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An antiproton-proton partial-wave analysis*

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I. INTRODUCTION

Partial-wave analyses (PWAs) have a long history in the fields of πN and NN scattering. Due to the poor quality of low-energy antiproton beams and the resulting absence of accurate experimental data, analogous model-independent studies of the much more complex $\overline{p}p$ system have in the past always been impossible. In recent years, however, experimental progress has been very significant, in particular due to the coming in 1983 of the Low-Energy Antiproton Ring (LEAR) facility at CERN. While in the pre-LEAR era spin-dependent observables and charge-exchange ($\overline{p}p \rightarrow \overline{n}n$) data were almost nonexistent, the situation between 400 and 925 MeV/c is now quite good: the LEAR collaborations PS172, PS173, PS198, and PS199 have measured a variety of observables with impressive accuracy. High-quality analyzing-power data have been obtained for the elastic [1] and charge-exchange [2] reactions. Very recently, even charge-exchange depolarization data have become available [3]. Unfortunately, the practical difficulties involved in constructing a high-quality "cooled" antiproton beam of lower momentum are large. Consequently, the $\overline{p}p$ database below about 400 MeV/c is still by far not as good as one would like, in striking contrast to the pp case where very accurate data exist as low as $T_L = 0.35$ MeV ($p_L = 25$ MeV/c). It also remains an outstanding experimental challenge to construct a polarized antiproton beam to further probe the spin structure of the interaction.

During the last 10 years a new method has been developed by the Nijmegen group to perform PWAs of the abundant and accurate NN (pp and np) scattering data below $T_L = 350$ MeV [4,5]. With the now available high-quality data from LEAR and KEK, we have been able to extend the methods used in these NN PWAs to perform an energy-dependent PWA of all $\overline{p}p$ scattering data below $p_L = 925$ MeV/c ($T_L = 379$ MeV). This work was started in 1987 [6] and has only recently been finished [7]. The same methods of PWA have also been applied [8] to the strangeness-exchange reaction $\overline{p}p \rightarrow \overline{\Lambda}\Lambda$, for which the PS185 group at LEAR has obtained beautiful data. In the next section we review the theoretical ideas behind these Nijmegen PWAs, and in section III we apply these ideas and methods to the case of $\overline{p}p$ scattering. In section IV some results of this $\overline{p}p$ PWA are presented and discussed.

After almost a decade of LEAR, it is fair to say that in this field theory has some catching up to do with respect to experiment. Since the partial-wave amplitudes or the phase-shift parameters are in a sense the meeting ground between theory and experiment, the results of the present PWA should be very useful in many ways. They can be used to improve models [9,10] for the $\overline{N}N$ interaction. Apart from the fact that this provides independent and complementary [7] information about the spin- and isospin structure of the NN force, the $\overline{N}N$ interaction is needed as input in many other $\overline{p}p$ subfields. Studies of for instance protonium (the $\overline{p}p$ atom) or specific annihilation processes like $\overline{p}p \rightarrow \pi^+\pi^-$, K^+K^- require a realistic treatment of the initial $\overline{p}p$ interaction. At the same time, this PWA could be helpful in planning new experiments at LEAR, the future of which is of course crucial to this field.

II. METHODS OF PARTIAL-WAVE ANALYSIS

The hallmark of the Nijmegen energy-dependent PWAs is the sophisticated manner in which the energy dependence of the partial-wave amplitudes is parametrized. At the basis of the PWA is the trivial observation that in the low-energy region (long wave lengths) the long-range interaction is very important. It is this long-range interaction that is responsible for the rapid variations with energy of the scattering amplitudes. Short-range interactions lead to much slower energy variations of the amplitudes. One usually looks for a function in the problem that one can parametrize as easily as possible, i.e. one that does not contain the contributions from these long-range processes. Because these long-range interactions are at the same time model independent (in the sense that they are or at least should be the same in all NN and $\overline{N}N$ models), they can then be taken into account separately and exactly.

