: Reidlike Nijmegen potential [14]. This model is an example of a new class of high-quality potential models, which are almost as good as the Nijmegen m.e. *pp* partial-wave analysis [15]. The model is a Reidlike version of the Nijm78 model [19] in the sense that we define a potential form for each partial wave separately. For each partial wave we only need to adjust a few parameters of the original Nijm78 model in order to arrive at a semiphenomenological potential model which gives an excellent description of the scattering data. In that sense this Reidlike potential model is another form of m.e. partial-wave analysis. Preliminary versions of this Reidlike Nijmegen model were presented in Refs. [37,38]. Comparison of the last column of Table I with the last column of Table II clearly demonstrates the excellent quality of this potential model.

Summarizing, only the Nijm78 and Bonn89 potentials give a rather good description of the *pp* scattering data over the entire 0–350 MeV energy range. When we do not include the very low-energy (0–2 MeV) data, also the Reid68 and Paris80 models are reasonably good, as can be read off from the last line of Table II. These four models are then roughly of the same quality, i.e., $\chi^2/N_{\rm data} \approx 2$. However, these "good" models are still not as good as the NijmRdl model which has $\chi^2/N_{\rm data} \approx 1$, which is very close to the *pp* partial-wave analysis. We therefore believe that one has to be very careful in drawing conclusions regarding the importance or unimportance of, e.g., three-nucleon forces in many-body calculations, when these conclusions are only based on calculations where the *NN* interaction is represented by an *NN* potential model which cannot even adequately describe the two-nucleon scattering data.

In the remaining part of this section we will focus on the four recent potential models which give a good (Nijm78, Paris80, Bonn89) or excellent (NijmRdl) description of the *pp* scattering data. It is very instructive to see how the different partial waves contribute to the total χ^2. For that purpose we start with the m.e. phase shifts and substitute the $^1S_0$ phase shifts of the different potential models. We then calculate the difference $\Delta\chi^2$ between this new $\chi^2_{\rm rep}$ and the χ^2 of the m.e. analysis. This is repeated for the other lower partial-wave phase shifts up to $J = 3$ as well. In this way we can judge the quality of the various partial waves of these models. The six separate contributions can be summed and compared with the χ^2 as obtained when we take all these potential phase shifts simultaneously as given in Table II, which gives some measure for the importance of the correlation between the different partial waves. The results are presented in Table III. The agreement between the sum of the $\Delta\chi^2$ contributions substituting the potential model phase shifts one at a time, and the $\Delta\chi^2$ contribution using all potential model phase shifts simultaneously is not very good, the result for the Bonn89 potential being the worst. This means that the correlation between the various phase shifts in the Bonn89 potential is very important.

The $^3P_1$ phase shift of the Nijm78 potential is found to be very close to the m.e. value. For the other phase shifts, the $\Delta\chi^2$ contributions are about the same for each of the separate contributions. The disagreement between the Nijm78 potential and the m.e. analysis is largest for the $^1D_2$ phase shift.

For the Paris80 potential the $^1D_2$ and coupled $^3P_2$-$^3F_2$ phase shifts are not too good, whereas the other phase shifts are in reasonable agreement with the m.e. analysis.

Similarly, for the Bonn89 potential the $^1S_0$, the $^3P_0$, and the coupled $^3P_2$-$^3F_2$ phase shifts are not very good. This is partially due to the fact that the isovector tensor force of the Bonn89 potential is too strong and its spin-orbit force is too weak, which can be concluded from comparing the tensor and spin-orbit combinations of the ^3P phase shifts with the corresponding combinations as obtained in the *pp* partial-wave analysis.

The $\Delta\chi^2$ differences of the NijmRdl model are much smaller. For the $^1D_2$ phase shift the difference is even negative, which means that for this partial wave the NijmRdl model is better than the m.e. analysis. Also the $^3F_3$ partial wave is slightly better than in the m.e. analysis.

IV. CONCLUDING REMARKS

We have tested the quality of a number of *NN* potentials with respect to the *pp* scattering data in the 0–350 MeV energy range. Of the older models only the Reid68, Nijm78, and Paris80 models give satisfactory results when confronted with the *pp* data. The new Bonn89 model, an adjustment of the full Bonn potential to explicitly fit the *pp* data, is of a similar quality as the Nijm78 and Paris80 potentials in the 2–350 MeV energy range.

If we also include the very low-energy data (0–2 MeV), only the Nijm78 and Bonn89 potentials still give a reasonable description of the data. The other models all give a large to very large contribution to χ^2 in this low-energy region. The reason is that the *pp* $^1S_0$ phase shift at $T_{\rm lab} = 382.54$ keV is very accurately known. So a small deviation for the $^1S_0$ prediction from one of these potential models will give rise to an enormous contribution to χ^2. However, this contribution should not be too large, since most potential models claim to give a good description of the scattering length and effective range parameters. Furthermore, the fact that some of the models give a rather poor description of the *pp* data is not only due to an incorrect $^1S_0$ phase shift. As an example we consider the Arg84 potential. In the 2–350 MeV energy range the Arg84 model gives $\chi^2/N_{\rm data} = 7.1$. When we replace the Arg84 $^1S_0$ phase shifts by our m.e. values (which roughly corresponds to having a model with "perfect" $^1S_0$ phase shifts), the quality of the model improves considerably. However, the resulting $\chi^2/N_{\rm data} \approx 4$ is still rather large. This demonstrates that the other phase shifts are not too good either.

