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Meson-baryon coupling constants from a chiral-invariant SU(3) Lagrangian and application to *NN* scattering

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Abstract

We present a chiral-invariant meson-baryon Lagrangian which describes the interactions of the baryon octet with the lowest-mass meson nonets. The nonlinear realization of the chiral symmetry generates pair-meson interaction vertices. The corresponding pair-meson coupling constants can all be expressed in terms of the meson-nucleon-nucleon pseudovector, scalar, and vector coupling constants, and their corresponding $F/(F+D)$ ratios, and for which empirical estimates are given. We show that it is possible to construct an NN potential of reasonable quality satisfying these theoretical and empirical constraints.

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I. INTRODUCTION

The construction of NN potentials has a long history, and a large number of different models have appeared in the literature. After a comparison of these NN potentials by confronting them first with the pp scattering data [1] and later also with the np scattering data [2], it was found that all existing models do a rather poor job (some worse than others) in properly describing these data. This is rather disturbing, since most of these models are used in many-body calculations and pp bremsstrahlung calculations to draw certain conclusions on three-body forces, relativistic effects, off-shell effects, etc. And so one can ask the question whether it is at all valid to conclude anything from a many-body calculation if the applied NN potential cannot even adequately describe the two-nucleon data.

This uncertainty has been recently cleared up with the construction of a number of high-quality NN potential models [3,4]. With high-quality we mean that these models describe the NN scattering data with the almost optimal $\chi^2/N_{\rm data} \approx 1$. Although based on very different functional forms, these potentials give remarkably similar results in many-body [5–7] and pp bremsstrahlung [8] calculations. One 'disadvantage' of these new potential models, however, is that they are largely phenomenological and only explicitly include the one-pion exchange. Our next task, therefore, is to try and construct a high-quality potential model which is less phenomenological in that it explicitly contains the heavier-meson exchanges (with coupling constants which can be compared with empirical data) and which, possibly, is consistent with the symmetries of QCD.

Recently, Ordóñez, Ray, and van Kolck [9] presented a nucleon-nucleon (NN) potential based on an effective chiral Lagrangian of pions, nucleons, and Δ isobars. Using 26 free parameters, the agreement with the experimental scattering data was found to be satisfactory up to lab energies of about 100 MeV. An extension to higher energies and a further improvement in the description of the data would require an expansion to higher orders in chiral perturbation theory, making the model much more complicated and introducing many new parameters. Hence, the authors conclude that it is not practical for potential models derived from effective chiral Lagrangians to compete with more phenomenological approaches.

In this paper we would like to advocate an alternative approach. We want to investigate whether it is possible to construct a potential model which not only gives a satisfactory description of the scattering data up to $\sim$350 MeV using a limited number of free parameters, but which also retains the salient features of chiral symmetry. For that purpose we here do not integrate out all mesons other than the pion [10], but rather adopt the successful approach used in one-boson-exchange models and keep all lowest-lying mesons with masses lower than 1 GeV, say. The NN potential model is then obtained by evaluating the standard one-boson-exchange contributions involving these mesons, but now including the contributions of the box and crossed-box two-meson diagrams [11] and of the pair-meson diagrams where at least one of the nucleon lines contains a pair-meson vertex [12]. The potential contributions are calculated up to order $1/M^2$ in the nucleon mass as is customary in the conventional one-boson-exchange approaches, and so we want to stress that here we are *not* doing chiral perturbation theory. Here we only use chiral symmetry to generate the interaction Lagrangians and to find constraints for the associated coupling constants. However, as we will see, the pair-meson diagrams arise as a direct consequence of chiral symmetry. The box and crossed-box diagrams then also have to be included because they are of the same order in the number of exchanged mesons as the pair-meson diagrams. The pair-meson interactions can be viewed [10] as the result of integrating out the heavy-meson (masses larger than 1 GeV, say) and resonance (e.g., Δ, N^*, Y^*) degrees of freedom in the two-meson-exchange processes. Also, according to "duality" [13], the resonance contributions to the various meson-nucleon amplitudes can be described approximately by heavy-meson exchanges. Treating the heavy-meson propagators as constants, which should be adequate at low energies, then leads directly to pair-meson exchanges.

In Refs. [11,12] we already showed that the inclusion of the two-meson (box, crossed-box, and pair) contributions provides a substantial improvement in the description of the scattering data as compared to a potential containing only the standard one-boson exchanges. Although originally [14,15] the meson-pair interaction Lagrangians were taken to be purely phenomenological, we later found that the pair-meson coupling constants could all be fixed using experimental input and chiral-symmetry constraints. In particular, in Ref. [12] the estimates for the pair-meson coupling constants were based on the linear σ model [16]. In order to appreciate this result, it should be realized that, by fixing the pair-meson coupling constants in this way, this improvement could be obtained without the introduction of any new parameters.

This remarkable result encourages us to go beyond the *NN* model and to investigate whether a similar approach will also be fruitful in the construction of an extended hyperon-nucleon (*YN*) potential. An important motivation for the development of an extended *YN* model is provided by the study of hypernuclei using one-boson-exchange models [17,18]. The problem with the construction of a *YN* potential is that there are only a few experimental data available, and these data are rather old and not very accurate. At present it is very difficult (if not impossible) to determine all the free parameters from the scattering data alone. A reduction in the number of free parameters is obtained by first fixing the parameters which also play a role in *NN* scattering (and which are easier to determine, since there are many accurate *NN* scattering data), and then using SU(3) symmetry to fix the coupling constants in the *YN* potential. This approach has been used successfully in the various hard-core [19] and soft-core [20] *YN* one-boson-exchange models of the Nijmegen group. The Jülich models [21,22] even impose an SU(6) symmetry. As a further reduction in the number of parameters, it would be convenient if we were able also to fix all (or at least most) single-meson coupling constants at their empirical values.

As a first step, in this paper we will construct a chiral-invariant Lagrangian for the meson-baryon sector. Rather than extending the linear σ model (the model we used in [12]) to the strange sector, we here find it more convenient to explore the nonlinear realization of the spontaneously broken chiral symmetry. This allows us to express all pair-meson coupling constants in terms of the single-meson vertex parameters. Using symmetry arguments and further experimental data we give estimates for these single-meson vertex parameters. We then have two options for constructing a potential model: (i) we can either strictly impose all the constraints on the coupling constants and investigate whether it is at all possible to impose chiral symmetry in a potential model or, (ii) we can relax at least some of the constraints and, hence, optimize the description of the scattering data. As an application we will here only explore the first option and show that a fully constrained *NN* model indeed allows for a very satisfactory description of the data. This is a remarkable result because, until now, in potential models it has never been possible to fix more than only a few coupling constants at their empirical values. The second option reflects that the imposed symmetries need not be exactly true, a fact which can be useful when fine-tuning the model to the scattering data. This option will be left for the future.

The outline of the paper is as follows. In Sec. II we list the baryons and mesons of the model. Section III gives a review of the linear σ model [16] and the nonlinear realization of chiral symmetry as introduced by Weinberg [23]. We briefly indicate how the model can be made to agree with experimental data. In Sec. IV we extend the nonlinear realization to SU(3) to obtain a chiral-invariant meson-baryon Lagrangian. This SU(3) Lagrangian describes the meson-baryon interactions of the scalar, pseudoscalar, vector, and axial-vector meson nonets with the baryon octet. Section V then lists estimates for the various single-meson vertex parameters (singlet and octet coupling constants, mixing angles, and $F/(F+D)$ ratios), using theoretical and experimental input. The coupling constants for the double-meson vertices can all be expressed in terms of these single-meson vertex parameters; this will be discussed in Sec. VI. Unfortunately, we have not been able to construct a potential model that satisfies all these constraints. Therefore, in Sec. VI C we briefly indicate how the interaction Lagrangian can be extended by introducing new free parameters without abandoning the empirical constraints on the single-meson coupling constants. This allows for much more flexibility in actually imposing these constraints in a baryon-baryon potential model. As an application, in Sec. VII we show that with this extension it is indeed possible to construct an NN potential that satisfies the constraints of chiral symmetry and which gives a very satisfactory description of the NN scattering data up to 350 MeV.

II. MESON AND BARYON CONTENT

In the following, the baryons are the members of the SU(3) octet: $N(940)$, $\Lambda(1115)$, $\Sigma(1192)$, and $\Xi(1318)$. The mesons consist of the standard pseudoscalar nonet (π, η, η', K) and vector nonet (ρ, ω, ϕ, K^*). The physical isoscalar mesons in these nonets come about as an admixture of the pure octet and pure singlet isoscalar states. That is, the physical η and η' are admixtures of the pure octet η_8 with the pure singlet η_0, and the physical ω and ϕ are admixtures of the pure octet ω_8 with the pure singlet ω_0.

In the SU(3)$\times$SU(3) chiral-invariant model, the extension of the global chiral symmetry to a local chiral symmetry introduces two octets of gauge fields, which can be reexpressed in terms of one octet of vector and one octet of axial-vector fields (see below). It seems natural to identify the octet of vector gauge fields with the octet of vector mesons. The singlet (which is not a gauge field) is then added to complete the nonet. Similarly, the octet of axial-vector fields can be identified with the octet of axial-vector mesons. Adding the singlet and mixing the pure octet and pure singlet isoscalar states is then assumed to give the physical nonet of axial-vector mesons [$a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and $K_1(1270)$]. In this scenario, the axial-vector mesons are a necessary ingredient in establishing the local symmetry, but one can argue that their role in low-energy baryon-baryon potential models is likely to be negligible. Their masses are well above 1 GeV, and so the corresponding short-range potentials are probably already effectively included by form factors at the meson-baryon vertices. For alternative approaches to include vector and axial-vector mesons in a chiral-invariant way, see Refs. [24,25].

The existence of the 0^{++} scalar nonet (octet plus singlet) is still controversial. The Particle Data Group [26] lists an isovector state $a_0(980)$ and an isoscalar state $f_0(980)$, while there appears to be evidence for a strange isodoublet of scalar mesons, denoted by $\kappa(887)$ [27]. These would make up the octet. There is also evidence for a broad isoscalar $0^{++}(760)$ state [28], which could be the scalar singlet. Alternatively, in analogy with the pseudoscalar and vector nonets, it is likely that also the $f_0(980)$ and $\varepsilon(760)$ are admixtures of the pure octet and pure singlet isoscalar states. The baryon-baryon potential due to the exchange of a broad meson can be approximated by the sum of two potentials where each potential is due to the exchange of a stable (effective) meson [29]. Due to the large width of the $\varepsilon(760)$, the low-mass pole in this two-pole approximation is rather small (∼500 MeV), and hence would represent a possible candidate for the low-mass σ meson which is a necessary ingredient in baryon-baryon potential models, and which is the ingredient of the linear σ model. It is beyond the scope of this paper to discuss whether or not a nonet of scalar mesons with masses below 1 GeV really exists, how its existence can be explained or disclaimed in a quark model, or whether a scalar one-boson exchange is nothing else but an effective representation of correlated two-pion exchange. Here we only mention that already some time ago Jaffe [30] presented a quark-bag model calculation of $\bar{q}^2q^2$ mesons where the members and decay modes of the lowest multiplet fit in remarkably well with what is observed for the scalar nonet discussed above. More recently, Törnqvist [31] has calculated the properties of a distorted $\bar{q}q$ nonet in a unitarized quark model, which also identifies the above scalar nonet reasonably well (except for the $\kappa(887)$ state). Finally, after an absence of almost 20 years, the latest edition of the Particle Data Group [26] now tentatively lists a very broad resonance as $f_0(400\text{–}1200)$ with a width of 600–1000 MeV. Here, we will stick to $\varepsilon(760)$ and assume a width of $\Gamma_\varepsilon \approx 800$ MeV.

III. SU(2) CHIRAL SYMMETRY

Although the SU(2) case has been discussed extensively in the literature, we believe it is still instructive to first review the SU(2) case in some detail before we turn to the SU(3) case. In the SU(2) case the results can be easily expressed in terms of the physical fields, which makes it easier to see what is going on. In the SU(3) case the expressions are much more involved.