It is, of course, not a good idea to try to parametrize the partial-wave S matrix itself, since it does contain all of these long-range effects. As a function of complex energy, the S matrix has a (kinematical) right-hand unitarity cut, other right-hand cuts due to the coupling to inelastic channels, and (dynamical) left-hand cuts due to particle exchanges. The left-hand cuts that are the closest to the origin $T_L = 0$ correspond to the longest-range processes. The left-hand cuts that start far away from the origin are due to the short-range interactions. For instance, the infinite-range Coulomb potential ($V \sim 1/r$) produces an essential singularity and a branch point at $T_L = 0$, vacuum polarization ($V \sim \exp(-2m_e r)/r^{5/2}$) produces a cut at $T_L = -0.6$ keV, and one-pion exchange ($V \sim \exp(-m_\pi r)/r$) leads to a cut starting at $T_L = -9.7$ MeV. One sees that the crux is to find a quantity in which the cuts nearest to the origin are not present. This quantity then allows an *analytical* parametrization in energy or k^2 in an enlarged domain up to the next left- or right-hand cut present.

A familiar example of such a quantity with improved analyticity properties is the modified effective-range function [4,11]. The Coulomb-modified effective-range function for the pp $^1S_0$ state was originally derived (in a rather intuitive way) by Landau and Smorodinsky [12]. When only the Coulomb potential is present the boundary condition for the radial wave function $\Phi(0) = 0$ is of course satisfied by F, the regular Coulomb wave function (for $\ell = 0$). Suppose that there is an additional short-range interaction. When the wave length is very large (very low energy), one can take the limit in which the range of this additional (strong) interaction goes to zero. Then its presence is *only* revealed by a modified boundary condition at $r = 0$, which is now satisfied by a linear combination of F and G, the irregular Coulomb wave function

$$\Phi(r) \; = \; F(r) \cot \delta_0 + G(r) \ , \tag{1}$$

where δ_0 is the nuclear phase shift in the presence of the Coulomb interaction ($\delta(^1S_0) = \delta_0 + \sigma_0$), as can be seen from the asymptotic behavior of Φ. An equation for $\cot \delta_0$ can then be obtained by evaluating the logarithmic derivative of the wave function, which we call $P(k, r)$, for $k \to 0$. In the np case this quantity $P(k, 0) = k \cot \delta_0$ approaches a constant:

$$P(k, \varepsilon) \; = \; \left(\frac{d\Phi}{dr} / \Phi \right)_{r=\varepsilon} \;\; \to \;\; -\frac{1}{a} \ . \tag{2}$$

In the pp case, the evaluation has to be done at $r = \varepsilon$ because of a term $\ln \varepsilon$ that appears due to the singular behavior of G. This term one absorbes in the constant $-1/a$, along with some further constant terms. Then one lets $\varepsilon \to 0$ and immediately obtains the Coulomb-modified ("zero-range") effective-range function. It can be shown that after these manipulations the resulting left-hand side of Eqn. (2) is an analytical (actually meromorphic) function of the energy, so that the right-hand side can be written as a power series in k^2 (this means dropping the zero-range approximation).

The analytical expansion of the Coulomb-modified effective-range function breaks down already at $T_L = -9.7$ MeV, where the one-pion–exchange cut starts. It is possible to derive a new "pion-modified" effective-range function from which also this cut has been removed [4]. Let the regular and irregular wave functions for the case where only the Coulomb and pion-exchange potentials are present be called F_π and G_π. (For the purpose of the present discussion, we ignore vacuum polarization.) The wave function can then be written as

$$\Phi(r) \; = \; F_\pi(r) \cot \delta_0 + G_\pi(r) \; , \tag{3}$$

where δ_0 is now the phase shift due to the short-range remainder of the strong interaction ($\delta(^1S_0) = \delta_0 + \pi_0 + \sigma_0$, where π_0 is the one-pion–exchange phase shift in the presence of the Coulomb potential). However, proceeding in similar fashion as above, one encounters an important problem here. The evaluation of $P(k, \varepsilon)$ has to be done numerically, since F_π and G_π are not known in analytical form. Due to the singular behavior of G_π when $\varepsilon \to 0$, it is very hard to maintain sufficient numerical accurary, especially for higher orbital angular momenta.

