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# Triton calculations with the new Nijmegen potentials

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## Abstract

Triton properties are calculated using new nucleon-nucleon potentials, which were fit to the world nucleon-nucleon data. Three of these models have a nearly optimal $\chi^2$ per degree of freedom and can therefore be considered as alternative partial-wave analyses, which in quality can almost compete with the Nijmegen partial-wave analysis. The triton binding energy obtained with three local models (Nijm II, Reid93, AV18) can be summarized as 7.62±0.01 MeV, which is nearly 900 keV lower than experiment. The non-local model Nijm I binds by 7.72 MeV.

In recent years one of the important problems of few-nucleon physics has been resolved. The non-relativistic Schrödinger (or Faddeev [1]) equation can now be solved numerically for an arbitrary (energy-independent) potential model, and with negligible error. Such complete or "exact" solutions have been carried out at the level of 1% (or less) error for the ground states [2] of $^3$He and $^3$H (including a Coulomb interaction in the former case), for the ground state [3] of $^4$He, for the low-lying (continuum) states [4] of $^5$He, for the zero-energy scattering states of the *n-d* and *p-d* systems [5] and for transitions [6] between these systems and the ground states, and for the *n-d* continuum states above breakup threshold [7].

The results of these calculations can be rather simply summarized. The ground states are underbound and their underbinding is correlated. The triton is underbound by an amount which varies from 0.2 to 1.1 MeV for a wide variety of potential models [2]. The sizes of these systems are strongly correlated with their binding, and reasonable extrapolations to the physical binding energies of $^3$H and $^3$He produce sizes which agree with experiment. The Coulomb energy [2] of $^3$He (properly extrapolated) accounts for about 85% of the $^3$He–$^3$H binding-energy difference, and a variety of small charge-symmetry-breaking mechanisms [8] produces the remaining 15%. The alpha particle is underbound by an amount which is correlated with the triton underbinding and correcting one problem will likely correct the other. Low-energy *n-d* and *p-d* scattering and capture reactions are in reasonable agreement [5,6] with experiment (after extrapolating), except for the *p-d* scattering length, whose experimental value may be suspect. Scattering calculations (above breakup threshold [7]) are almost entirely in agreement with experiment. Those few areas where disagreements exist could be the result of using rather poor two-body forces, comparing *n-d* calculations to (Coulomb-modified) *p-d* data, inaccurate data, or an incompletely understood nuclear force.

Among the interesting physics which might manifest itself in these comparisons is three-nucleon forces [9]. Traditionally, the nuclear force is described by a sum of two-body (pair-wise) potentials. However, more complicated three-body forces are also present. Three-nucleon forces are expected to contribute a rather small amount of binding ($\sim 1$ MeV out of the total triton potential energy of roughly 50 MeV). This is also the scale for relativistic corrections. A representative momentum scale in the few-nucleon systems is the pion mass: $m_\pi c$. An estimate for the size of the relativistic corrections for the nucleons is then $(v/c)^2 \sim (m_\pi/M)^2 \sim 2\%$. Calculations performed to date typically find corrections of small magnitude from special relativity ($\sim 0.2$–$0.3$ MeV), but are not otherwise in agreement [10].

In addition to the uncertainties from relativity and three-body-force mechanisms, there is the uncertainty in the two-nucleon force. All of the calculations described above were performed with two-nucleon forces which are called "realistic." Such forces contain OPEP (the one-pion-exchange potential) and give at least a qualitative fit to the scattering data. The $\chi^2$ per degree of freedom for such a potential compared to the scattering data can nevertheless vary greatly. This is particularly true when a potential which has been fit to the *np* data is compared to the *pp* data [11]. One of the reasons being that charge dependence between the $T = 1$ *np* and *pp* partial waves in such a fit is not always accounted for properly (e.g., effects due to the presence of the Coulomb interaction are included, whereas equally important effects due to the neutral-to-charged pion mass difference are completely neglected). Differences in the quality of the fit to the data will produce differences in the predicted triton binding. In addition, differences in the assumed form of the potential, or of its type (local, or momentum dependent), might also affect the binding energy and yet not affect the fit to the scattering data in the 0–350 MeV energy region, which is the traditional domain of nucleon-nucleon ($NN$) potential models.