The classic example for a model with chiral SU(2) symmetry is the linear σ model [16]. The linear σ model contains an isotriplet of pseudoscalars, $\boldsymbol{\pi}$, and an isosinglet scalar, σ, which can be grouped into

$$\Sigma = \sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}, \tag{3.1}$$

and which transforms under global $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ as

$$\Sigma \rightarrow L\Sigma R^\dagger. \tag{3.2}$$

The nucleon field ψ has left and right components, $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$, transforming as

$$\psi_L \rightarrow L\psi_L, \quad \psi_R \rightarrow R\psi_R. \tag{3.3}$$

Given these transformation properties, we can construct the chiral-invariant Lagrangian $\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_\Sigma + \mathcal{L}_I$, where

$$\begin{aligned}
\mathcal{L}_\psi &= \overline{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \overline{\psi}_R i\gamma^\mu \partial_\mu \psi_R, \\
\mathcal{L}_\Sigma &= \tfrac{1}{4}\mathrm{Tr}(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) - \tfrac{1}{4}\mu^2 \mathrm{Tr}(\Sigma^\dagger \Sigma) - \tfrac{1}{8}\lambda^2 \mathrm{Tr}(\Sigma^\dagger \Sigma)^2, \\
\mathcal{L}_I &= -g(\overline{\psi}_L \Sigma \psi_R + \overline{\psi}_R \Sigma^\dagger \psi_L),
\end{aligned} \tag{3.4}$$

and μ, λ, and g free parameters. Obviously, the ground state cannot have the full $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry, since in that case all hadrons (nucleon, pion, sigma in this model) would have partners of equal mass but with opposite parity, which is not the case in the real world. Hence, the chiral symmetry has to be spontaneously broken down to the vectorial subgroup $\mathrm{SU}(2)_V$. This can be achieved by adding a term linear in the σ field, choosing the ground state as $\langle\sigma\rangle = f_0$, and shifting the scalar field to $\sigma = s + f_0$. We then find the familiar Lagrangian

$$\begin{aligned}
\mathcal{L} &= \overline{\psi}(i\gamma^\mu \partial_\mu - M)\psi - g\overline{\psi}(s + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi \\
&\quad + \tfrac{1}{2}(\partial^\mu s \partial_\mu s - m_s^2 s^2) + \tfrac{1}{2}(\partial^\mu \boldsymbol{\pi}\cdot\partial_\mu \boldsymbol{\pi} - m_\pi^2 \boldsymbol{\pi}^2) \\
&\quad - \frac{m_s^2 - m_\pi^2}{2f_0}\, s(s^2 + \boldsymbol{\pi}^2) - \frac{m_s^2 - m_\pi^2}{8f_0^2}\,(s^2 + \boldsymbol{\pi}^2)^2,
\end{aligned} \tag{3.5}$$

where the nucleon mass is given by $M = gf_0$, and the scalar and pseudoscalar masses are given by $m_s^2 = \mu^2 + 3\lambda^2 f_0^2$ and $m_\pi^2 = \mu^2 + \lambda^2 f_0^2$, respectively. The PCAC (partially-conserved axial-vector current) condition for the axial-vector Noether current, $\partial_\mu \mathbf{A}^\mu = m_\pi^2 f_\pi \boldsymbol{\pi}$, determines the relation between $\langle\sigma\rangle = f_0$ and $f_\pi = 92.4 \pm 0.3$ MeV [26], the pion decay constant. As we will see below, the introduction of vector and axial-vector fields enforces a renormalization of the pion field, and so we cannot make the identification $f_0 = f_\pi$ (a true identity in the absence of vector and axial-vector gauge fields).

Weinberg has shown [32] that the linear σ model has the major disadvantage that it hides the fact that soft pions are emitted in clusters by derivative couplings from external lines, and so it is more convenient to transform the nonderivative $\overline{\psi} i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}\psi$ and $\sigma\boldsymbol{\pi}^2$ interactions away. For that purpose, Weinberg defines a nonlinear transformation of the nucleon fields, given by [23]

$$\psi = (1 + \boldsymbol{\xi}^2)^{-1/2}(1 - i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\xi})N, \tag{3.6}$$

with $\boldsymbol{\xi}$ chosen such that

$$\overline{\psi}(M + gs + ig\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi = \overline{N}(M + gs')N. \tag{3.7}$$

It will be convenient to write this transformation in the form

$$\left.\begin{aligned} N_R &= u\psi_R \\ N_L &= u^\dagger \psi_L \end{aligned}\right\}, \quad u(\boldsymbol{\xi}) = \frac{1 + i\boldsymbol{\tau}\cdot\boldsymbol{\xi}}{(1 + \boldsymbol{\xi}^2)^{1/2}}, \tag{3.8}$$

in which case we find

$$\Sigma' \equiv f_0 + s' = u^\dagger \Sigma u^\dagger. \tag{3.9}$$

Solving Eq. (3.7) gives

$$\begin{aligned}
\boldsymbol{\xi} &= \boldsymbol{\pi}\left[f_0 + s + \sqrt{(f_0+s)^2 + \boldsymbol{\pi}^2}\right]^{-1} \equiv \frac{1}{2f_0}\boldsymbol{\pi}', \\
s' &= [(f_0+s)^2 + \boldsymbol{\pi}^2]^{1/2} - f_0 = (\Sigma^\dagger\Sigma)^{1/2} - f_0,
\end{aligned} \tag{3.10}$$

where the chiral rotation vector $\boldsymbol{\xi}$ is proportional to a new pion field, $\boldsymbol{\pi}'$. We should mention that the condition (3.7) also removes the $\sigma\boldsymbol{\pi}^2$ interaction, and the new Lagrangian $\mathcal{L}_{\Sigma'}$ only contains three-point and four-point interactions for the s' scalar field.

In this nonlinear realization of the spontaneously broken chiral symmetry, the new fields $u(\boldsymbol{\xi})$ of the coset space $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R/\mathrm{SU}(2)_V$ transform as [33]

$$u \to LuH^\dagger = HuR^\dagger, \tag{3.11a}$$

$$H = \sqrt{Lu^2R^\dagger}Ru^\dagger = \sqrt{Ru^{\dagger 2}L^\dagger}Lu, \tag{3.11b}$$

where the equality in Eq. (3.11a) is due to parity. The left and right components of the new doublet nucleon field N both transform with the same SU(2) matrix H, and so the nucleon part of the Lagrangian can now be written as

$$\mathcal{L}_N = \overline{N}(i\gamma^\mu D_\mu - M)N - g\overline{N}Ns' - g_A\overline{N}\gamma^5\gamma^\mu u_\mu N, \tag{3.12}$$

where we defined $D_\mu = \partial_\mu + i\Gamma_\mu$, and

$$\begin{aligned}
\Gamma_\mu &= -\tfrac{i}{2}\left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right) = \frac{1}{4f_0^2}\,\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}'\times\partial_\mu\boldsymbol{\pi}'}{1+\boldsymbol{\xi}^2}, \\
u_\mu &= -\tfrac{i}{2}\left(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right) = \frac{1}{2f_0}\,\frac{\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}'}{1+\boldsymbol{\xi}^2}.
\end{aligned} \tag{3.13}$$

The transformation rule for the connection Γ_μ,

$$\Gamma_\mu \to H\Gamma_\mu H^\dagger - iH\partial_\mu H^\dagger, \tag{3.14}$$

ensures the invariance of the kinetic-energy term. The transformation rule for the N field, $N \to HN$, means that $\overline{N}N$ is already an invariant in itself, and so the mass M can be treated as a free parameter. Similarly, the transformation rule for u_μ,

$$u_\mu \to Hu_\mu H^\dagger, \tag{3.15}$$

allows for the introduction of the free parameter g_A in Eq. (3.12). Choosing it to be $g_A = 1.2601 \pm 0.0025$ [26], which is the value for the weak interaction axial-vector coupling constant, and substituting for $\boldsymbol{\pi}'$, the pion is seen to couple to the nucleon via the pseudovector coupling with strength $g_A/2f_\pi$. This relation between the weak axial-vector and the pion pseudovector coupling constants is known as the Goldberger-Treiman relation [34]. Although this relation only exactly holds in the chiral limit, experimentally it is found to hold within about 2%. Note that in the ψ representation of Eq. (3.5), this coupling is found to be $g_A = 1$, which is off by 25%.

The next step is to add the isotriplets of vector ($\boldsymbol{\rho}$) and axial-vector ($\mathbf{a}_1$) fields. One way to do this is to extend the global chiral symmetry to a local one and to define the corresponding left- and right-handed gauge fields as

$$
\begin{aligned}
l_\mu &\equiv \tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{l}_\mu = \tfrac{1}{2}\boldsymbol{\tau}\cdot(\boldsymbol{\rho}_\mu + \mathbf{a}_\mu), \\
r_\mu &\equiv \tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{r}_\mu = \tfrac{1}{2}\boldsymbol{\tau}\cdot(\boldsymbol{\rho}_\mu - \mathbf{a}_\mu),
\end{aligned}
\tag{3.16}
$$

with their field strength tensors

$$
l_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu + i g_V [l_\mu, l_\nu], \tag{3.17}
$$

and similarly for $r_{\mu\nu}$. Their transformation properties are

$$
\begin{aligned}
l_\mu &\to L l_\mu L^\dagger - \frac{i}{g_V} L\partial_\mu L^\dagger, \qquad l_{\mu\nu} \to L l_{\mu\nu} L^\dagger, \\
r_\mu &\to R r_\mu R^\dagger - \frac{i}{g_V} R\partial_\mu R^\dagger, \qquad r_{\mu\nu} \to R r_{\mu\nu} R^\dagger,
\end{aligned}
\tag{3.18}
$$

and so the proper field strength tensors for the nonlinear transformation are $u^\dagger l_{\mu\nu} u$ and $u r_{\mu\nu} u^\dagger$. Invariance under local chiral transformations implies the extensions

$$
\begin{aligned}
\Gamma_\mu &= -\tfrac{i}{2}\left[u^\dagger(\partial_\mu + i g_V l_\mu)u + u(\partial_\mu + i g_V r_\mu)u^\dagger\right], \\
u_\mu &= -\tfrac{i}{2}\left[u^\dagger(\partial_\mu + i g_V l_\mu)u - u(\partial_\mu + i g_V r_\mu)u^\dagger\right].
\end{aligned}
\tag{3.19}
$$

The locally chiral-invariant Lagrangian can now be written as [35]

$$
\mathcal{L}_N = \overline{N}(i\gamma^\mu\partial_\mu - M)N - g_s\overline{N}Ns' - \tfrac{1}{2}g_V\overline{N}\gamma^\mu\boldsymbol{\tau}N\cdot\boldsymbol{\rho}'_\mu - \tfrac{1}{2}\lambda\overline{N}\gamma^5\gamma^\mu\boldsymbol{\tau}N\cdot\mathbf{a}'_\mu, \tag{3.20}
$$

where g_s, g_V, and λ are free parameters, and where we defined the field abbreviations

$$
\begin{aligned}
\boldsymbol{\rho}'_\mu &= \boldsymbol{\rho}_\mu + \frac{2}{1+\boldsymbol{\xi}^2}\,\boldsymbol{\xi}\times\mathbf{a}_\mu + \frac{2}{g_V}\frac{1}{1+\boldsymbol{\xi}^2}\,\boldsymbol{\xi}\times(\partial_\mu\boldsymbol{\xi} + g_V\boldsymbol{\xi}\times\boldsymbol{\rho}_\mu), \\
\mathbf{a}'_\mu &= \mathbf{a}_\mu - \frac{2}{1+\boldsymbol{\xi}^2}\,\boldsymbol{\xi}\times(\mathbf{a}_\mu\times\boldsymbol{\xi}) + \frac{2}{g_V}\frac{1}{1+\boldsymbol{\xi}^2}\,(\partial_\mu\boldsymbol{\xi} + g_V\boldsymbol{\xi}\times\boldsymbol{\rho}_\mu).
\end{aligned}
\tag{3.21}
$$

These field combinations are seen to give rise to pair ($\pi\pi$, $\pi\rho$, πa_1) vertices, and other higher-order multiple-meson vertices.