At this point one has to realize that this numerical problem of the modified effective-range function is really an *artificial* problem: it crops up due to the singular behavior of the irregular function of the *long-range* potential *near the origin*. However, it is precisely this short-range interaction that one wants to parametrize, since it is essentially unknown, very complicated, and leads to only slow energy variations of the scattering amplitudes. Looking at Eqn. (3), one observes that it is valid for *any* r, so why not evaluate $P(k, r)$ at a *finite* value $r = b$, instead of at $r = \varepsilon$?

This is essentially what is done in the Nijmegen PWAs. The wave functions are obtained by solving the (relativistic) Schrödinger equation. Suppose one starts at a point r_∞ where only the Coulomb potential is present. Integrating inwards, one picks up sequentially the contributions (varying rapidly with energy) from the electromagnetic potentials, one-pion exchange, and contributions (varying slower with energy) from other meson exchanges. One then stops at a point $r = b$. If there are no additional interactions for $r < b$, the boundary condition $P(k, b)$ at $r = b$ is obviously satisfied by the regular wave function corresponding to precisely this potential tail. For small enough b the model used for $r > b$ will of course not be correct, and the boundary condition has to be modified, as in the above examples. In practice, it works the other way: one starts integrating at $r = b$ and $P(k, b)$ is parametrized as a function of energy. Also the best value for b is determined by fitting the data. In general multichannel problems $P(k, b)$ becomes a matrix. This P matrix has the required improved analyticity properties. When there are no nearby right-hand cuts, it is an analytical (again: actually meromorphic) function of k^2 in a domain bounded by the nearest left-hand cut not removed by including (or including incorrectly) the corresponding exchange in the potential tail for $r > b$. It can happen, of course, that short-range dynamics gives rise to a rapid energy variation of the amplitudes, as in the case of a resonance. This would have to be taken into account in the P matrix, for instance by including a pole in the parametrization. It is seen that the formalism used in the Nijmegen PWAs is similar to the boundary-condition approach to the strong interactions that goes back to the work of Feshbach and Lomon [13] and earlier. The philosophy, however, is very different. The term P matrix (for "pole" matrix) was introduced by Jaffe and Low [14] in the framework of the bag model.

III. AN ANTIPROTON-PROTON PARTIAL-WAVE ANALYSIS

Let us now be more specific and apply the foregoing ideas to the case of $\overline{p}p$ scattering. In all the Nijmegen PWAs, the two-body scattering process is described with the relativistic Schrödinger equation [15,16], which is essentially a coordinate-space version of the Blankenbecler-Sugar equation. It reads the same as the ordinary Schrödinger equation

$$\left(\Delta + k^2 - 2mV\right)\psi(\mathbf{r}) \;=\; 0 \;, \tag{4}$$

except that the proper relativistic relation between energy and momentum is used. It is well known how to derive the potentials for use in this equation [15,16]. In this relativistic framework, there is no known quantum-mechanical interpretation for the "wave function" $\psi(\mathbf{r})$. It is perhaps best to regard it as just a tool that allows one to compute the correct relativistic scattering amplitude (e.g. the poles are the correct bound states). We solve Eqn. (4) for the coupled $\overline{p}p$ and $\overline{n}n$ channels. The mass difference between proton and neutron is included in order to account for the $\overline{n}n$ threshold at $p_L = 99$ MeV/c.

The interaction in the region $r > b$ is described by a theoretically well-founded $\overline{N}N$ potential. This potential is given by

$$V \;=\; V_C + V_{MM} + V_N \;\;, \tag{5}$$

where V_C and V_{MM} are the relativistic Coulomb and magnetic-moment interaction respectively. V_N is the $\overline{N}N$ meson-exchange potential. It consists of one-pion exchange and the (charge-conjugated) heavy-meson and pomeron exchanges from the 1978 Nijmegen NN potential [17]. As argued in the previous section, the rapid energy variations of the amplitudes due to the long-range electromagnetic interactions and one-pion exchange are now included *exactly*.