While it is not yet possible to resolve the three-nucleon force and relativistic correction problems, it should be possible to eliminate part of the uncertainty associated with the two-nucleon force. In the triton the charge dependence of the nuclear force can be taken into account by using an effective charge-symmetric force (for each $T = 1$ wave) given by [12]

$$V_{\text{eff}} = \frac{2}{3} V_{pp} + \frac{1}{3} V_{np} \ , \tag{1}$$

which prescription has an error the order of a few keV. Potential models whose $^1S_0$ forces are fit only to *pp* scattering generate a triton binding energy approximately 100 keV too low, while those fit only to *np* scattering are roughly 200 keV too high. Charge dependence in higher waves accounts for only a few keV. After applying this correction to previous triton calculations, potential models still underbind the triton by 0.4 to 1.0 MeV. This spread of values is larger than most estimates of relativistic effects and comparable to the contribution of most three-nucleon forces.

It has been known for many years that weakening the $T = 0$ tensor force (but still maintaining a fit to the two-nucleon data) increases the triton binding energy. The reason is that the deuteron is even more sensitive to the tensor force than is the triton, in spite of the fact that more than half of the triton's potential energy has that source. Consequently, when one is fitting the $NN$ potential to the two-nucleon scattering data, slight variations in the tensor force (increases or decreases) must be compensated by opposite variations (decreases or increases) in the central force. The triton is relatively more sensitive to the central force. Thus a weaker tensor force increases the triton binding because the deuteron binding is *fixed*.

Until recently, published measures of the tensor force, such as the $\varepsilon_1$ mixing parameter, were of poor quality, and almost any force seemed acceptable. This situation has been changed with the completion of the new and comprehensive Nijmegen partial-wave analysis (PWA), which has been described in detail elsewhere [13–15]. However, this PWA is largely irrelevant to the present work, since we construct our potentials by directly fitting the potential parameters to the data. Of primary importance is our charge-dependent OPEP produced by using different charged and neutral pion masses [15].

The original Nijmegen potential [16] (Nijm78) is a one-boson-exchange potential which incorporates the non-strange mesons of the pseudoscalar, vector, and scalar nonets. Using 13 parameters, the description of the *pp* data is reasonably good [11], whereas the description of the *np* data is rather poor. In order to improve its quality, we are currently constructing an update using 15 parameters. The preliminary version (Nijm92) gives a much better description of the *pp* as well as the *np* data with a $\chi^2$ per datum of 1.92. However, its quality is still not as good as the quality of the Nijmegen analysis, Nijm PWA93, which has a $\chi^2$ per datum of 0.99. We therefore also followed a different approach in that we constructed Reid-like models, where each partial wave is parametrized independently. Introducing as many parameters as necessary, it is then easy to arrive at a model with a (nearly) optimal $\chi^2$ per datum. The Reid-like Nijmegen model constructed this way is denoted by Nijm I.

A feature of relativistic origin in the Nijmegen potentials is the momentum-dependent part of the central potential, which follows from field theory. It gives rise to a non-local structure $(\Delta\phi(r) + \phi(r)\Delta)$ to the potential in configuration space. Such a term might be expected to behave differently in the triton, which is one of the reasons we constructed two Reid-like versions of the Nijmegen potential: The non-local Nijm I potential, which contains these momentum-dependent terms (as do the Nijm78 and Nijm92 potentials), and a local Nijm II potential, where these terms are intentionally omitted. We also constructed updates of two other *local* potentials. The Reid soft-core potential [17] was reparametrized using sums of regularized Yukawa functions and is here denoted by Reid93. The Argonne potential [18] was extended to include charge-independence breaking in the phenomenological parametrization of the short-range interaction and is here denoted by AV18. All these models will be discussed in more detail elsewhere [19]. Here we only want to mention that all potentials explicitly include the charge-dependent OPEP described earlier.

It is important to emphasize that the Nijmegen analysis Nijm PWA93 has not been used in constructing these potentials. The parameters of all models have been optimized in a direct fit to the data. In fact, both the Nijmegen potentials Nijm I and Nijm II, as well as the regularized Reid soft-core potential Reid93, are alternative partial-wave analyses: They have roughly the same number of fit parameters as the original analysis Nijm PWA93, they are fit to the same data, and they achieve nearly the same values of $\chi^2_{\rm min}$, which is significantly better than any other previous potential model. It is even significantly better than other multi-energy partial-wave analyses. The $\chi^2_{\rm min}$ per datum for the new models is given in table I.