As an example of how this model can be made to agree with the actual experimental results, we next give some attention to the meson sector. The axial-vector field can be given a mass by adding to the Lagrangian a chiral-invariant term proportional to $\mathrm{Tr}(u_\mu u^\mu)$; this will also generate the kinetic-energy term for the pion field. The vector field receives its mass from a term proportional to $\mathrm{Tr}(l_\mu l^\mu + r_\mu r^\mu)$, which is only invariant under *global* chiral transformations. This term also contributes to the mass of the axial-vector fields. However, in order to get an acceptable agreement with experiment (not only with respect to the empirical masses, but also with respect to some of the empirical meson coupling constants and decay widths), we have to extend the Lagrangian even further. A very general form in the context of the linear σ model is given by Ko and Rudaz [36]. Translating their results in terms of the fields of the nonlinear transformation (3.8), the masses for the vector and axial-vector mesons are obtained from the Lagrangian

$$\mathcal{L}_m = f_0^2 \mathrm{Tr}(u^\mu u_\mu) + \tfrac{1}{2} m_0^2 \mathrm{Tr}(l^\mu l_\mu + r^\mu r_\mu) - c\, g_V^2 (f_0 + s')^2 \mathrm{Tr}(l^\mu u^2 r_\mu u^{\dagger 2}). \tag{3.22}$$

We find that there is a mixing between the $\boldsymbol{\pi}$ and $\mathbf{a}_\mu$ fields, which can be removed by making a field redefinition for the $\mathbf{a}_\mu$ field, $\mathbf{a}_\mu = \mathbf{A}_\mu - h(\partial_\mu \boldsymbol{\pi} + g_V \boldsymbol{\pi} \times \boldsymbol{\rho})$, and then choosing h appropriately. Also, the kinetic-energy term for the pion field is no longer of the canonical form, and so we have to do a wave function renormalization: $\boldsymbol{\pi} = Z_\pi^{-1/2} \boldsymbol{\pi}_r$. The final result reads (for details, see Ref. [36])

$$\begin{aligned} m_\rho^2 &= m_0^2 - c\, g_V^2 f_0^2, \\ m_{a_1}^2 &= m_0^2 + (c+1) g_V^2 f_0^2, \\ Z_\pi &= 1 - g_V^2 f_0^2 / m_{a_1}^2, \end{aligned} \tag{3.23}$$

and the PCAC condition gives $f_\pi = Z_\pi^{1/2} f_0 = 92.4$ MeV. The parameter c is needed to get a simultaneous agreement for the ρ mass ($m_\rho = 770$ MeV), the a_1 mass ($m_{a_1} = 1.23$ GeV), and the ρNN coupling constant ($g_V = 5.04$ from $\rho^0 \to e^+ e^-$); all data from Ref. [26]. Substituting f_0 in the equation for Z_π leads to the two solutions $(Z_\pi, c) = (0.173, -0.131)$ and $(0.827, 1.25)$. Similarly, the numerical value for the $\rho\pi\pi$ coupling constant ($g_{\rho\pi\pi} = 6.05$ from $\rho \to \pi^+ \pi^-$ [26]) can be enforced by adding yet another term to the Lagrangian, introducing a new parameter κ_6 (see Ref. [36]). Having fixed both c and κ_6, some of the other meson properties such as the root-mean-square pion radius and a_1 decay widths also come out very favorably [36].

Although a full discussion of the meson sector is outside the scope of this paper, it is important to note that it is indeed possible to construct a chiral-invariant Lagrangian (sometimes at the expense of introducing new parameters and small chiral-symmetry violating terms) which does a reasonably good job in describing a variety of experimental results. Furthermore, the introduction of the axial-vector field as a gauge boson enforces a renormalization of the pion field, and so we explicitly need part of the Lagrangian for the meson sector to define this renormalization constant.

IV. SU(3) CHIRAL SYMMETRY

The extension to SU(3) is easily obtained by replacing the Pauli isospin matrices, τ_i, by the Gell-Mann matrices, λ_a. The two-component nucleon field is replaced by a 3×3 traceless baryon-octet matrix Ψ, where the left- and right-handed components transform as

$$\Psi_L \to L \Psi_L L^\dagger, \qquad \Psi_R \to R \Psi_R R^\dagger. \tag{4.1}$$

The meson content of the model still consists of the isosinglet scalar, σ, but the pseudoscalar triplet is extended to an octet by including the η_8 and the four K mesons. They are collectively denoted by π_a. These nine mesons are grouped into

$$\Sigma = \sigma + i \lambda_a \pi_a, \quad (a = 1, \ldots, 8), \tag{4.2}$$

which still transforms according to Eq. (3.2), but where L and R are now elements of SU(3).

Translating the definition (3.8) to the SU(3) case, we write

$$\left.\begin{array}{l} B_R = u\Psi_R u^\dagger \\ B_L = u^\dagger \Psi_L u \end{array}\right\}, \quad u(\xi_a) = \exp[i\lambda_a \xi_a] \equiv \exp\left[\frac{i\lambda_a \pi'_a}{2f_0}\right], \tag{4.3}$$

where we used a more convenient representation for $u(\xi_a)$, the elements of the coset space $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R/\mathrm{SU}(3)_V$. We will identify the primed fields with the octet of physical pseudoscalar fields, which is given by

$$\frac{1}{\sqrt{2}}\lambda_a \pi'_a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \tag{4.4}$$

and where the fields transform according to the SU(3) analogue of Eq. (3.11a). Following the phase convention of Ref. [37], the new octet of baryon fields reads

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & -\Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \tag{4.5}$$

which transforms as

$$B \to HBH^\dagger. \tag{4.6}$$

The slightly different transformation property of the baryon octet matrix requires that the SU(3) analogue to the nucleon derivative, $D_\mu N = (\partial_\mu + i\Gamma_\mu)N$ in Eq. (3.12), reads

$$D_\mu B = \partial_\mu B + i[\Gamma_\mu, B]. \tag{4.7}$$

Due to the transformation properties (4.1) of the original octet fields, we cannot simply copy the interaction Lagrangian of Eq. (3.4) to the SU(3) case. To ensure invariance under chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, the Σ field would then have to appear quadratically; i.e., the interaction Lagrangian has to be of the form $\mathrm{Tr}(\overline{\Psi}_L \Sigma \Psi_R \Sigma^\dagger + \overline{\Psi}_R \Sigma^\dagger \Psi_L \Sigma)$. Alternatively, we note that the field combinations $u\Sigma^\dagger u$ and $u^\dagger \Sigma u^\dagger$ both transform according to Eq. (4.6). Defining the combination

$$\chi_\pm = \tfrac{1}{2}\left(u^\dagger \Sigma u^\dagger \pm u\Sigma^\dagger u\right), \tag{4.8}$$

we thus find that the most simple interaction Lagrangian is given by

$$\mathcal{L}_I = -g_{s,1}\,\mathrm{Tr}(\overline{B}\chi_+ B) - g_{s,2}\,\mathrm{Tr}(\overline{B}B\chi_+), \tag{4.9}$$

where $g_{s,1}$ and $g_{s,2}$ are arbitrary constants. In principle, we also could have included a term of the form $g_p\mathrm{Tr}(\overline{B}\gamma_5\chi_- B)$, but that would have re-introduced a nonderivative pseudoscalar interaction[1], which we wanted to get rid of in the first place.

Unfortunately, the interaction Lagrangian (4.9) is not good enough if we want it to generate the empirical baryon masses, since it gives the same mass, $M = (g_{s,1} + g_{s,2})f_0$, for all baryons in the baryon octet. This problem can be solved by adding an octet of scalar fields, $\lambda_a\sigma_a$, where the isoscalar octet member is given a nonvanishing vacuum expectation value. We can then extend Σ by writing ($\lambda_0 = \sqrt{2/3}1\!\!1_3$)

$$\Sigma = F + \lambda_0 s_0 + \lambda_a s_a + i\lambda_a\pi_a, \qquad (a = 1, \ldots, 8), \tag{4.10}$$

with π_a the *original* pseudoscalar fields and F the vacuum expectation value of the scalar-nonet fields, i.e.

$$F = \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_1 & 0 \\ 0 & 0 & f_2 \end{pmatrix}, \tag{4.11}$$

with

$$\begin{aligned} f_1 &= \sqrt{\tfrac{2}{3}}\langle\sigma_0\rangle + \sqrt{\tfrac{1}{3}}\langle\sigma_8\rangle, \\ f_2 &= \sqrt{\tfrac{2}{3}}\langle\sigma_0\rangle - 2\sqrt{\tfrac{1}{3}}\langle\sigma_8\rangle. \end{aligned} \tag{4.12}$$

In the Appendix we show that now it is still possible to find a transformation $u(\xi)$ that transforms away the octet of original pseudoscalar fields while leaving the vacuum expectation matrix F invariant, and where the ξ_a fields can be identified with an octet of new pseudoscalar fields. As before, this transformation generates the combinations $\chi_\pm$ of Eq. (4.8). Both combinations contain the *original* pseudoscalar fields in a complicated way. However, χ_+ behaves like a set of scalar fields, and so we can simply *define* these new fields to be the physical scalar fields and drop any reference to the original scalar fields. Clearly, to zeroth order in the new pseudoscalar fields, the old and new scalar fields are the same. Dropping primes, the nonet of new scalar fields is given by

$$\chi_+ = F + \lambda_0 s_0 + \lambda_a s_a, \tag{4.13}$$

where s_0 now denotes the new scalar singlet and the octet is given by

$$\frac{1}{\sqrt{2}}\lambda_a s_a = \begin{pmatrix} \dfrac{a_0^0}{\sqrt{2}} + \dfrac{s_8}{\sqrt{6}} & a_0^+ & \kappa^+ \\ a_0^- & -\dfrac{a_0^0}{\sqrt{2}} + \dfrac{s_8}{\sqrt{6}} & \kappa^0 \\ \kappa^- & \overline{\kappa^0} & -\dfrac{2s_8}{\sqrt{6}} \end{pmatrix}. \tag{4.14}$$

---

[1]This in contrast to the SU(2) case, where we can choose $u(\xi_i)$ such that $\chi_- = 0$, and so all nonderivative pseudoscalar interactions are transformed away. The difference is due to the fact that $(\tau_i\pi_i)(\tau_j\pi_j)$ is proportional to $1\!\!1_2$, whereas $(\lambda_a\pi_a)(\lambda_b\pi_b)$ is not proportional to $1\!\!1_3$.

The pure octet isoscalar s_8 and pure singlet s_0 are mixed to give the physical $f_0(980)$ and $\varepsilon(760)$. Similarly, $-i\chi_-$ can be identified as a new isosinglet pseudoscalar field, not present before. Because it transforms as $H(-i\chi_-)H^\dagger$, it can be formally added to the octet pseudoscalar matrix, which completes the nonet. It must be remembered, however, that group transformations such as given for instance in Eq. (3.11a) and which involve different transformation matrices, are only valid for the (traceless) octet matrices.

As discussed for the SU(2) case at the end of the previous section, also in the SU(3) case the global chiral symmetry can be easily extended to a local chiral symmetry. The required gauge fields are now given by two octets of combinations of vector and axial-vector fields. The vector octet is given by

$$\frac{1}{\sqrt{2}}\lambda_a\rho_a = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}}+\frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}}+\frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \overline{K^{*0}} & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix}, \tag{4.15}$$

and a similar matrix for the axial-vector octet. Adding a mass term of the form (3.22) breaks the local symmetry and introduces a mixing between the pseudoscalar and axial-vector fields, which requires a redefinition of the axial-vector fields. This in turn means that the kinetic-energy term for the pseudoscalar fields is no longer of the canonical form, which is taken care of by renormalizing the pseudoscalar fields. Unfortunately, then it is not possible to get a satisfactory agreement for both the vector and axial-vector masses simultaneously. But also this problem can be solved. One way is to introduce yet another term [36] proportional to $\mathrm{Tr}(l_{\mu\nu}\Sigma r^{\mu\nu}\Sigma^\dagger)$, which renormalizes the vector and axial-vector fields. Another option is to modify the term proportional to m_0^2 by inserting the combinations $\Sigma\Sigma^\dagger$ and $\Sigma^\dagger\Sigma$. The advantage of the latter extension is that it does not involve a new parameter, while it does allow for a very satisfactory description of both vector and axial-vector masses. Therefore, a convenient spin-1 mass Lagrangian reads

$$\begin{aligned}\mathcal{L}_m^{(1)} = {}& f_1^2\mathrm{Tr}(u^\mu u_\mu) + \frac{m_0^2}{2f_1^2}\,\mathrm{Tr}(l^\mu\Sigma\Sigma^\dagger l_\mu + r^\mu\Sigma^\dagger\Sigma r_\mu) \\ & -c\, g_V^2\mathrm{Tr}(l^\mu\Sigma r_\mu\Sigma^\dagger).\end{aligned} \tag{4.16}$$

Using this Lagrangian, we find that the set $m_0 = 653$ MeV, $c = -0.131$, $f_1 = 222$ MeV, and $f_2 = 290$ MeV gives very satisfactory results for the ρ, a_1, K^*, and K_1 masses, as well as for the pion and kaon decay constants. Given these parameters, the renormalization constants are $Z_\pi = 0.173$, $Z_K = 0.225$, and $Z_{\eta_8} = 0.236$. Note that (Z_π, c) is the same as in the SU(2) case, and so the good phenomenology in the ρ–a_1 sector [36] is still valid. To make a further connection with the real physical world, we next also introduce isosinglet vector and axial-vector fields, which complete the experimentally observed nonets. The isosinglet fields are taken to be SU(3) invariants. This means that we can formally include them on the diagonal in the respective octet matrix representations.

Finally, to be complete, we should also list the kinetic-energy and mass terms for the scalar and pseudoscalar fields. The kinetic-energy term for the scalar fields is given by

$\mathrm{Tr}\,(D^\mu \chi_+^\dagger D_\mu \chi_+)$, with the covariant derivative defined as in Eq. (4.7) to ensure the chiral invariance. The kinetic-energy term for the pseudoscalar fields is already contained in $\mathrm{Tr}(u^\mu u_\mu)$. The simplest possible mass term for the scalar fields is of the form

$$\mathcal{L}^{(0)}_{m,s} = f_1^3 \mathrm{Tr}(\chi_+ A_s) - c_2 f_1^2 \mathrm{Tr}(\chi_+^\dagger \chi_+) - c_4 \mathrm{Tr}(\chi_+^\dagger \chi_+)^2, \tag{4.17}$$

where the first term with the diagonal matrix $A_s = \mathrm{diag}(x, x, y)$ breaks the chiral symmetry. This term has to be included in order to remove the terms linear in the s_0 and s_8 fields as generated by the last two terms. This condition fixes (x, y) in terms of (c_2, c_4). Fitting to the scalar masses as discussed in Sec. I, we find $(c_2, c_4) = (7.62, -0.46)$.