Let us next turn to the parametrization of the short-range interactions for $r < b$ by way of the P-matrix boundary condition at $r = b = 1.3$ fm. Due to the coupling to the annihilation channels, the S matrix has a right-hand cut starting already to the left of $T_L = 0$. (In the pp case this cut starts only at the $pp \to pp\pi^0$ threshold at $T_L = 280$ MeV.) As these annihilation processes are of short range (and so give rise to slow energy variations of the amplitudes), this right-hand cut has to be present in the P matrix, which we therefore take to be complex. (Similarly, the effective-range parameters for the $\overline{N}N$ case are complex.) The choice of the value for b is rather critical, more so than in the NN case (where it was taken to be $b = 1.4$ fm). The best results are obtained for $b = 1.3$ fm. Since for $r > b$ we use only a real potential, the coupling to the annihilation channels is completely represented by the boundary condition. We conclude therefore that the range of the annihilation process is in fact about 1.3 fm [7].

The electromagnetic interactions that we use are adapted from the improved Coulomb potential [16]. This potential, designed specifically for use in the relativistic Schrödinger equation, contains relativistic corrections to the static Coulomb potential and (in its off-shell behavior) the main contributions from the two-photon–exchange diagrams. All these effects are included in the Nijmegen pp PWA [4,5], as well as the vacuum-polarization potential. In our case it suffices to use the following spin-dependent one-photon–exchange potentials

$$V_\gamma(r) \;=\; -\frac{\alpha'}{r} \;+\; \frac{\mu_p^2}{4M_p^2}\,\frac{\alpha}{r^3}\,S_{12} \;+\; \frac{8\mu_p - 2}{4M_p^2}\frac{\alpha}{r^3}\,\mathbf{L}\cdot\mathbf{S} \quad \text{for } \overline{p}p \to \overline{p}p \ , \tag{6}$$

and

$$V_\gamma(r) \;=\; \frac{\mu_n^2}{4M_n^2}\frac{\alpha}{r^3}\,S_{12} \quad \text{for } \overline{n}n \to \overline{n}n \ . \tag{7}$$

The magnetic moments of the proton and neutron are $\mu_p = 1 + \kappa_p = 2.793$ and $\mu_n = \kappa_n = -1.913$, respectively. The use of α' in the central potential for $\overline{p}p \to \overline{p}p$ takes care of the main relativistic corrections to the Coulomb potential. It is given by $\alpha'/\alpha = 2k/Mv_L$ where v_L is the velocity of the antiproton in the laboratory system. At 600 MeV/c one has for instance $v_L = 0.54$ and $\alpha'/\alpha = 1.135$, a correction of 13.5% to the static Coulomb potential. The spin-orbit potential comes from the interaction of the magnetic moment of one particle with the Coulomb field of the other particle (and is consequently absent in $\overline{n}n \to \overline{n}n$). It includes a relativistic correction due to the Thomas precession. The tensor potential comes from the interaction of the two magnetic moments. Vacuum polarization and two-photon–exchange effects are negligible in our case. The proper treatment of these electromagnetic effects in the evaluation of the scattering amplitudes is a nontrivial matter [7]. The following simple one-pion–exchange potential without a form factor is used

$$V_\pi(r) \;=\; f_{NN\pi}^2 \frac{M}{\sqrt{k^2+M^2}}\,\frac{m^2}{m_{\pi^\pm}^2}\,\frac{1}{3}\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + S_{12}\left(1+\frac{3}{(mr)}+\frac{3}{(mr)^2}\right)\right]\frac{e^{-mr}}{r} \ , \tag{8}$$

where m is the mass of the pion and $f_{NN\pi}^2 = 0.0745$ is the rationalized pion-nucleon coupling constant [18]. The mass difference between the π^0 and $\pi^\pm$ is included.

