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The Status of the Pion-Nucleon Coupling Constant*

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Abstract

A review is given of the various determinations of the different πNN coupling constants in analyses of the low-energy pp, np, $\overline{p}p$, and πp scattering data. The most accurate determinations are in the energy-dependent partial-wave analyses of the NN data. The recommended value is $f^2 = 0.075$. A recent determination of f^2 by the Uppsala group from backward np cross sections is shown to be model dependent and inaccurate, and therefore completely uninteresting. We also argue that an accurate determination of f^2 using pp forward dispersion relations is not a realistic option.

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I. INTRODUCTION

It is exactly 50 years ago that the pion was discovered in cosmic rays by Lattes, Occhialini, and Powell [1] in Bristol. Since then we have come to know quite a lot about the pion and its interactions. We will concentrate here on the pion-nucleon-nucleon (πNN) coupling constants, and review the various determinations of the πNN coupling constants in analyses of the low-energy pp, np, $\overline{p}p$, and πp scattering data.

One can distinguish in principle 4 different πNN coupling constants. The coupling constant $f(p\pi^- \rightarrow n) = \sqrt{2}\, f_-$ denotes the strength at the vertex at which a proton is annihilated and a neutron created and where either a π^- is annihilated or a π^+ created. The coupling constant $f(n\pi^+ \rightarrow p) = \sqrt{2}\, f_+$ denotes the strength at the vertex at which a neutron is annihilated and a proton created and where either a π^+ is annihilated or a π^- created. The coupling constant $f(p\pi^0 \rightarrow p) = f_p$ denotes the strength at the vertex at which a proton is annihilated and another proton created and where a π^0 is either annihilated or created. The coupling constant $f(n\pi^0 \rightarrow n) = -f_n$ denotes the strength at the vertex at which a neutron is annihilated and another neutron created and where a π^0 is either annihilated or created. Important combinations are $f_c^2 = f_+\, f_-$ and $f_0^2 = f_p\, f_n$. The factors in front of f_+ , f_- , and f_n are chosen in such a way that we get in the case of *charge independence*:

$$f_p = f_n = f_+ = f_- = f \ . \tag{1}$$

Because the strong interactions are invariant under charge conjugation there is a relation between the pion-nucleon-nucleon coupling constants and the pion-antinucleon-antinucleon coupling constants. By using the charge-conjugation operator C we can define the antiproton $\overline{p} = Cp$ and the antineutron $\overline{n} = Cn$. The neutral pion is its own antiparticle and is an eigenstate of the charge-conjugation operator $C\pi^0 = \eta_c \pi^0$ with as eigenvalue the charge parity $\eta_c = 1$. The coupling constants of the neutral pion to the antinucleons are related to the coupling constants of the neutral pion to the nucleons by

$$f(\overline{p}\pi^0 \rightarrow \overline{p}) = \eta_c f(p\pi^0 \rightarrow p) \quad \text{and} \quad f(\overline{n}\pi^0 \rightarrow \overline{n}) = \eta_c f(n\pi^0 \rightarrow n) \ . \tag{2}$$

This leads to the following relations

$$\begin{aligned} f(\overline{p}\pi^0 \rightarrow \overline{p}) &= f_p \ , \ f(\overline{p}\pi^+ \rightarrow \overline{n}) = \sqrt{2} f_- \\ f(\overline{n}\pi^0 \rightarrow \overline{n}) &= -f_n \ , \ f(\overline{n}\pi^- \rightarrow \overline{p}) = \sqrt{2} f_+ \ . \end{aligned} \tag{3}$$

The pion is a pseudoscalar ($J^{PC} = 0^{-+}$) particle. The coupling of the pion to the nucleons can be described by either the PS (pseudoscalar) interaction lagrangian $\mathcal{L}_{PS}$ or the PV (pseudovector) interaction lagrangian $\mathcal{L}_{PV}$, where

$$\mathcal{L}_{PS} = g\sqrt{4\pi}(i\overline{\psi}\gamma_5\psi)\phi \quad \text{and} \quad \mathcal{L}_{PV} = \frac{f}{m_s}\sqrt{4\pi}(i\overline{\psi}\gamma_\mu\gamma_5\psi)\partial^\mu\phi \ . \tag{4}$$

We prefer to use the PV coupling, because of chiral symmetry and because of SU(3,F) flavor symmetry of the pseudoscalar-meson couplings to the baryon octet [2]. The scaling mass m_s was introduced to make the PV coupling constant dimensionless. It is convention [3] to choose $m_s = m_+$, where m_+ is the mass of the π^+. Unfortunately, there are physicists [4] who take m_s equal to the mass of the quanta of the field ϕ. This leads to an unnecessary large charge-independence breaking in the coupling constants. We strongly advice to use the convention. The PS coupling constant g and the PV coupling constant f are related by [5] the *equivalence relation*

$$\frac{g}{M_1 + M_2} = \frac{f}{m_s} \ . \tag{5}$$

This means $g_p^2 = 180.773 f_p^2$, $g_c^2 = 181.022 f_c^2$, and $g_n^2 = 181.271 f_n^2$.

One must realize that charge independence or SU(2,I) isospin symmetry for the PV coupling constants leads to broken SU(2,I) symmetry for the PS coupling constants. For example, when we take $f^2 = 0.0747$, then we get $g_p^2 = 13.50$, $g_+^2 = g_-^2 = 13.52$, and $g_n^2 = 13.54$. For most purposes charge independence for the values of the coupling constants can be assumed. We recommend to use

$$\text{either} \quad f^2 = 0.075 \ , \quad \text{or} \quad g^2 = 13.5 \ . \tag{6}$$

This implies that the Goldberger-Treiman discrepancy is only $\Delta_{\pi N} \simeq 2\%$.