To generate the octet pseudoscalar masses, we again need a term that breaks the chiral symmetry. The singlet pseudoscalar mass is generated by $\mathrm{Tr}(\chi_-^\dagger \chi_-)$, which is chiral invariant. Hence, an appropriate Lagrangian for the pseudoscalar meson masses looks like

$$\mathcal{L}^{(0)}_{m,p} = \tfrac{1}{4} f_1^2 \mathrm{Tr}\left[(u^2 + u^{\dagger 2}) A_p\right] - \tfrac{1}{4} m^2_{\eta_0} \mathrm{Tr}(\chi_-^\dagger \chi_-), \tag{4.18}$$

with the diagonal matrix $A_p = \mathrm{diag}(x', x', y')$, where $x' = m_\pi^2/Z_\pi$ and $y' = 2m_K^2/Z_K - m_\pi^2/Z_\pi$. Note that with this choice the quadratic Gell-Mann–Okubo mass formula, $3m^2_{\eta_8} + m_\pi^2 - 4m_K^2 = 0$, is approximately satisfied. The mass m_{η_0} is chosen such that, with the proper mixing angle, we get the empirical η and η' masses.

The main achievement of the field transformations as discussed above is that we have constructed various 3×3 matrices of the general form $\Phi = (1/\sqrt{2})\lambda_c \phi_c$, where $c = 0, \ldots, 8$. These matrices contain scalar, pseudoscalar, vector, and axial-vector meson fields, and (except for the vector mesons) they all transform in the same way as the baryon fields. This allows us to define the following chiral-invariant combinations

$$\begin{aligned}
\left[\overline{B}B\Phi\right]_F &= \mathrm{Tr}(\overline{B}\Phi B) - \mathrm{Tr}(\overline{B}B\Phi), \\
\left[\overline{B}B\Phi\right]_D &= \mathrm{Tr}(\overline{B}\Phi B) + \mathrm{Tr}(\overline{B}B\Phi) - \tfrac{2}{3}\,\mathrm{Tr}(\overline{B}B)\mathrm{Tr}(\Phi), \\
\left[\overline{B}B\Phi\right]_S &= \mathrm{Tr}(\overline{B}B)\mathrm{Tr}(\Phi).
\end{aligned} \tag{4.19}$$

For a traceless matrix Φ (i.e., only octets of mesons), the S-type coupling $[\overline{B}B\Phi]_S$ vanishes, and the F- and D-type couplings can be written as $\mathrm{Tr}(\overline{B}[\Phi, B])$ and $\mathrm{Tr}(\overline{B}\{\Phi, B\})$, respectively; a notation often encountered in the literature. Defining a baryon-baryon-meson octet coupling constant g^{oct} and a baryon-baryon-singlet coupling constant g^{sin}, a general interaction Lagrangian which satisfies chiral symmetry can now be written in the form

$$\mathcal{L}_I = -g^{\mathrm{oct}}\sqrt{2}\left\{\alpha\left[\overline{B}B\Phi\right]_F + (1-\alpha)\left[\overline{B}B\Phi\right]_D\right\} - g^{\mathrm{sin}}\sqrt{\tfrac{1}{3}}\left[\overline{B}B\Phi\right]_S, \tag{4.20}$$

where α is known as the $F/(F+D)$ ratio, and the square-root factors are introduced for later convenience. The various field matrices are given by

$$\Phi_{\mathrm{sc}} = \frac{1}{\sqrt{2}}\left[F + \lambda_c s_c\right], \tag{4.21a}$$

$$\Phi_{\mathrm{vc}} = \frac{-i}{\sqrt{2}g_V}\gamma_\mu\left[u^\dagger\left(\partial^\mu + \tfrac{1}{2}ig_V\lambda_c(\rho^\mu + A^\mu - hD^\mu\pi)_c\right)u + u\left(\partial^\mu + \tfrac{1}{2}ig_V\lambda_c(\rho^\mu - A^\mu + hD^\mu\pi)_c\right)u^\dagger\right], \tag{4.21b}$$

$$\Phi_{\mathrm{ax}} = \frac{-i}{\sqrt{2}g_V}\gamma_5\gamma_\mu\left[u^\dagger\left(\partial^\mu + \tfrac{1}{2}ig_V\lambda_c(\rho^\mu + A^\mu - hD^\mu\pi)_c\right)u - u\left(\partial^\mu + \tfrac{1}{2}ig_V\lambda_c(\rho^\mu - A^\mu + hD^\mu\pi)_c\right)u^\dagger\right], \tag{4.21c}$$

where $D^\mu(\lambda\pi) = \partial^\mu(\lambda\pi) - \frac{1}{2} i g_V [(\lambda\pi), (\lambda\rho_\mu)]$, and h chosen such that the mixing between the axial-vector and pseudoscalar fields in the meson sector vanishes. Note that the pseudovector coupling of the pseudoscalar fields is already included in the axial-vector field matrix $\Phi_{\rm ax}$.

In addition to the electric coupling $\gamma^\mu \rho'_\mu$ [where ρ'_μ is a shorthand for the fields appearing in Eq. (4.21b)], it is also possible [35] to include a chiral-invariant magnetic coupling $\sigma^{\mu\nu}\rho'_{\mu\nu}$, where $\rho'_{\mu\nu}$ is the field strength tensor for the ρ'_μ field combination. This is due to the fact that ρ'_μ transforms according to Eq. (3.14), and so we can define a chiral-invariant field strength tensor $\rho'_{\mu\nu}$, as in Eq. (3.17). The transformation (3.14) also imposes the constraint that the chiral-variant D-type coupling $[\overline{B}B\Phi_{\rm vc}]_D$ should vanish, i.e., the electric $\alpha_V^e = 1$. This represents the so-called universality condition proposed by Sakurai [38]. Hence, the assumption that the ρ meson couples universally to the isospin current in this model is a direct consequence of chiral SU(3) symmetry. On the other hand, the magnetic α_V^m is still a free parameter.

Until now, it appears that we have not gained much by imposing the SU(3)×SU(3) chiral symmetry: the form of the interaction Lagrangian (4.20) can be written down immediately by assuming only an SU(3) symmetry, and has been known for a long time (see, e.g., Ref. [39]). However, the important difference is that the 3×3 matrices Φ are not just simple representations of the scalar, vector, or axial-vector meson fields, but they contain the pseudoscalar fields in a nonlinear way as well. This means that the chiral Lagrangian contains all kinds of multiple-meson (pair, triple, etc.) interactions not envisaged before. The coupling constants for these multiple-meson interactions can all be expressed in terms of the single-meson interaction coupling constants. This will be the subject of Sec. VI. Furthermore, the pseudoscalar fields are renormalized due to the introduction of the axial-vector fields. Hence, already in leading order the chiral Lagrangian gives interactions between baryons and pseudoscalar mesons that are slightly different from what is obtained with the standard (i.e., nonchiral) Lagrangian. Also, the coupling constants of the pseudoscalar mesons are directly related to those of the axial-vector mesons. Finally, the imposed symmetry allows us to fit all the octet meson masses with only two parameters for each octet.

V. SINGLE-MESON VERTICES

We will first look at the single-meson baryon-baryon coupling constants, i.e., the interaction terms linear in the meson fields. Let us drop for a moment the Lorentz character of the interaction vertices ($\mathbb{1}_4$ in spinor space for scalar mesons, $\gamma_5\gamma_\mu\partial^\mu$ for pseudoscalar mesons, γ_μ and $\sigma_{\mu\nu}\partial^\nu$ for vector mesons, and $\gamma_5\gamma_\mu$ for axial-vector mesons) and take as an example the nonet of pseudoscalar mesons. The derivative (pseudovector-coupled) pseudoscalar-meson interaction Lagrangian to lowest order in the fields is then of the form

$$\mathcal{L}_{\rm pv} = \mathcal{L}_{\rm pv}^{\{1\}} + \mathcal{L}_{\rm pv}^{\{8\}}, \tag{5.1}$$

where the S-type coupling in Eq. (4.20) gives the singlet interaction Lagrangian

$$\begin{aligned} m_\pi \mathcal{L}_{\rm pv}^{\{1\}} = & -f_{NN\eta_0}(\overline{N}N)\eta_0 - f_{\Xi\Xi\eta_0}(\overline{\Xi}\Xi)\eta_0 \\ & -f_{\Sigma\Sigma\eta_0}(\overline{\mathbf{\Sigma}}\cdot\mathbf{\Sigma})\eta_0 - f_{\Lambda\Lambda\eta_0}(\overline{\Lambda}\Lambda)\eta_0, \end{aligned} \tag{5.2}$$

with the (derivative) pseudovector coupling constants

$$f_{NN\eta_0} = f_{\Xi\Xi\eta_0} = f_{\Sigma\Sigma\eta_0} = f_{\Lambda\Lambda\eta_0} = f_{\rm pv}^{\rm sin}, \tag{5.3}$$

and where we introduced the charged-pion mass as a scaling mass to make the pseudovector coupling f dimensionless. The interaction Lagrangian for the meson octet is obtained by evaluating the F- and D-type couplings in Eq. (4.20), and can be written as

$$\begin{aligned} m_\pi \mathcal{L}_{\rm pv}^{\{8\}} = & -f_{NN\pi}(\overline{N}\boldsymbol{\tau}N)\cdot\boldsymbol{\pi} - f_{\Xi\Xi\pi}(\overline{\Xi}\boldsymbol{\tau}\Xi)\cdot\boldsymbol{\pi} - f_{\Lambda\Sigma\pi}(\overline{\Lambda}\boldsymbol{\Sigma}+\overline{\boldsymbol{\Sigma}}\Lambda)\cdot\boldsymbol{\pi} + if_{\Sigma\Sigma\pi}(\overline{\boldsymbol{\Sigma}}\times\boldsymbol{\Sigma})\cdot\boldsymbol{\pi} \\ & -f_{NN\eta_8}(\overline{N}N)\eta_8 - f_{\Xi\Xi\eta_8}(\overline{\Xi}\Xi)\eta_8 - f_{\Lambda\Lambda\eta_8}(\overline{\Lambda}\Lambda)\eta_8 - f_{\Sigma\Sigma\eta_8}(\overline{\boldsymbol{\Sigma}}\cdot\boldsymbol{\Sigma})\eta_8 \\ & -f_{\Lambda NK}\left[(\overline{N}K)\Lambda + \overline{\Lambda}(\overline{K}N)\right] - f_{\Xi\Lambda K}\left[(\overline{\Xi}K_c)\Lambda + \overline{\Lambda}(\overline{K_c}\Xi)\right] \\ & -f_{\Sigma NK}\left[\overline{\boldsymbol{\Sigma}}\cdot(\overline{K}\boldsymbol{\tau}N) + (\overline{N}\boldsymbol{\tau}K)\cdot\boldsymbol{\Sigma}\right] - f_{\Xi\Sigma K}\left[\overline{\boldsymbol{\Sigma}}\cdot(\overline{K_c}\boldsymbol{\tau}\Xi) + (\overline{\Xi}\boldsymbol{\tau}K_c)\cdot\boldsymbol{\Sigma}\right]. \end{aligned} \tag{5.4}$$

Here we introduced the doublets

$$N = \begin{pmatrix} p \\ n \end{pmatrix}, \quad \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad K_c = \begin{pmatrix} \overline{K^0} \\ -K^- \end{pmatrix}, \tag{5.5}$$

and $\boldsymbol{\Sigma}$ and $\boldsymbol{\pi}$ are isovectors with phases chosen [39] such that

$$\boldsymbol{\Sigma}\cdot\boldsymbol{\pi} = \Sigma^+\pi^- + \Sigma^0\pi^0 + \Sigma^-\pi^+. \tag{5.6}$$

The octet coupling constants are given by the following expressions ($f \equiv f_{\rm pv}^{\rm oct}$)

$$\begin{aligned} f_{NN\pi} &= f, & f_{NN\eta_8} &= \tfrac{1}{\sqrt{3}}f(4\alpha-1), & f_{\Lambda NK} &= -\tfrac{1}{\sqrt{3}}f(1+2\alpha), \\ f_{\Xi\Xi\pi} &= -f(1-2\alpha), & f_{\Xi\Xi\eta_8} &= -\tfrac{1}{\sqrt{3}}f(1+2\alpha), & f_{\Xi\Lambda K} &= -\tfrac{1}{\sqrt{3}}f(4\alpha-1), \\ f_{\Lambda\Sigma\pi} &= \tfrac{2}{\sqrt{3}}f(1-\alpha), & f_{\Lambda\Lambda\eta_8} &= -\tfrac{2}{\sqrt{3}}f(1-\alpha), & f_{\Sigma NK} &= f(1-2\alpha), \\ f_{\Sigma\Sigma\pi} &= 2f\alpha, & f_{\Sigma\Sigma\eta_8} &= \tfrac{2}{\sqrt{3}}f(1-\alpha), & f_{\Xi\Sigma K} &= f. \end{aligned} \tag{5.7}$$

Similar relations (without the scaling mass m_π) are found for the coupling constants of the scalar, vector, and axial-vector mesons. Note, however, that for the pseudoscalar mesons the relations (5.3) and (5.7) should be slightly modified due to the difference in renormalization factors Z_π, Z_K, Z_η, and $Z_{\eta'}$ (see Sec. IV).