Let us finish this section with some more general remarks about PWAs. Even for the pp case, where the database is of high quality and the observables are very well mapped out, a PWA is impossible without a substantial amount of theoretical input or constraints. For the np and $\overline{p}p$ PWAs, this is true a fortiori. For instance, one has to make some assumptions about the validity of symmetries like charge independence or (as in our case) charge conjugation. Obviously, one has to careful here: sometimes general physical principles are inspired by local renormalizable field theories and not strictly valid for extended objects like hadrons. A good example can be found in πN PWAs, where one usually implements full Mandelstam analyticity [19]. The amplitudes are assumed to be analytic functions of the two complex variables s and t except for singularities from the mass spectrum and unitarity. These amplitudes then exhibit crossing symmetry. It is not clear at all to what extent low-energy hadron dynamics actually satisfies this symmetry.

Using strong and mostly model-independent theoretical constraints it has turned out to be possible to perform an *energy-dependent* or *multienergy* PWA of the $\overline{p}p$ data. However, it is quite a different ballgame to perform *energy-independent* or *single-energy* $\overline{p}p$ PWAs. In a single-energy $\overline{p}p$ PWA one has to determine in principle 20 phase-shift parameters for each $J \neq 0$ (8 for $J = 0$), which is four times as many as in a single-energy np PWA [7]! Almost certainly the present $\overline{p}p$ database does not allow satisfactory energy-independent PWAs. One has to realize, however, that even in the *NN* field the usefulness of energy-independent PWAs is more limited than is perhaps generally thought. When one has a *good* energy-dependent PWA, the best value for a phase shift (or the pion-nucleon coupling constant!) is definitely the one determined in the energy-dependent PWA, and not the one from an energy-independent PWA. One reason is that an energy-independent PWA contains no information about the energy dependence of the amplitudes. This makes it for instance less stable than an energy-dependent PWA with respect to the addition of new data to the database. Also, a set of energy-independent PWAs is usually overparametrized compared to a good energy-dependent PWA in the same energy region. It thus almost certainly contains noise. For an extensive discussion of this important point, see Ref. [5].

IV. SOME RESULTS OF THE ANALYSIS

While in *NN* PWAs there is essentially agreement on the correct database (especially for *pp*), we had to spend a lot of time and effort into collecting, scrutinizing, and cleaning up the world set of $\overline{p}p$ scattering data, which contains a lot of flaws and contradictory data. Exactly the same statistical arguments were used in this process as were used in the set-up of the Nijmegen *NN* database [4,5]. This means for instance that data with a very improbable high *or low* χ^2 are rejected on statistical grounds. The resulting Nijmegen 1993 $\overline{p}p$ database in the momentum interval 119–923 MeV/c is unique in the world and consists of $N_{\rm data} = 3646$ $\overline{p}p$ data. It is extensively discussed in Ref. [7]. In the final fit to this database we reach $\chi^2 = 3801.0$ or $\chi^2/N_{\rm data} = 1.04$. The number of boundary-condition parameters needed is 30, which is a reasonable number, in view of the fact that 21 parameters were needed in the Nijmegen *pp* PWA and an additional 18 in the *np* PWA. The total number of degrees of freedom is $N_{\rm df} = 3503$, which means that $\chi^2/N_{\rm df} = 1.09$.

If the database is a correct statistical ensemble and if the theoretical model is correct, one expects that $\langle\chi^2\rangle = N_{\rm df} = 3503$ with an error of $\sqrt{2N_{\rm df}} = 84$. This means that in our PWA we are 298 or only 3.5 standard deviations away from the expectation value for χ^2. We conclude that although there is still room for improvement, our *energy-dependent solution is essentially correct statistically.* As a consequence, the values for the phase-shift parameters (and also for the pion-nucleon coupling constant) and the statistical errors (obtained in the standard manner from the error matrix) are essentially correct.