The Faddeev equations for the triton bound state were solved for these new potentials using up to 34 channels (three-nucleon partial-wave states), which guarantees partial-wave convergence at the level of 10 keV. This includes all partial waves of the $NN$ interaction with $J \leq 4$. The result for the non-local Nijm I potential for 34 channels is 7.72 MeV, which is nearly 800 keV lower than the experimental value of 8.48 MeV. The charge radii for $^3$H and $^3$He, and the Coulomb energy of $^3$He are not significantly different from what one would expect for these binding energies. This result is comparable to that for the original Nijm78 potential, whose binding energy for the triton is 7.63 MeV. That potential, however, had a *pp*-type $^1S_0$ force, so that the result should be increased by roughly 100 keV. This similarity should not be too unexpected in view of a rather similar tensor force in the two models. The Nijm78 and Nijm I models have deuteron D-state probabilities of 5.39% and 5.66%, respectively, and have an identical structural form. We also note that the Nijm I potential generates 72% of the total triton potential energy (48 MeV) from the tensor force and about 74% of that energy from the (iterated) OPEP component (i.e., $\langle V_\pi \rangle$). The bulk of the potential energy typically comes from the tensor force and OPEP.

The quality of the local Nijm II potential is equally as good as that of the non-local Nijm I potential, both having a $\chi^2$ per datum of 1.03. The deuteron D-state probability is 5.64%, which is virtually the same as that of the Nijm I potential. The triton binding energy for the Nijm II potential is 7.62 MeV for 34 channels. This is a strong indication that replacing local structure in configuration space by a non-local one can affect the triton binding energy, even when the quality of the fit to the $NN$ data remains unchanged. This potential generates 52% and 67% of the triton potential energy from the tensor and (iterated) OPEP forces, respectively.

A significant question which can be posed is the following: how important are differences in the specific functional forms used to parameterize the radial dependence in the various parts of the potential? In order to answer this question in part, we also solved the triton bound state for the Reid93 and AV18 potentials. The Reid93 potential has an equally good fit to the *NN* data. The deuteron D-state probability is 5.70%, slightly higher than the other potentials. The triton binding energy for this potential is 7.63 MeV, which is very close to that of the Nijm II potential. It is worth noting that the original Reid68 potential had a triton binding energy of 7.35 MeV. The 300 keV difference in binding is the result of different quality fits to the *NN* data, and to a very different current data set than existed in 1968. The updated Argonne potential AV18 fits the *NN* data with a $\chi^2$ per datum of 1.30, has a D-state probability of 5.65%, and produces a triton binding energy of 7.62 MeV.

Finally, we mention that the (non-local) preliminary update of the original Nijm78 potential, denoted by Nijm92, gives a triton binding energy of 7.68 MeV. The difference with the Nijm I result is at least partially due to the difference in amount of non-locality. The results for all five potential models are summarized in table I. Note that the Nijm92 potential has the largest $\langle V_T \rangle$ and $\langle V_\pi \rangle$, indicating a very strong tensor force. Surprisingly, its values for $P_d$ and $\varepsilon_1$ at 50 MeV are the smallest, implying an anticorrelation.

All of the local potentials treated here have virtually the same triton binding energy (7.62 MeV) and deuteron D-state probability (5.66%). This similarity may not be coincidental. If one were to fit the partial waves of the potential to the partial-wave "data" which result from a PWA, and if one further assumes that the potential is local, the resulting fit to the relevant deuteron properties and phase-shift data *at all energies* yields a unique potential [20]. In practice one does not fit to data at all energies, of course, but it has long been an article of faith in nuclear physics that scattering data at high energies are unimportant to calculations of the nuclear bound states. Equivalently, these data primarily determine details of the potential at very short distances, which are known to be rather unimportant. Our results here are consistent with this picture. The local potential results suggest (but obviously do not prove) that local potentials which fit the *NN* scattering data very well, bind the triton by a unique value of about 7.62 MeV. Should this prove to be true, the physics issues in the triton problem (besides the question of three-nucleon forces) then shift to the origin and presence of nonlocalities in the *NN* force. Indeed, the 100 keV difference between the Nijm I and Nijm II models is the effect of just such a nonlocality.

In general, the local potentials tend to have a fairly large binding energy difference between the 5- and 34-channel cases ($>$ 200 keV), while the opposite is true for the non-local potentials. The results for the non-local Nijm I potential for 5, 9, 18, 26, and 34 channels are: 7.70, 7.77, 7.67, 7.72, and 7.72 MeV. The results for the local Nijm II potential are 7.39, 7.56, 7.51, 7.61, and 7.62 MeV for 5, 9, 18, 26, and 34 channels, respectively. This trend is consistent with most other potential models [2].