For each type of meson there are only four parameters: the singlet coupling, the octet coupling, the $F/(F+D)$ ratio, and the mixing angle to generate the physical isoscalar mesons from the pure octet and singlet fields. In most cases we can impose theoretical and experimental constraints on these parameters. This will be discussed next. As already mentioned before, the axial-vector mesons are very heavy, and hence are not expected to play an important role in low-energy potential models, but we will still discuss them here for reasons of completeness.

A. Scalar mesons

Because the existence of a nonet of scalar mesons with masses below 1 GeV is still controversial, the constraints on the SU(3) parameters solely depend on the particular theoretical model one wants to use to describe the scalar mesons. As a matter of fact, it is not at all clear whether it is indeed valid to impose an SU(3) symmetry. However, in order to limit the number of free parameters, we will here assume the standard SU(3) relations and assume that the physical $\varepsilon(760)$ and $f_0(980)$ mesons are admixtures of the pure octet σ_8 and pure singlet σ_0 fields, in terms of the scalar mixing angle θ_S,

$$\begin{aligned} |f_0\rangle &= \sin\theta_S\,|\sigma_8\rangle + \cos\theta_S\,|\sigma_0\rangle, \\ |\varepsilon\rangle &= \cos\theta_S\,|\sigma_8\rangle - \sin\theta_S\,|\sigma_0\rangle. \end{aligned} \tag{5.8}$$

A possible value for the mixing angle is obtained by assuming that the scalar mesons are $\bar{q}^2q^2$ states [30], and that the $\varepsilon(760)$ does not contain any strange quarks (hence, its low mass). This implies an ideal mixing angle $\theta_S = 35.3°$.

A value for α_S can be determined by assuming that the baryon masses are generated by the nonvanishing vacuum expectation value of the σ_0 and σ_8 scalar fields. Defining the vacuum expectation values f_1 and f_2 as in Eq. (4.12), we have

$$\begin{aligned} \sqrt{3}\langle\sigma_0\rangle &= \sqrt{\tfrac{1}{2}}\,(2f_1 + f_2), \\ \sqrt{3}\langle\sigma_8\rangle &= (f_1 - f_2), \end{aligned} \tag{5.9}$$

or, assuming the ideal mixing $\theta_S = 35.3°$,

$$f_1 = \langle f_0(980)\rangle, \quad f_2 = -\sqrt{2}\langle\varepsilon(760)\rangle. \tag{5.10}$$

From the interaction Lagrangians (5.2) and (5.4) for the scalar fields we find the following relations for the baryon masses,

$$\begin{aligned} M_N &= M_0 - \tfrac{1}{3}g_{\rm sc}^{\rm oct}(4\alpha_S - 1)(f_2 - f_1), \\ M_\Lambda &= M_0 - \tfrac{2}{3}g_{\rm sc}^{\rm oct}(\alpha_S - 1)(f_2 - f_1), \\ M_\Sigma &= M_0 + \tfrac{2}{3}g_{\rm sc}^{\rm oct}(\alpha_S - 1)(f_2 - f_1), \\ M_\Xi &= M_0 + \tfrac{1}{3}g_{\rm sc}^{\rm oct}(2\alpha_S + 1)(f_2 - f_1), \end{aligned} \tag{5.11}$$

with $M_0 = g_{\rm sc}^{\rm sin}\,(2f_1 + f_2)/\sqrt{6} = \frac{1}{2}(M_\Lambda + M_\Sigma)$. According to the relations given above, these masses satisfy the equality $2M_N + 2M_\Xi = 3M_\Lambda + M_\Sigma$, which is indeed approximately true experimentally (4516 MeV versus 4537 MeV). Solving for α_S, we find $\alpha_S = 1.42$ and $g_{\rm sc}^{\rm oct}(f_2 - f_1) = 136.5$ MeV.

Finally, using the estimates for f_1 and f_2 as given in Sec. IV, we find $g_{\rm sc}^{\rm sin} = 3.8$ and $g_{\rm sc}^{\rm oct} = 2.0$.

B. Pseudoscalar mesons

The mixing angle θ_{PS} for the pseudoscalar mesons is defined by

$$\begin{aligned} |\eta\rangle &= \cos\theta_{PS}\,|\eta_8\rangle - \sin\theta_{PS}\,|\eta_0\rangle, \\ |\eta'\rangle &= \sin\theta_{PS}\,|\eta_8\rangle + \cos\theta_{PS}\,|\eta_0\rangle. \end{aligned} \tag{5.12}$$

The linear and quadratic Gell-Mann–Okubo mass formulas give [26] $\theta_{PS} \approx -23^\circ$ and $\theta_{PS} \approx -10^\circ$, respectively. The current experimental evidence, however, seems to favor [40] $\theta_{PS} \approx -20^\circ$.

The axial-vector current F/D ratio, obtained from the Cabibbo theory of semileptonic decays of baryons, gives [41] 0.575±0.0165, or $\alpha_{PS} = 0.365 \pm 0.007$.

The πNN pseudovector coupling constant is obtained from the Goldberger-Treiman relation [34], which gives $f^{\rm oct}_{\rm pv} = f_{NN\pi} = g_A(0)m_\pi/2f_\pi = 0.952 \pm 0.003$. It can also be extracted from multienergy partial-wave analyses of pp, np, and $\bar{p}p$ scattering data [42], which yields for the coupling constant at the pion pole $f^2_{NN\pi}/4\pi = 0.0745 \pm 0.0006$. This latter result has to be extrapolated to $t = 0$ before comparing it with the Goldberger-Treiman result.

Finally, we can also give an estimate for the singlet pseudovector coupling constant. The estimate is based on the fact that we can rewrite the η_8 and η_0 in a nonstrange-strange basis, rather than the standard $\{u, d, s\}$ quark basis, and then assume that the purely strange-quark state does not couple to the nucleon. To be specific, in the standard quark basis

$$
\begin{aligned}
|\eta_8\rangle &= \sqrt{\tfrac{1}{6}}|\bar{u}u + \bar{d}d - 2\bar{s}s\rangle, \\
|\eta_0\rangle &= \sqrt{\tfrac{1}{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle,
\end{aligned}
\tag{5.13}
$$

and so the purely strange-quark state can be expressed as $|S\rangle = |\bar{s}s\rangle = (|\eta_0\rangle - \sqrt{2}|\eta_8\rangle)/\sqrt{3}$. This state does not couple to the nucleons provided that $f_{NN\eta_0} = \sqrt{2}\, f_{NN\eta_8}$ or, equivalently, $f^{\rm sin}_{\rm pv} = \sqrt{\tfrac{2}{3}}(4\alpha_{PS} - 1)f^{\rm oct}_{\rm pv}$.

C. Vector mesons

The mixing angle θ_V for the vector mesons is defined by

$$
\begin{aligned}
|\omega\rangle &= \sin\theta_V\,|\omega_8\rangle + \cos\theta_V\,|\omega_0\rangle, \\
|\phi\rangle &= \cos\theta_V\,|\omega_8\rangle - \sin\theta_V\,|\omega_0\rangle.
\end{aligned}
\tag{5.14}
$$

Ideal mixing (i.e., the ϕ meson is a pure $\bar{s}s$ state) gives $\theta_V = 35.3^\circ$, which is very close to the experimental value ($\theta_V \approx 35^\circ$) and the values found for the linear ($\theta_V \approx 36^\circ$) and quadratic ($\theta_V \approx 39^\circ$) Gell-Mann–Okubo mass formulas [26].

As mentioned earlier, the transformation property of the connection Γ_μ requires that $\alpha^e_V = 1$ for the electric coupling of the vector mesons, which is the universality assumption [38].

The ρNN coupling constant is obtained from the electromagnetic decay $\rho^0 \to e^+e^-$. The vector-meson dominance (VMD) hypothesis assumes that this decay proceeds via the photon and that the ρ^0–γ coupling is proportional to the ρNN coupling [38]. The decay width [26] $\Gamma(\rho^0 \to e^+e^-) = 6.77 \pm 0.32$ keV then gives $g^{\rm oct}_{\rm vc} = g_{NN\rho} = 2.52 \pm 0.06$. Note the factor of 2 difference between the definition of $g^{\rm oct}_{\rm vc}$ in Eq. (4.20) and g_V in Eq. (4.21b).

If we assume that the ϕ meson is a pure $\bar{s}s$ state, which does not couple to the nucleons (i.e., an ideal mixing angle $\theta_V = 35.3^\circ$), then we find for the singlet coupling $g^{\rm sin}_{\rm vc} = \sqrt{6}g^{\rm oct}_{\rm vc}$ or, equivalently, $g_{NN\omega} = 3g_{NN\rho}$. Perhaps a better estimate is obtained from the decay width [26] $\Gamma(\omega \to e^+e^-) = 0.60 \pm 0.02$ keV, which suggests

$$\frac{g_{NN\omega}}{g_{NN\rho}} = \left[\frac{m_\omega \Gamma_{\rho^0 \to e^+e^-}}{m_\rho \Gamma_{\omega \to e^+e^-}}\right]^{1/2} = 3.4 \pm 0.1.$$

Keeping only the leading-order contribution proportional to $\sigma_{\mu\nu}\partial^\nu \rho^\mu$, the magnetic coupling of the vector mesons is defined as $f_{\rm vc}^{\rm oct}/2\mathcal{M}$, where the scaling mass $\mathcal{M}$, taken to be the proton mass, is included to make $f_{\rm vc}^{\rm oct}$ dimensionless. Following Ref. [43], the SU(6) result for α_V^m can be expressed as $\alpha_V^m = (4M_8 - m_{v8})/(10M_8 + 2m_{v8})$, where M_8 denotes the average mass in the baryon octet and m_{v8} the average mass in the vector-meson octet. This gives $\alpha_V^m \approx 0.28$ for the relativistic SU(6) case, while $\alpha_V^m = \frac{2}{5}$ for the static SU(6) case.

Again applying the VMD hypothesis and assuming that the lowest-mass vector mesons (ρ, ω, ϕ) saturate the nucleon electromagnetic form factors, the magnetic couplings are given in terms of the anomalous magnetic moments of the proton and neutron. This gives $(f/g)_{NN\rho} = \kappa_p - \kappa_n = 3.71$, and the isoscalar values are expected to be close to $(f/g)_{NN\omega} + (f/g)_{NN\phi} \approx \kappa_p + \kappa_n = -0.12$.

D. Axial-vector mesons

We follow Ref. [44] in estimating the value for the mixing angle θ_A, defined by

$$\begin{aligned} |f_1(1285)\rangle &= \cos\theta_A\,|a_8\rangle - \sin\theta_A\,|a_0\rangle, \\ |f_1(1420)\rangle &= \sin\theta_A\,|a_8\rangle + \cos\theta_A\,|a_0\rangle. \end{aligned} \tag{5.15}$$

For that purpose we rewrite the $f_1(1285)$ and $f_1(1420)$ mesons in the nonstrange-strange basis,

$$\begin{aligned} |f_1(1285)\rangle &= \cos\phi_A\,|A_{NS}\rangle - \sin\phi_A\,|A_S\rangle, \\ |f_1(1420)\rangle &= \sin\phi_A\,|A_{NS}\rangle + \cos\phi_A\,|A_S\rangle. \end{aligned} \tag{5.16}$$

The observed decay widths give a mixing angle $\phi_A \approx 12^\circ$. In the singlet-octet basis the mixing angle is then given by $\theta_A = \phi_A - \arctan\sqrt{2} = -42.7^\circ$.