In our 1991 preliminary PWA [6] we were able to determine the charged-pion–nucleon coupling constant $f_c^2 \equiv f_{pn\pi^+} f_{np\pi^-}/2$ from the data on the charge-exchange reaction $\overline{p}p \rightarrow \overline{n}n$, in which only isovector mesons can be exchanged. The result found was $f_c^2 = 0.0751(17)$, at the pion pole. The error is purely statistical. In our final analysis, we have repeated the determination of f_c^2, but this time from the complete 1993 Nijmegen database. The coupling constants of the neutral pion were kept at the value of $f^2_{pp\pi^0} = f^2_{nn\pi^0} = 0.0745$ [18]. We now find $f_c^2 = 0.0732(11)$, at the pion pole. This result supersedes our previous value from Ref. [6]. Again, the error is of statistical origin only. In view of the enormous amount of

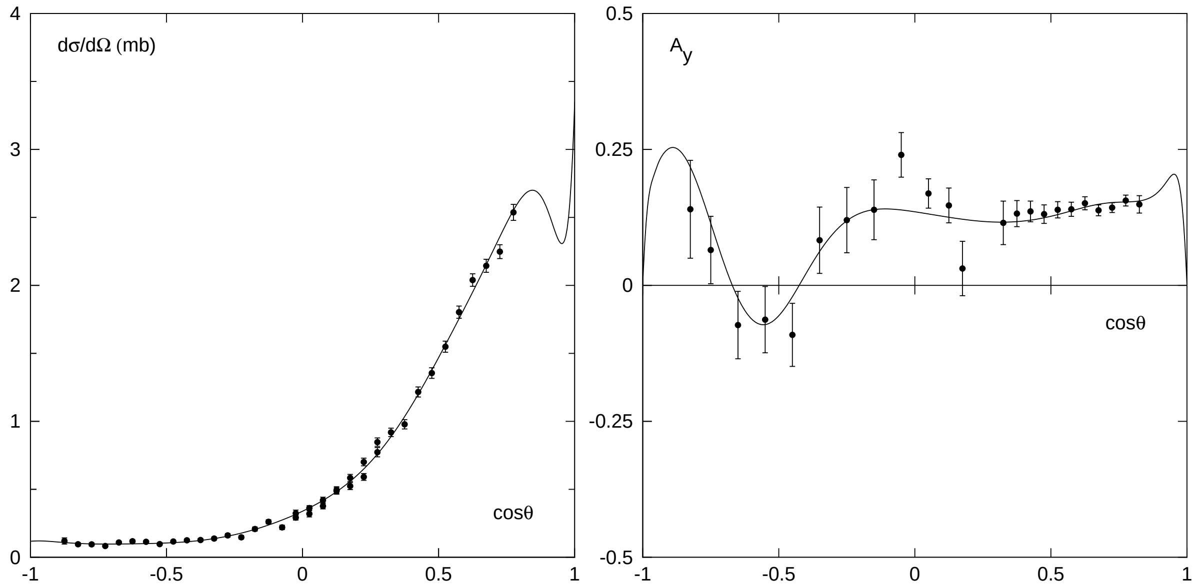

FIG. 1. Differential cross section at 693 MeV/c and analyzing power at 875 MeV/c for the charge-exchange reaction $\overline{p}p \rightarrow \overline{n}n$. The data are from PS199 [2]. The curves are from the Nijmegen PWA [7].

work involved, it is very hard to estimate possible systematic errors on this result. We have checked that there are no systematic errors due to form-factor effects or due to uncertainties in $\rho^{\pm}(770)$ exchange. In the Nijmegen pp PWA systematic errors could be more thoroughly investigated and they were found to be small [18]. In our case the systematic errors are probably larger than for the pp case, but it is very encouraging that the result for f_c^2 is in good agreement with recent determinations $f_c^2 = 0.0735(15)$ from $\pi^{\pm}p$ [20] scattering and $f_{pp\pi^0}^2 = 0.0745(6)$ and $f_c^2 = 0.0748(3)$ from NN scattering [18]. Very probably the new LEAR experiment PS206 on $\overline{p}p \rightarrow \overline{n}n$ will further constrain the charged-pion–nucleon coupling constant.