An important conclusion which can be drawn from the potential comparison with the *pp* scattering data discussed in this paper is that only the potential models which were explicitly fitted to the *pp* data (Nijm78, Paris80, Bonn89) give a reasonable description of these data. Here we have to keep in mind that the Nijm78 and Paris80 models were fitted to the 1969 Livermore database [28]. Our present database contains a large number of new and more accurate data, which are still described rather well by these two models. The Bonn89 potential was fitted to a much more recent database, not too different from our present database. Apparently, a good fit to the *np* data does not automatically guarantee a good fit to the *pp* data. One of the reasons is that the *np* data are less accurate than the *pp* data, so the constraints on the *np* phase shifts are not so large. Also, the difference between the *pp* and *np* $^1S_0$ phase shifts should be included explicitly.

We therefore conclude that if a potential model is claimed to give a good description of the *pp* scattering data, this claim should be based on an explicit confrontation of the model with these *pp* data, either directly or using Eq. (1).

Part of this work was included in the research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM) with financial support from the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

REFERENCES

[1] M. Taketani, S. Nakamura, and M. Sasaki, Prog. Theor. Phys. **6**, 581 (1951).
[2] M. Taketani, S. Machida, and S. Ohnuma, Prog. Theor. Phys. **7**, 45 (1952).
[3] K.A. Brueckner and K.M. Watson, Phys. Rev. **92**, 1023 (1953).
[4] J.L. Gammel, R.S. Christian, and R.M. Thaler, Phys. Rev. **105**, 311 (1957).
[5] J.L. Gammel and R.M. Thaler, Phys. Rev. **107**, 291, 1337 (1957).
[6] P.S. Signell and R.E. Marshak, Phys. Rev. **109**, 1229 (1958).
[7] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).
[8] T. Hamada and I.D. Johnston, Nucl. Phys. **34**, 382 (1962); T. Hamada, Y. Nakamura, and R. Tamagaki, Prog. Theor. Phys. **33**, 769 (1965).
[9] K.E. Lassila, M.H. Hull, Jr., H.M. Ruppel, F.A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).
[10] R.V. Reid, Jr., Ann. Phys. (NY) **50**, 411 (1968).
[11] N. Hoshizaki, I. Lin, and S. Machida, Prog. Theor. Phys. **26**, 680 (1961).
[12] D.Y. Wong, Nucl. Phys. **55**, 212 (1964).
[13] R.A. Bryan and B.L. Scott, Phys. Rev. **135**, B434 (1964).
[14] V.G.J. Stoks, R.A.M. Klomp, and J.J. de Swart, unpublished.
[15] J.R. Bergervoet, P.C. van Campen, R.A.M. Klomp, J.-L. de Kok, T.A. Rijken, V.G.J. Stoks, and J.J. de Swart, Phys. Rev. C **41**, 1435 (1990).
[16] J. Haidenbauer and K. Holinde, Phys. Rev. C **40**, 2465 (1989); K. Holinde, private communication.
[17] V.G.J. Stoks and J.J. de Swart, Nucl. Phys. **A514**, 309 (1990).
[18] J.R. Bergervoet, P.C. van Campen, W.A. van der Sanden, and J.J. de Swart, Phys. Rev. C **38**, 15 (1988).
[19] M.M. Nagels, T.A. Rijken, and J.J. de Swart, Phys. Rev. D **17**, 768 (1978).
[20] J. Smyrski, St. Kistryn, J. Lang, J. Liechti, H. Lüscher, Th. Maier, R. Müller, M. Simonius, J. Sromicki, F. Foroughi, and W. Haeberli, Nucl. Phys. **A501**, 319 (1989).
[21] G. Breit, M.H. Hull, Jr., K.E. Lassila, K.D. Pyatt, Jr., and H.M. Ruppel, Phys. Rev. **128**, 826 (1962); M.H. Hull, Jr., K.E. Lassila, H.M. Ruppel, F.A. McDonald, and G. Breit, *ibid.* **128**, 830 (1962).
[22] J.W. Humberston and J.B.G. Wallace, Nucl. Phys. **A141**, 362 (1970).
[23] R.A. Arndt and M.M. MacGregor, Phys. Rev. **141**, 873 (1966).
[24] B.D. Day, Phys. Rev. C **24**, 1203 (1981).
[25] R. de Tourreil, B. Rouben, and D.W.L. Sprung, Nucl. Phys. **A242**, 445 (1975); J. Côté, B. Rouben, R. de Tourreil, and D.W.L. Sprung, *ibid.* **A273**, 269 (1976).
[26] R. de Tourreil and D.W.L. Sprung, Nucl. Phys. **A201**, 193 (1973).
[27] T. Obinata and M. Wada, Prog. Theor. Phys. **53**, 732 (1975); **57**, 1984 (1977); T. Obinata, Prog. Theor. Phys. Prog. Lett. **73**, 1270 (1985).
[28] M.M. MacGregor, R.A. Arndt, and R.M. Wright, Phys. Rev. **182**, 1714 (1969).
[29] T.A. Rijken, R.A.M. Klomp, and J.J. de Swart, Nijmegen Report THEF-NIJM 91.05, 1991 (unpublished).
[30] M. Lacombe, B. Loiseau, J.M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).