The axial-vector coupling constants are closely related to the pseudovector coupling constants due to the mixing between the axial-vector and pseudoscalar fields; see the end of Sec. III. The redefinition of the axial-vector fields and the renormalization of the pseudoscalar fields gives the relation between the coupling constants as

$$g_{\rm ax}^{\rm oct} = g_{\rm vc}^{\rm oct}\,\frac{g_A(0)}{Z_\pi} = g_{\rm vc}^{\rm oct} g_A(0)\left(1 - \frac{g_V^2 f_1^2}{m_{a_1}^2}\right)^{-1}. \tag{5.17}$$

The above relation can be recast into the form

$$g_{NNa_1} = \frac{m_{a_1}}{m_\pi} f_{NN\pi}\sqrt{\frac{1-Z_\pi}{Z_\pi}}, \tag{5.18}$$

where the square root equals 1 for $Z_\pi = 1/2$. The latter choice gives the familiar result [23,35,45] $g_{NNa_1} = (m_{a_1}/m_\pi) f_{NN\pi} \approx 8.4$. But this assumes $m_{a_1} = \sqrt{2}m_\rho$, which is not supported by experiment. With our previous choice $(Z_\pi, c) = (0.173, -0.131)$, the axial-vector coupling constant comes out to be much larger, $g_{\rm ax}^{\rm oct} \approx 18$. A more moderate value is obtained by choosing the alternative solution to Eq. (3.23), $(Z_\pi, c) = (0.827, 1.25)$, which gives $g_{\rm ax}^{\rm oct} \approx 3.8$. An estimate for the a_1NN coupling constant from experiment is based on the idea of axial-vector meson dominance, which relates it to the axial-vector coupling constant of the weak interaction, $g_A(0)$, and the decay constant f_{a_1}, as defined by the isovector a_1-to-vacuum matrix element of the hadronic axial-vector current. With our definition (4.20) this gives [44] $g_{\rm ax}^{\rm oct} = 4.7 \pm 0.6$. Hence, we find contradictory results: the larger $|c|$ solution gives reasonable agreement with the empirical g_{NNa_1} coupling constant, whereas the smaller $|c|$ solution gives better phenomenology for a_1 decay widths and the pion charge radius [36]. This needs to be further explored.

VI. DOUBLE-MESON VERTICES

A. Vector double-meson vertices

The vector double-meson vertices are obtained from an expansion of Eq. (4.21b). The expansion to second order in the meson fields is given by

$$\Phi_{\rm vc} = \frac{1}{\sqrt{2}}\,\gamma^\mu(\lambda\rho_\mu) - \frac{i(1-2g_V f_1 h)}{4\sqrt{2}g_V f_1^2}\,\gamma^\mu\left[(\lambda\pi), \partial_\mu(\lambda\pi)\right] - \frac{i}{2\sqrt{2}f_1}\,\gamma^\mu\left[(\lambda\pi), (\lambda A_\mu)\right] + \ldots \quad (6.1)$$

Hence, it should be obvious that the pair interaction Lagrangians can be obtained by a simple replacement of the meson fields in the single-meson interaction Lagrangians. For example, the two-pion interaction Lagrangian is obtained from the vector version of Eq. (5.4) with the replacement

$$\gamma^\mu \boldsymbol{\rho}_\mu \longrightarrow \gamma^\mu(\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi}), \quad (6.2)$$

which gives

$$\begin{aligned} m_\pi^2 \mathcal{L}_{(\pi\pi)} = & -g_{NN(\pi\pi)}(\overline{N}\gamma^\mu\boldsymbol{\tau}N)\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}) - g_{\Xi\Xi(\pi\pi)}(\overline{\Xi}\gamma^\mu\boldsymbol{\tau}\Xi)\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}) \\ & -g_{\Lambda\Lambda(\pi\pi)}(\overline{\Lambda}\gamma^\mu\boldsymbol{\Sigma} + \overline{\boldsymbol{\Sigma}}\gamma^\mu\Lambda)\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}) + ig_{\Sigma\Sigma(\pi\pi)}(\overline{\boldsymbol{\Sigma}}\times\gamma^\mu\boldsymbol{\Sigma})\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}). \end{aligned} \quad (6.3)$$

Here we introduced the square of the charged-pion mass to make the coupling constants dimensionless. Substituting the appropriate renormalization factors and $\frac{1}{2}g_V = g_{NN\rho}$, the coupling constants are given by

$$g_{B'B(\pi\pi)} = \frac{m_\pi^2}{4f_1^2}\,\frac{2Z_\pi - 1}{Z_\pi}\,\frac{g_{B'B\rho}}{g_{NN\rho}}, \quad (6.4)$$

for $B'B = NN$, $\Xi\Xi$, $\Lambda\Sigma$, and $\Sigma\Sigma$. Note that by choosing [23,35,45] $Z_\pi = \frac{1}{2}$, i.e., making the assumption that $m_{a_1} = \sqrt{2}m_\rho$, all the $(\pi\pi)$ pair interactions are absent. This agrees with the assumption of vector meson dominance which in its most stringent form states that those multiple-meson interactions which can arise through the exchange of one single vector meson should not also occur directly [35]. However, experimentally $m_{a_1} \neq \sqrt{2}m_\rho$, and so here the $(\pi\pi)$ pair interactions are still present in the interaction Lagrangian. We can also view the $(\pi\pi)$ pair exchange as to proceed via ρ exchange where the ρ-meson propagator is approximated by a constant (which should be adequate for a heavy meson at low energies). Comparing the coupling constant for this and any other pair vertex with the effective coupling constant as obtained in such a meson saturation picture then gives an indication of how good this picture really is.

The (πK) and $(\eta_8 K)$ combinations occur at the same place in the $\Phi_{\rm vc}$ matrix as the K^* fields. Therefore, the (πK) and $(\eta_8 K)$ interactions are obtained by substituting, respectively,

$$\begin{aligned} \gamma^\mu K^*_\mu &\longrightarrow -i\gamma^\mu \boldsymbol{\tau}\cdot(\boldsymbol{\pi}\partial_\mu K - K\partial_\mu \boldsymbol{\pi}), \\ \gamma^\mu K^*_\mu &\longrightarrow -i\gamma^\mu (\eta_8\partial_\mu K - K\partial_\mu \eta_8). \end{aligned} \tag{6.5}$$

This gives the coupling constants

$$\begin{aligned} g_{B'B(\pi K)} &= \frac{m_\pi^2}{4\sqrt{2}f_1^2}\,\frac{2Z_{K\pi}-1}{Z_{K\pi}}\,\frac{g_{B'BK^*}}{g_{NN\rho}}, \\ g_{B'B(\eta_8 K)} &= \frac{m_\pi^2}{4\sqrt{2}f_1^2}\,\frac{2Z_{K\eta_8}-1}{Z_{K\eta_8}}\,\frac{\sqrt{3}g_{B'BK^*}}{g_{NN\rho}}, \end{aligned} \tag{6.6}$$

for $B'B = \Lambda N$, $\Xi\Lambda$, ΣN, and $\Xi\Sigma$. Here we defined averaged renormalization constants $Z_{ab}^2 = Z_a Z_b$ to simplify the expressions.

In the $\Phi_{\rm vc}$ matrix, the two-kaon interactions occur on the diagonal and on the same off-diagonal places as the $\boldsymbol{\rho}$ fields. We can therefore split up the contributions into two parts. One part behaves like an isoscalar octet field, and so we have the replacement

$$\gamma^\mu \omega_{8\mu} \longrightarrow i\gamma^\mu(\overline{K}\partial_\mu K - \overline{K_c}\partial_\mu K_c), \tag{6.7}$$

and the coupling constants

$$g_{BB(KK)} = \frac{m_\pi^2}{8f_1^2}\,\frac{2Z_K-1}{Z_K}\,\frac{\sqrt{3}g_{BB\omega_8}}{g_{NN\rho}}. \tag{6.8}$$

The remaining part behaves like an isovector field $\mathbf{K}_{2\mu}$ which can be written as

$$\mathbf{K}_{2\mu} = (\overline{K}\boldsymbol{\tau}\partial_\mu K) + (\overline{K_c}\boldsymbol{\tau}\partial_\mu K_c), \tag{6.9}$$

The Lagrangian for this type of two-kaon interaction is obtained by making the substitution $\boldsymbol{\rho}_\mu \to i\mathbf{K}_{2\mu}$, which gives the coupling constants

$$g_{B'B(K\tau K)} = \frac{m_\pi^2}{8f_1^2}\,\frac{2Z_K-1}{Z_K}\,\frac{g_{B'B\rho}}{g_{NN\rho}}, \tag{6.10}$$

where again $B'B = NN$, $\Xi\Xi$, $\Lambda\Sigma$, and $\Sigma\Sigma$.

The double-meson interactions consisting of a pseudoscalar and an axial-vector meson are analogously obtained by the substitutions

$$\begin{aligned}
\gamma^\mu \boldsymbol{\rho}_\mu &\longrightarrow \gamma^\mu(\boldsymbol{\pi}\times\mathbf{A}_\mu),\\
\gamma^\mu K^*_\mu &\longrightarrow -i\gamma^\mu \boldsymbol{\tau}\cdot(\boldsymbol{\pi} K_{1\mu} - K\mathbf{A}_\mu),\\
\gamma^\mu K^*_\mu &\longrightarrow -i\gamma^\mu(\eta_8 K_{1\mu} - K a_{8\mu}),\\
\gamma^\mu \omega_{8\mu} &\longrightarrow i\gamma^\mu(\overline{K} K_{1\mu} - \overline{K_c} K_{1\mu\, c}),\\
\gamma^\mu \boldsymbol{\rho}_\mu &\longrightarrow i\gamma^\mu(\overline{K}\boldsymbol{\tau} K_{1\mu} + \overline{K_c}\boldsymbol{\tau} K_{1\mu\, c}).
\end{aligned} \tag{6.11}$$

Including averaged renormalization constants, the coupling constants are given by

$$\begin{aligned}
g_{B'B(\pi a_1)} &= \frac{m_\pi}{\sqrt{Z_\pi} f_1}\, g_{B'B\rho},\\
g_{B'B(\pi K_1)} &= -g_{B'B(K a_1)} = \frac{m_\pi}{\sqrt{Z_{K\pi}} f_1}\, \sqrt{\tfrac{1}{2}} g_{B'BK^*},\\
g_{B'B(\eta_8 K_1)} &= -g_{B'B(K a_8)} = \frac{m_\pi}{\sqrt{Z_{K\eta_8}} f_1}\, \sqrt{\tfrac{3}{2}} g_{B'BK^*},\\
g_{B'B(K K_1)} &= \frac{m_\pi}{2\sqrt{Z_K} f_1}\, \sqrt{3} g_{B'B\omega_8},\\
g_{B'B(K\tau K_1)} &= \frac{m_\pi}{2\sqrt{Z_K} f_1}\, g_{B'B\rho},
\end{aligned} \tag{6.12}$$

with the relevant substitutions for the $B'B$ baryon combinations.

B. Axial-vector double-meson vertices

The expansion of Eq. (4.21c) to second order in the meson fields gives

$$\Phi_{\rm ax} = \frac{1}{\sqrt{2}}\gamma^5\gamma^\mu(\lambda A_\mu) + \frac{1-g_V f_1 h}{\sqrt{2} g_V f_1}\gamma^5\gamma^\mu\partial_\mu(\lambda\pi) - \frac{i(1-g_V f_1 h)}{2\sqrt{2} f_1}\gamma^5\gamma^\mu\left[(\lambda\pi),(\lambda\rho_\mu)\right] + \dots \tag{6.13}$$

Completely analogous to the previous section, we make the substitutions

$$\begin{aligned}
\gamma^5\gamma^\mu \mathbf{A}_\mu &\longrightarrow \gamma^5\gamma^\mu(\boldsymbol{\pi}\times\boldsymbol{\rho}_\mu),\\
\gamma^5\gamma^\mu K_{1\mu} &\longrightarrow -i\gamma^5\gamma^\mu \boldsymbol{\tau}\cdot(\boldsymbol{\pi} K^*_\mu - K\boldsymbol{\rho}_\mu),\\
\gamma^5\gamma^\mu K_{1\mu} &\longrightarrow -i\gamma^5\gamma^\mu(\eta_8 K^*_\mu - K\omega_{8\mu}),\\
\gamma^5\gamma^\mu a_{8\mu} &\longrightarrow i\gamma^5\gamma^\mu(\overline{K} K^*_\mu - \overline{K_c} K^*_{\mu\, c}),\\
\gamma^5\gamma^\mu \mathbf{A}_\mu &\longrightarrow i\gamma^5\gamma^\mu(\overline{K}\boldsymbol{\tau} K^*_\mu + \overline{K_c}\boldsymbol{\tau} K^*_{\mu\, c}).
\end{aligned} \tag{6.14}$$

The coupling constants are most easily expressed in terms of $g_{NN\rho}$ and the pseudovector coupling constants:

$$\begin{aligned}
g_{B'B(\pi\rho)} &= 2 g_{NN\rho} f_{B'B\pi},\\
g_{B'B(\pi K^*)} &= -g_{B'B(K\rho)} = \sqrt{2} g_{NN\rho} f_{B'BK},\\
g_{B'B(\eta_8 K^*)} &= -g_{B'B(K\omega_8)} = \sqrt{6} g_{NN\rho} f_{B'BK},\\
g_{B'B(KK^*)} &= \sqrt{3} g_{NN\rho} f_{B'B\eta_8},\\
g_{B'B(K\tau K^*)} &= g_{NN\rho} f_{B'B\pi}.
\end{aligned} \tag{6.15}$$

C. Possible extensions

Given the transformation property of the $\Phi_{\rm sc}$ and $\Phi_{\rm ax}$ fields, we can of course arbitrarily add more interaction Lagrangians of the type (4.20) with higher orders in the meson fields. In general this will introduce a number of new free parameters. Because there is no guarantee that we can actually construct a baryon-baryon potential without any free parameters for the coupling constants, it might be very useful to still have some flexibility (i.e., free parameters) in the model. It will be convenient, of course, if at the same time we can still satisfy the empirical constraints for the single-meson coupling constants as given in Sec. V. With this purpose in mind, we therefore now discuss some possible extensions in more detail.