Summarizing, solutions to the Faddeev equations for the triton ground state were obtained for five new *NN* potentials, three of which fit the *NN* data with a nearly optimal $\chi^2$ per datum of 1.03. The three local potentials (Nijm II, Reid93, and AV18) bind the triton by 7.62$\pm$0.01 MeV. The non-local potential Nijm I, whose quality is similar to Nijm II, is more bound by roughly 100 keV. An update of the original non-local Nijm78 potential, denoted by Nijm92, binds the triton by 7.68 MeV. With respect to their fit to the scattering data, these new Nijmegen potentials are the best ever constructed, and triton binding calculations with them provide a benchmark against which calculations with other potential models should be compared.

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## REFERENCES


[1] L.D. Faddeev, Zh. Eksp. Teor. Fiz. **39**, 1459 (1960) [Sov. Phys. - JETP **12**, 1014 (1961)].

[2] C.R. Chen, G.L. Payne, J.L. Friar, and B.F. Gibson, Phys. Rev. C **31**, 2266 (1985); **33**, 1740 (1986); J.L. Friar, B.F. Gibson, and G.L. Payne, *ibid*, **35**, 1502 (1987) and **37**, 2869 (1988); these references contain a representative set of potential model results.

[3] J. Carlson, Phys. Rev. C **36**, 2026 (1987).

[4] J. Carlson, to be published.

[5] C.R. Chen, G.L. Payne, J.L. Friar, and B.F. Gibson, Phys. Rev. C **44**, 50 (1991).

[6] J.L. Friar, B.F. Gibson, H.C. Jean, and G.L. Payne, Phys. Rev. Lett. **66**, 1827 (1991).

[7] W. Glöckle, H. Witała, and Th. Cornelius, Nucl. Phys. **A508**, 115c (1990); W. Glöckle and H. Witała, Kerntechnik **57**, 228 (1992).

[8] Y. Wu, S. Ishikawa, and T. Sasakawa, Phys. Rev. Lett. **64**, 1875 (1990); **66**, 242(E) (1991).

[9] S.A. Coon, M.D. Scadron, P.C. McNamee, B.R. Barrett, D.W.E. Blatt, and B.H.J. McKellar, Nucl. Phys. **A317**, 242 (1979); S.A. Coon and J.L. Friar, Phys. Rev. C **34**, 1060 (1986).

[10] J. Carlson, V.R. Pandharipande, and R. Schiavilla, Phys. Rev. C **47**, 484 (1993); G. Rupp and J.A. Tjon, *ibid.* **45**, 2133 (1992); W. Glöckle, T.-S. H. Lee, and F. Coester, *ibid.* **33**, 709 (1986); F. Sammarruca, D.P. Xu, and R. Machleidt, *ibid.* **46**, 1636 (1992).

[11] V. Stoks and J.J. de Swart, Phys. Rev. C **47**, 761 (1993).

[12] J.L. Friar, B.F. Gibson, and G.L. Payne, Phys. Rev. C **36**, 1140 (1987).

[13] J.R. Bergervoet, P.C. van Campen, W.A. van der Sanden, and J.J. de Swart, Phys. Rev. C **38**, 15 (1988); J.R. Bergervoet, P.C. van Campen, R.A.M. Klomp, J.-L. de Kok, T.A. Rijken, V.G.J. Stoks, and J.J. de Swart, *ibid.* **41**, 1435 (1990).

[14] V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, and J.J. de Swart, Phys. Rev. C **48**, 792 (1993).

[15] R.A.M. Klomp, V.G.J. Stoks, and J.J. de Swart, Phys. Rev. C **44**, R1258 (1991).

[16] M.M. Nagels, T.A. Rijken, and J.J. de Swart, Phys. Rev. D **17**, 768 (1978).

[17] R.V. Reid, Ann. Phys. (NY) **50**, 411 (1968).

[18] R.B. Wiringa, R.A. Smith, and T.A. Ainsworth, Phys. Rev. C **29**, 1207 (1984).