[31] W.N. Cottingham, M. Lacombe, B. Loiseau, J.M. Richard, and R. Vinh Mau, Phys. Rev. D **8**, 800 (1973).
[32] I.E. Lagaris and V.R. Pandharipande, Nucl. Phys. **A359**, 331 (1981).
[33] R.A. Arndt, R.H. Hackman, and L.D. Roper, Phys. Rev. C **15**, 1002 (1977).
[34] R.B. Wiringa, R.A. Smith, and T.L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[35] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).
[36] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[37] J.J. de Swart, R. Klomp, R. Timmermans, and T.A. Rijken, in *Proceedings International Workshop on Quark-Gluon Structure of Hadrons and Nuclei, Shanghai, China, 1990*, edited by L.S. Kisslinger and Xijun Qiu (International Academic Publishers, Beijing, 1991), p. 112.
[38] J.J. de Swart, R.A.M. Klomp, T.A. Rijken, and V.G.J. Stoks, in *Proceedings XIIIth European Conference on Few-Body Problems, Elba, Italy, 1991* (Springer Verlag, Wien, 1992).

TABLES

TABLE I. The χ^2 results of the pp partial-wave analyses for the 10 single-energy bins.

Energy	Bin	N_{data}	$\chi^2_{\text{m.e.},n}$	$\chi^2_{\text{s.e.},n}$	$\chi^2_{\text{rep},n}$
0.38254	0.0–0.5	134	134.5	129.2	5.3
1.0	0.5–2	63	39.7	37.4	2.3
5.0	2–8	48	44.6	30.9	13.9
10.0	8–17	108	102.9	87.8	14.9
25.0	17–35	59	63.1	62.0	1.1
50.0	35–75	243	212.9	206.4	6.6
100.0	75–125	167	170.8	150.8	19.7
150.0	125–183	343	377.9	356.7	21.5
215.0	183–290	239	286.1	265.8	20.7
320.0	290–350	383	353.7	349.3	4.5
	0–350	1787	1786.4	1676.3	110.5
	2–350	1590	1612.2	1509.8	102.8

TABLE II. The $\chi^2_{\text{rep},n}$ results at the 10 single-energy bins for various NN potential models. The shorthand notation for each model is defined in Sec. III. In order to arrive at the total χ^2 one has to add the $\chi^2_{\text{s.e.},n}$ contributions of the analyses listed in Table I. The χ^2/N_{data} in the bottom line refers to the data in the 2–350 MeV energy range.

Bin	HJ62	Reid68	TRS75	OBEG75	Nijm78	Paris80	Urb81	Arg84	Bonn87	Bonn89	NijmRdl
0.0–0.5	6620	880	480	25500	62	3660	980	845000	665000	71	5.3
0.5–2	1960	132	100	610000	8	773	20	230000	195000	7	2.3
2–8	29	63	20	5370	51	18	115	1960	2400	8	15.2
8–17	103	206	55	3980	76	33	275	1470	2540	46	13.6
17–35	201	9	23	960	67	13	575	675	1950	20	2.1
35–75	6370	300	980	5330	555	333	1920	1365	6090	346	8.0
75–125	110	128	332	320	131	41	470	265	840	57	26.1
125–183	305	242	630	6540	222	415	3280	3060	1870	284	17.9
183–290	227	110	980	2750	202	174	995	700	1420	309	13.2
290–350	835	395	1500	4500	412	560	1080	335	2660	510	11.6
0–350	16760	2465	5100	665000	1786	6020	9710	1085000	880000	1658	115.3
2–350	8180	1453	4520	29750	1716	1587	8710	9830	19770	1580	107.7
χ^2/N_{data}	6.1	1.9	3.8	20	2.0	1.9	6.4	7.1	13	1.9	1.0

TABLE III. The difference $\Delta\chi^2$ (see text) in the 2–350 MeV energy range of the potential models using all potential phase shifts, or using one particular phase shift only.

Model	All phases	One particular phase shift $^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$-$^3F_2$	$^1D_2$	$^3F_3$	Sum
Nijm78	1614	283	396	5	462	570	378	2094
Paris80	1480	165	215	139	709	600	232	2060
Bonn89	1478	720	481	87	695	340	84	2407
NijmRdl	4.9	2.9	1.1	0.4	4.5	–6.6	–0.6	1.7