Although most of the scalar meson masses are already close to 1 GeV, the low mass of $\sim$500 MeV in the two-pole approximation [29] to the broad $\varepsilon(760)$ meson makes the double-scalar interactions worthwhile to be investigated. We can extend the scalar interaction Lagrangian by adding $\sqrt{2}(g_{ss}/m_\pi)(\Phi_{\rm sc})^2$ to $\Phi_{\rm sc}$ of Eq. (4.21a), where the charged-pion mass is introduced to make the coupling constant dimensionless. Since the octet and singlet parts in the interaction Lagrangian (4.20) have independent coupling constants, we can also define two independent scalar-scalar coupling constants. Hence, writing $X = \lambda_c s_c$ we have the substitution

$$\begin{aligned} g_{\rm sc}^{\rm oct}\Phi_{\rm sc}^{(8)} \longrightarrow \frac{1}{\sqrt{2}}\Big\{ & g_{\rm sc}^{\rm oct}\,(F+X) \\ & +\frac{g_{ss}^{(8)}}{m_\pi}\Big(F^2+(FX+XF)+X^2\Big)\Big\}, \end{aligned} \tag{6.16}$$

and a similar expression for the singlet part. We still want to satisfy the baryon-mass relations of Sec. V A, which gives the constraints

$$\begin{aligned} \sqrt{\tfrac{1}{6}}\left[g_{\rm sc}^{\rm sin}(2f_1+f_2)+\frac{g_{ss}^{(1)}}{m_\pi}(2f_1^2+f_2^2)\right] &= 1153.5\text{ MeV}, \\ \left[g_{\rm sc}^{\rm oct}(f_2-f_1)+\frac{g_{ss}^{(8)}}{m_\pi}(f_2^2-f_1^2)\right] &= 136.5\text{ MeV}, \end{aligned} \tag{6.17}$$

where we substituted $M_0 = 1153.5$ MeV. Proceeding along the lines of the previous section, the evaluation of $(\lambda_c s_c)^2$ under the assumption of ideal mixing allows us to read off the double-scalar interactions by making the following substitutions

$$\begin{aligned} g_{\rm sc}^{\rm oct}\mathbf{a}_0 &\to g_{ss}^{(8)}\left[2f_0\mathbf{a}_0+\tfrac{1}{2}(\overline{\kappa}\boldsymbol{\tau}\kappa-\overline{\kappa_c}\boldsymbol{\tau}\kappa_c)\right], \\ g_{\rm sc}^{\rm oct}\kappa &\to g_{ss}^{(8)}\left[(\boldsymbol{\tau}\cdot\mathbf{a}_0)\kappa+(f_0-\sqrt{2}\varepsilon)\kappa\right], \\ g_{\rm sc}^{\rm oct}s_8 &\to g_{ss}^{(8)}\sqrt{\tfrac{1}{3}}\left[\mathbf{a}_0^2+f_0^2-2\varepsilon^2-\overline{\kappa}\kappa\right], \\ g_{\rm sc}^{\rm sin}s_0 &\to g_{ss}^{(1)}\sqrt{\tfrac{2}{3}}\left[\mathbf{a}_0^2+f_0^2+\varepsilon^2+2\overline{\kappa}\kappa\right], \end{aligned} \tag{6.18}$$

where we included the relevant coupling constants. Note that the vacuum expectation matrix F does not commute with $\lambda_c s_c$, and so also the scalar single-meson coupling constants need some modification. In terms of the original coupling constants, we have

$$
\begin{aligned}
g_{B'Ba_0} &\longrightarrow g_{B'Ba_0}\left[1+\frac{2f_1}{m_\pi}\,\frac{g^{(8)}_{ss}}{g^{\rm oct}_{\rm sc}}\right], \\
g_{B'B\kappa} &\longrightarrow g_{B'B\kappa}\left[1+\frac{f_1+f_2}{m_\pi}\,\frac{g^{(8)}_{ss}}{g^{\rm oct}_{\rm sc}}\right], \\
g_{B'Bs_8} &\longrightarrow g_{B'Bs_8}\left[1+\frac{2}{3}\,\frac{f_1+2f_2}{m_\pi}\,\frac{g^{(8)}_{ss}}{g^{\rm oct}_{\rm sc}}\right] \\
&\qquad +\frac{2\sqrt{2}}{3}\,\frac{f_1-f_2}{m_\pi}\,\tilde{g}_{B'Bs_8}, \\
g_{B'Bs_0} &\longrightarrow g_{B'Bs_0}\left[1+\frac{2}{3}\,\frac{2f_1+f_2}{m_\pi}\,\frac{g^{(1)}_{ss}}{g^{\rm sin}_{\rm sc}}\right] \\
&\qquad +\frac{2\sqrt{2}}{3}\,\frac{f_1-f_2}{m_\pi}\,\tilde{g}_{B'Bs_0}, \qquad (6.19)
\end{aligned}
$$

where $\tilde{g}_{B'Bs_8}$ is obtained from the scalar version of Eq. (5.3) replacing s_0 by s_8 and $g^{\rm sin}_{\rm sc}$ by $g^{(1)}_{ss}$. Similarly, $\tilde{g}_{B'Bs_0}$ is obtained from the scalar version of Eq. (5.7) replacing s_8 by s_0 and $g^{\rm oct}_{\rm sc}$ by $g^{(8)}_{ss}$. Clearly, this type of extension allows for much more freedom in the scalar one-boson exchanges, without the need of abandoning the generation of the proper baryon masses. Note, however, that because of the second constraint in Eq. (6.17), the resulting numerical values for $g_{B'B\kappa}$, of some importance in YN potentials, remain unaffected by these changes.

A similar example is the case where we include the matrix for the scalar fields in the axial-vector interaction Lagrangian. This gives both scalar-pseudoscalar and scalar–axial-vector pair interactions. Since the axial-vector meson masses are well above 1 GeV and the scalar meson masses are close to 1 GeV, the scalar–axial-vector exchanges are expected to be completely negligible in baryon-baryon potential models, and we will not consider them here. The scalar-pseudoscalar interactions are generated by the combination $\{\Phi_{\rm sc},\Phi_{\rm ax}\}$. In principle, another possible combination is $i[\Phi_{\rm sc},\Phi_{\rm ax}]$, where the i is required by hermiticity. However, due to the fact that the vacuum expectation matrix F does not commute with $\Phi_{\rm ax}$, the commutator generates a complex contribution to the baryon-baryon-kaon coupling constants; we therefore drop this combination. As before, by including the anticommutator combination we can introduce two coupling constants $g^{(8)}_{sp}$ and $g^{(1)}_{sp}$, for the octet and singlet part of the interaction Lagrangian, respectively. The form of the various interaction Lagrangians and the expressions for the pair coupling constants are now easy to derive. Note that the baryon-baryon-pseudovector coupling constants are modified in an analogous manner to Eq. (6.19).

As a final example we discuss the class of pair-meson interactions which, within the present formalism, do not have any theoretical constraint on the overall coupling constants. The most simple interaction of this type contains $(\Phi_{\rm ax})^2$ or, equivalently, $u_\mu u_\nu$. This is also the most important one since it contains the lightest meson (the pion). Because the axial-vector mesons are already rather heavy, in the following we will only consider the pseudoscalar-pseudoscalar contributions. We have the possibility for two types of field combinations, one symmetric and one antisymmetric in the fields:

$$
\phi_s \sim -\tfrac{1}{2}g^{\mu\nu}\left[\partial_\mu(\lambda\pi)\partial_\nu(\lambda\pi)+\partial_\nu(\lambda\pi)\partial_\mu(\lambda\pi)\right],
$$

$$\phi_a \sim +\tfrac{i}{2}\sigma^{\mu\nu}\left[\partial_\mu(\lambda\pi)\partial_\nu(\lambda\pi) - \partial_\nu(\lambda\pi)\partial_\mu(\lambda\pi)\right]. \tag{6.20}$$

It will be convenient to identify the two-pseudoscalar contributions with a matrix of scalar fields as in Eq. (4.14). For the symmetric combination this implies the following substitutions

$$\begin{aligned}
\mathbf{a}_0 &\to -\tfrac{1}{2}g^{\mu\nu}\Big[\sqrt{\tfrac{1}{3}}(\partial_\mu\boldsymbol{\pi}\partial_\nu\eta_8 + \partial_\nu\boldsymbol{\pi}\partial_\mu\eta_8) + \sqrt{\tfrac{2}{3}}(\partial_\mu\boldsymbol{\pi}\partial_\nu\eta_0 + \partial_\nu\boldsymbol{\pi}\partial_\mu\eta_0) + \tfrac{1}{2}(\partial_\mu\overline{K}\boldsymbol{\tau}\partial_\nu K + \partial_\nu\overline{K}\boldsymbol{\tau}\partial_\mu K)\Big], \\
\kappa &\to -\tfrac{1}{2}g^{\mu\nu}\Big[\sqrt{\tfrac{1}{2}}\boldsymbol{\tau}\cdot(\partial_\mu\boldsymbol{\pi}\partial_\nu K + \partial_\nu\boldsymbol{\pi}\partial_\mu K) - \sqrt{\tfrac{1}{6}}(\partial_\mu\eta_8\partial_\nu K + \partial_\nu\eta_8\partial_\mu K) + \sqrt{\tfrac{4}{3}}(\partial_\mu\eta_0\partial_\nu K + \partial_\nu\eta_0\partial_\mu K)\Big], \\
s_8 &\to -\tfrac{1}{2}g^{\mu\nu}\Big[\sqrt{\tfrac{1}{3}}\partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi} - \sqrt{\tfrac{1}{3}}\partial_\mu\eta_8\partial_\nu\eta_8 + \sqrt{\tfrac{2}{3}}(\partial_\mu\eta_8\partial_\nu\eta_0 + \partial_\nu\eta_8\partial_\mu\eta_0) - \sqrt{\tfrac{1}{12}}(\partial_\mu\overline{K}\partial_\nu K + \partial_\nu\overline{K}\partial_\mu K)\Big], \\
s_0 &\to -\tfrac{1}{2}g^{\mu\nu}\sqrt{\tfrac{2}{3}}\Big[\partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi} + \partial_\mu\eta_8\partial_\nu\eta_8 + \partial_\mu\eta_0\partial_\nu\eta_0 + (\partial_\mu\overline{K}\partial_\nu K + \partial_\nu\overline{K}\partial_\mu K)\Big],
\end{aligned} \tag{6.21}$$

while for the antisymmetric combinations we have

$$\begin{aligned}
\mathbf{a}_0 &\to +\tfrac{i}{2}\sigma^{\mu\nu}\Big[i\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi} - \tfrac{1}{2}(\partial_\mu\overline{K}\boldsymbol{\tau}\partial_\nu K - \partial_\nu\overline{K}\boldsymbol{\tau}\partial_\mu K)\Big], \\
\kappa &\to +\tfrac{i}{2}\sigma^{\mu\nu}\Big[\sqrt{\tfrac{1}{2}}\boldsymbol{\tau}\cdot(\partial_\mu\boldsymbol{\pi}\partial_\nu K - \partial_\nu\boldsymbol{\pi}\partial_\mu K) + \sqrt{\tfrac{3}{2}}(\partial_\mu\eta_8\partial_\nu K - \partial_\nu\eta_8\partial_\mu K)\Big], \\
s_8 &\to +\tfrac{i}{2}\sigma^{\mu\nu}\Big[-\sqrt{\tfrac{3}{4}}(\partial_\mu\overline{K}\partial_\nu K - \partial_\nu\overline{K}\partial_\mu K)\Big].
\end{aligned} \tag{6.22}$$

These matrices can be substituted in the interaction Lagrangian (4.20), which contains the free parameters $g^{(8)}_{\rm sym}$, $g^{(1)}_{\rm sym}$, and $\alpha_{\rm sym}$ for the symmetric case, and $g^{(8)}_{\rm asym}$ and $\alpha_{\rm asym}$ for the antisymmetric case. Including the renormalization factors it is now straightforward to find the pair coupling constants for each of the two-pseudoscalar contributions, expressed in terms of these free parameters.