In Fig. 1 the differential cross section at 693 MeV/c and the analyzing power at 875 MeV/c are shown for $\overline{p}p \rightarrow \overline{n}n$. The data are from PS199 [2]. One can see the truly remarkable accuracy of the cross-section data and the analyzing-power data in the forward region. The "dip-bump" structure in $d\sigma/d\Omega$ at forward angles is due to the interference of one-pion exchange and a smooth background.

The fact that the available charge-exchange data already pin down the charged-pion coupling constant with a remarkable small statistical error is only one example of how at present *quantitative* information can be extracted from the $\overline{p}p$ system. We can mention some subtle effects that are also visible in the data. The use of α' instead of α, i.e. the main relativistic correction to the static Coulomb potential, gives a drop of $\Delta\chi^2 = 30$, or 5.5 standard deviations. The inclusion of the magnetic-moment interaction gives a drop of $\Delta\chi^2 = 14$, or 3.7 standard deviations. Even the use of the correct pion masses instead of an average mass of 138 MeV is a 3 standard-deviation effect.

Since the present $\overline{p}p$ PWA is the first of its kind, we have also proposed a convention for extracting phase-shift and inelasticity parameters from the S matrix. In the presence of coupling to annihilation channels the S matrix describing $\overline{N}N$ scattering is only a submatrix of the much larger multichannel S matrix. It is therefore still symmetric, but no longer unitary. This doubles the number of parameters needed. For the partial waves with $\ell = J$, $s = 0, 1$ one obviously writes $S = \eta \exp(2i\delta)$. For the states with $\ell = J \pm 1$, $s = 1$, coupled by the tensor force, six parameters are needed to parametrize the 2×2 S matrix. In this case it is not so easy to think of a convenient parametrization which satisfies all constraints from unitarity, is completely general, and free from nontrivial ambiguities. We have used a generalization [21] of the "bar-phase" convention commonly used in NN scattering. One writes (with the notation $\overline{\delta}_{\ell J}$ for the phase shift)

$$S^J = \exp(i\overline{\delta}) \begin{pmatrix} \cos\overline{\varepsilon}_J & i\sin\overline{\varepsilon}_J \\ i\sin\overline{\varepsilon}_J & \cos\overline{\varepsilon}_J \end{pmatrix} H^J \begin{pmatrix} \cos\overline{\varepsilon}_J & i\sin\overline{\varepsilon}_J \\ i\sin\overline{\varepsilon}_J & \cos\overline{\varepsilon}_J \end{pmatrix} \exp(i\overline{\delta}) \ , \tag{9}$$

where $\overline{\delta} = \mathrm{diag}(\overline{\delta}_{J-1,J}, \overline{\delta}_{J+1,J})$ and $\overline{\varepsilon}_J$ is the mixing parameter. H^J is a three-parameter real and symmetric matrix representing inelasticity. It can be diagonalized in Blatt-Biedenharn fashion

$$H^J = \begin{pmatrix} \cos\omega_J & -\sin\omega_J \\ \sin\omega_J & \cos\omega_J \end{pmatrix} \begin{pmatrix} \eta_{J-1,J} & 0 \\ 0 & \eta_{J+1,J} \end{pmatrix} \begin{pmatrix} \cos\omega_J & \sin\omega_J \\ -\sin\omega_J & \cos\omega_J \end{pmatrix} \ , \tag{10}$$

where the diagonal matrix contains the "eigeninelasticities" $\eta_{J-1,J}$ and $\eta_{J+1,J}$, and ω_J is again a mixing parameter. We are presently in the process of doing a careful evaluation of these phase-shift and inelasticity parameters and their errors. Unfortunately, this involves a large amount of work. These and other issues will be the subject of future publications.

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