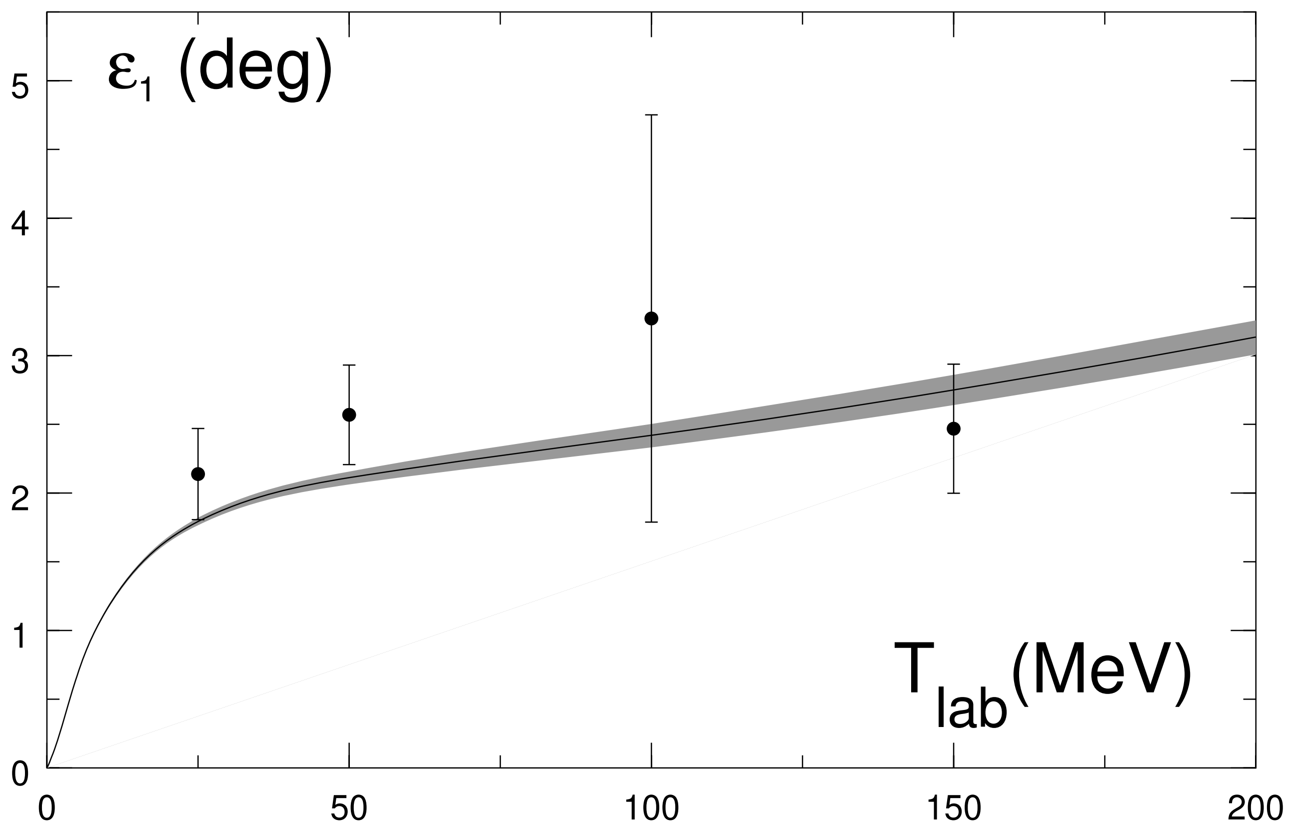
[19] V.G.J. Stoks, R.A.M. Klomp, and J.J. de Swart, in preparation.

[20] K. Chadan and P.C. Sabatier, *Inverse Problems in Quantum Scattering Theory*, (Springer-Verlag, New York, 1977).

# TABLES

TABLE I. Potential models used in this work, the values of $\chi^2_{\rm min}$ per datum obtained in producing them, values of $\varepsilon_1$ in degrees at 50 MeV and the deuteron D-state percentages, the corresponding triton binding energies, and percentage contributions of the tensor force and OPEP to the total potential energy. The Nijmegen partial-wave analysis [14] (Nijm PWA93) is given for comparison.

| Model | $\chi^2_{\rm min}/N_d$ | $\varepsilon_1$(50 MeV) | $P_{\rm d}$(%) | $E_{\rm B}$(MeV) | $\langle V_T \rangle$(%) | $\langle V_\pi \rangle$(%) |
|---|---|---|---|---|---|---|
| Nijm PWA93 | 0.99 | 2.11±0.05 | – | – | – | – |
| Nijm I | 1.03 | 2.09 | 5.66 | 7.72 | 72 | 74 |
| Nijm92 | 1.92 | 1.98 | 5.61 | 7.68 | 77 | 113 |
| Nijm II | 1.03 | 2.00 | 5.64 | 7.62 | 52 | 67 |
| Reid93 | 1.03 | 2.03 | 5.70 | 7.63 | 57 | 71 |
| AV18 | 1.30 | 2.16 | 5.65 | 7.62 | 52 | 77 |


ε1 (deg)
Tlab(MeV)
0
1
2
3
4
5
0
50
100
150
200