VII. APPLICATION TO *NN*

As a first application of the chiral-symmetry constraints given in this paper, we like to investigate whether with the values for the coupling constants as given in the previous sections it is indeed possible to construct a baryon-baryon potential model which gives a satisfactory description of the baryon-baryon scattering data. Of course, the imposed constraints need not all be exactly true. For example, the vector-dominance assumption that $\kappa_\rho = 3.71$ and $\kappa_\omega + \kappa_\phi = -0.12$ is only true if these mesons fully saturate the nucleon electromagnetic form factors. The presence of heavier vector-meson nonets likely changes these relations [46]. Also, the SU(3) relations in Sec. V need not be true in an exact sense. This is already clear from the fact that we have to introduce symmetry-violating terms to generate the empirical meson masses. The existence of a scalar meson nonet and its quark content is still under debate, and so the assumption of an SU(3) symmetry for the scalar mesons might even be incorrect. On the other hand, relaxing too many constraints introduces too many free parameters – something we would like to avoid. Therefore, at this stage we choose to impose *all* the constraints and only show that the resulting *NN* potential model then already gives a very reasonable description of the scattering data.

The experience with *NN* potential models that have appeared in the literature suggests that a fully constrained potential model of the one-boson-exchange type is unlikely to succeed [2]. On the other hand, we have already demonstrated [11,12] that by including two-meson-exchange contributions a major improvement in the description of the *NN* scattering data can be obtained. In order to arrive at a model which at least gives a reasonable description of the scattering data, we found that we had to include the double-scalar and double-pseudoscalar extensions as outlined in Sec. VI C. The single-meson coupling constants satisfy the empirical constraints as discussed in Sec. V and the pair-meson coupling constants satisfy the relations as given in Sec. VI. The one-boson-exchange part of the potential is standard but includes the diffractive contribution [47], while the two-meson part can be found in, or easily derived from, Refs. [11,12]. The potential is regularized with exponential form factors, one for each type of meson (scalar, pseudoscalar, or vector). The single-meson coupling constants and the exponential cutoffs for each type are given in Table I. The pair-meson coupling constants are given in Table II. Note that we only include meson pairs with a total mass below ~1 GeV. However, since the $\eta\eta$-exchange contributions did not significantly improve the fit, we decided not to include them at this stage.

The 12 free parameters of the model (Λ_S, Λ_P, Λ_V, g_{A_2}, g_P, $g^{(8)}_{ss}$, $g^{(1)}_{ss}$, $g^{(8)}_{\rm sym}$, $g^{(1)}_{\rm sym}$, $\alpha_{\rm sym}$, $g^{(8)}_{\rm asym}$, and $\alpha_{\rm asym}$) were determined in a fit to the Nijmegen representation [48] of the χ^2 hypersurface of the NN scattering data below $T_{\rm lab} = 350$ MeV, updated with the inclusion of new data which have been published since then. The effective diffractive mass, $m_P = 310$ MeV, was fixed at the (rounded-off) value as used in the old Nijm78 potential [47]. The resulting $\chi^2/N_{\rm data}$ for each of the ten energy bins is shown in Table III, in comparison with the (updated) Nijmegen partial-wave analysis.

The $\chi^2/N_{\rm data} = 1.75$ for the 0–350 MeV energy interval actually compares very favorably to other potential models that have appeared in the literature [2]. As a matter of fact, it should be realized that in this model *all* coupling constants satisfy constraints as imposed by chiral symmetry, or empirical constraints as discussed in Sec. V. This in contrast to any other model that has appeared in the literature. The model even gives a much better description of the data below 300 MeV ($\chi^2/N_{\rm data} = 1.36$), whereas it rapidly worsens at higher energies. This sudden rise is probably due to the nonadiabatic expansion in the two-meson contributions [11,12] which, strictly speaking, is only valid below the pion production threshold ($T_{\rm lab} \approx 280$ MeV). The nonadiabatic expansion is an artifact of us working in coordinate space, and the sudden rise in χ^2 at higher energies will be further investigated when we have developed a momentum-space version where we can retain the full energy dependence in the propagators[2]. At this stage we prefer to work in coordinate space for several reasons. First, the (already rather time consuming) fit in coordinate space is much faster than a fit in momentum space. Second, here we only wanted to investigate whether it is indeed possible to construct a potential model which incorporates all constraints and still gives a satisfactory description of the data. Modifications such as keeping the full energy dependence in the propagators rather than making a nonadiabatic expansion can be investigated at a later stage. Third, ultimately it is our goal to produce a high-quality potential model which is exactly equivalent in both coordinate space and momentum space. This allows equivalent applications in both spaces, but requires certain approximations to

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[2]Alternatively, we can decide that the model should only be used up to $T_{\rm lab} \approx 300$ MeV, say. Without pursuing this option any further, a quick and not too thorough refit already shows that a $\chi^2/N_{\rm data} = 1.3$ seems easily feasible.

be made in order to arrive at analytical expressions in coordinate space.

We should point out that, although the number of free parameters is not too large, the fully constrained fit is far from trivial. Due to the constraints, the relation between the change in a parameter and the corresponding change in the phase shifts is highly nonlinear. It turned out to be impossible to fit all the free parameters at the same time, and so we had to do numerous fit cycles where we fit only an (arbitrary) subset of the parameters. This makes it very complicated and time consuming (but not impossible) to arrive at a satisfactory fit, and we cannot guarantee that the fit presented here is the most optimal one. However, the present result already clearly illustrates our main objective, viz. to try to construct a potential model which gives a reasonable description of the scattering data, and at the same time incorporates a number of empirical and chiral-symmetry constraints. Improvements by investigating different parameter sets, adding more parameters, or relaxing some of the constraints will be left for the future.

VIII. SUMMARY

We have constructed a chiral-invariant Lagrangian for the meson-baryon sector, where the mesons consist of the nonets of scalar, pseudoscalar, vector, and axial-vector mesons, and the baryons are the members of the baryon octet. Although we mainly focussed on the meson-baryon interaction Lagrangian, we briefly indicated what the Lagrangian in the meson sector looks like, and how it can be made to reproduce the correct phenomenology by allowing for some small contributions which violate the chiral symmetry.

In the meson-baryon sector, the chiral symmetry imposes constraints on the coupling constants of the meson-pair vertices. This allows us to express all meson-pair coupling constants in terms of the single-meson vertex coupling constants. These single-meson vertex coupling constants, in principle, can all be fixed by using experimental data such as meson masses, baryon masses, meson decay parameters, and meson mixing angles.

As a first step in the application of the meson-baryon Lagrangian, we demonstrate that it is possible to construct an NN potential which has all the theoretical and empirical constraints as discussed in Secs. V and VI. The model gives a very satisfactory description of the NN scattering data. It even has a slightly better quality than other (often more phenomenological) potential models that have appeared in the literature, where this is the first model where such strict constraints on the coupling constants have successfully been imposed. We should mention that this result could only be achieved after the inclusion of the two-meson contributions. Improvements and an extension to the YN sector are presently under investigation. The success of the present model gives us hope that it might indeed be possible to ultimately arrive at a potential model which not only provides a high-quality description of the scattering data, but which is also consistent with the symmetries of QCD.

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APPENDIX

We follow Ref. [49] and show that it is possible to find a nonlinear transformation that transforms away the octet of original pseudoscalar fields, while leaving the scalar vacuum expectation matrix F invariant. We write the scalar nonet as $\lambda_c \sigma_c = F + \Sigma$, where the diagonal matrix $F = (f_1, f_2, f_3)$ is the nonvanishing vacuum expectation matrix of the scalar fields. The parameters f_i can be anything, but in the following we assume that $f_i + f_j \neq 0$ for all $\{i, j\}$. The pseudoscalar octet is written as $\Pi = \lambda_a \pi_a$. We want to find a traceless matrix P (to be identified as an octet of new pseudoscalar fields), such that

$$F + \Sigma + i\Pi = e^{iP}(F + X + iY)e^{iP}, \tag{A1}$$

where X and Y are sets of different scalar and pseudoscalar fields, respectively. For that purpose, we formally write [49]

$$P = \alpha \sum_{k=0}^{\infty} \alpha^k P_k, \quad X = \sum_{k=0}^{\infty} \alpha^k X_k, \quad Y = \sum_{k=0}^{\infty} \alpha^k Y_k, \tag{A2}$$

and try to solve for

$$F/\alpha + \Sigma + i\Pi = e^{iP}(F/\alpha + X + iY)e^{iP}, \tag{A3}$$

in each order of α. To order α^{-1} we have an identity. To order α^0 we can write

$$\begin{aligned} &X_0 = \Sigma, \qquad Y_0 = \frac{\Pi_{11} f_2 f_3 + \Pi_{22} f_1 f_3 + \Pi_{33} f_1 f_2}{f_2 f_3 + f_1 f_3 + f_1 f_2} \, \mathbb{1}_3, \\ &(P_0)_{ij} = (f_i + f_j)^{-1} \left[\Pi - Y_0\right]_{ij}, \end{aligned} \tag{A4}$$

where we chose Y_0 such that $\mathrm{Tr}(P) = 0$. This is not the only possible choice for Y_0, but it will turn out to be a very convenient one. (Note that in Ref. [49] a different choice is made, but there P_0 is not traceless, contrary to what is claimed.) This procedure can be extended to all orders in α, and we always find that

$$Y_n + \{F, P_n\} = g_n, \tag{A5}$$

with g_n a nonlinear function of P_k, X_k, and Y_k with $k = 0, \ldots, n-1$. Hence, in analogy to the $n = 0$ case, we can always define a traceless matrix P_n and at the same time keep Y_n proportional to $\mathbb{1}_3$. Substituting $u = \exp(iP)$, the definition (4.8) gives

$$\chi_+ = F + X, \qquad \chi_- = iY, \tag{A6}$$

where X represents the new scalar octet and where Y (because it is proportional to $\mathbb{1}_3$) can be identified with a new isosinglet pseudoscalar field, not present before. Clearly, different choices for Y_n, not proportional to $\mathbb{1}_3$, will introduce unwanted contributions to the isovector (π) and/or isoscalar (η_8) pseudoscalar fields.

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TABLES

TABLE I. Single-meson NN coupling constants and exponential cutoff masses for the constrained NN model. The single-meson coupling constants are divided by $\sqrt{4\pi}$.

g-scalar			f-pseudosc.			g-vector			f-vector			g-diffr.	
a_0	ε	f_0	π	η	η'	ρ	ω	ϕ	ρ	ω	ϕ	A_2	P
2.599	3.273	–0.978	0.270	0.117	0.075	0.711	2.408	–0.183	2.634	0.297	0.045	2.497	2.126
$\Lambda_S = 590.0$ MeV			$\Lambda_P = 870.0$ MeV			$\Lambda_V = 845.0$ MeV							

TABLE II. Pair-meson coupling constants for the constrained NN model. The coupling constants are divided by 4π. For the double-pseudoscalar $\pi\pi$ and KK contributions we distinguish interactions symmetric and antisymmetric in the fields.

							symmetric			antisymmetric		
$\pi\pi$	KK	$K\tau K$	$\pi\rho$	$\varepsilon\varepsilon$	$\pi\eta$	$\pi\eta'$	$\pi\pi$	KK	$K\tau K$	$\pi\pi$	KK	$K\tau K$
–0.030	–0.028	–0.009	0.384	2.463	0.007	0.004	0.045	0.007	0.005	–0.050	0.082	0.033

TABLE III. χ^2 and χ^2 per datum ($\chi^2_{\text{p.d.p.}}$) at the 10 energy bins for the updated partial-wave analysis (PWA) and the constrained NN potential. N_{data} lists the number of data within each energy bin. The bottom line gives the results for the total 0–350 MeV interval.

		PWA		potential	
Bin(MeV)	N_{data}	χ^2	$\chi^2_{\text{p.d.p.}}$	χ^2	$\chi^2_{\text{p.d.p.}}$
0.0–0.5	145	144.45	0.996	155.73	1.07
0.5–2	68	43.08	0.633	56.48	0.83
2–8	110	105.11	0.956	157.41	1.43
8–17	294	275.24	0.936	335.07	1.14
17–35	359	294.58	0.821	387.39	1.08
35–75	585	565.30	0.966	1045.22	1.79
75–125	399	409.62	1.027	443.18	1.11
125–183	760	823.82	1.084	1249.44	1.64
183–290	1046	1022.72	0.978	1306.90	1.25
290–350	992	993.49	1.002	3197.26	3.22
0–350	4758	4677.42	0.983	8334.09	1.75