II. EARLY DETERMINATIONS

It is impossible for us to give a proper and historically correct review of the various determinations of f^2. To get an impression about what was going on we will give a short, incomplete, and at certain places possibly incorrect, description.

In the early fifties pion-photoproduction experiments were much more accurate than the πN scattering experiments. Also the NN scattering experiments were still in their infancy. To our knowledge, the first estimate $f_c^2 \simeq 0.3$ of the πNN coupling constant came in 1950 from the photoproduction of charged pions [6]. The Kroll-Ruderman Theorem [7] shows that f_c^2 can be determined directly from the photoproduction reactions near threshold. Using this method Bernardini and Goldwasser [8] found in 1954 the already much better value $f_c^2 = 0.065$.

Perhaps the best place to determine the πNN coupling constants is from the NN scattering data. Probably the first determination in NN scattering was made in 1952 by M.M. Lévy [9]. In order to reproduce the low-energy parameters (binding energy of the deuteron and the s-wave scattering lengths) he needed $f^2 = 0.054$. In 1955 Chew and Low's static nucleon extended-source interaction Hamiltonian (PV) was applied by Gartenhaus [10] in his calculation of the one- and two-pion-exchange NN potential. This potential with $f^2 = 0.089$ described the low-energy NN data well.

Encouraged by Lévy's result Chew [11] studied around 1953 in a series of papers the low-energy πN scattering data using the nonrelativistic Tamm-Dancoff method. He found $f_c^2 = 0.058$. At about the same time Sartori and Watighin [12] used the Cini-Fubini method to study the same data. They came up with the value $f_c^2 = 0.065$.

The very first applications of dispersion relations to the πN data to determine the pion-nucleon coupling were by Haber-Schaim [13], Davidon and Goldberger [14], and Gilbert [15].

The value obtained by Haber-Schaim was $f_c^2 = 0.08(1)$. Using forward dispersion relations Schnitzer and Salzman [16] obtained in a careful analysis of the πN data the value $f^2 = 0.08(1)$.

Then there is the determination of f_c^2 by Jackson [17]. Using the Chew-Low effective-range plot for the P_{33} phase shift he found $f_c^2 = 0.08(2)$.

In 1958 Chew [18] pointed out that the backward *np* scattering was possibly a good place to determine f_c^2. As will be pointed out later, that "this is a good place" has *incorrectly* become part of our folklore. This method was for the first time applied to the *np* data by Cziffra and Moravcsik [19] with the result that f_c^2 is between 0.06 and 0.07 with a very large error. They already pointed out that a partial-wave analysis of *NN* scattering data would not suffer from the disadvantages of an extrapolation method and give more reliable and realistic results.

Around this time the outstanding series of *NN* partial-wave analyses (PWA) by the Livermore-Berkeley group was started. They regularly produced a value for the πNN coupling constant. In a single-energy phase-shift analysis of the *pp* data at 310 MeV MacGregor *et al.* [20] found in 1959 either $f_p^2 = 0.062(11)$ or $f_p^2 = 0.069(17)$. In 1968 they [21] found for this coupling constant $f_p^2 = 0.081(5)$. Applying forward dispersion relations to the *pp* data Bugg [22] determined in 1968 the value $f_p^2 = 0.075(4)$.

In a study of the πN data in 1973 Bugg *et al.* [23] used fixed-t dispersion relations to determine the coupling constant $f_c^2 = 0.079(1)$. They stated over-optimistically that their small error is "believed to cover both statistical and systematic uncertainties." This determination has been the standard for almost two decades.

The outstanding Karlsruhe-Helsinki partial-wave analyses [24] of the πN data used the value of Bugg *et al.* as input. In 1980 Koch and Pietarinen [25] used fixed-t dispersion relations and found again that $f_c^2 = 0.079(1)$. However, this is more a consistency check than a real determination, because the value of the coupling constant was used as *input* in the analyses. Other values of f_c^2 were not tried as input. It is interesting to read the claim made by Koch and Pietarinen that "the error is our estimate due to systematic effects. The statistical error is completely negligible"! It is incomprehensible to us how theory can remove or reduce the statistical errors. It should also be remarked that the Karlsruhe-Helsinki analyses of the πN data did *not* include a treatment of the normalization errors of experimental data.

In 1981 Kroll [26] applied *pp* forward dispersion relations to determine f_p^2. He came to the conclusion that $f_p^2 = 0.080(2)$. Later on, more attention will be paid to this special method.

We see that around 1980 all dispersion-relation determinations from πN data as well as from *NN* data seem to agree on the charge-independent value $f^2 = 0.079(1)$. Dispersion relations were then considered by many people *the* method to be used in determining coupling constants.

For references to other work on the determination of coupling constants the reader is referred to the classic book [27] by H.A. Bethe and F. de Hoffmann and to the various editions [3,28–32] of the "Compilation of Coupling Constants and Low-Energy Parameters."

III. NEWER DETERMINATIONS

In 1975 the Nijmegen group [33] published the Nijmegen D *NN* potential. This is a hard-core potential, which reproduced the then available *NN* data very well. It required $f^2 = 0.0741$. It is interesting to see that already then the *NN* data favored a small value for the coupling constant and that the deuteron properties of that potential were excellent. The soft-core Nijmegen *NN* potential Nijm78 was published [34] in 1978. Also in this potential the coupling constant preferred a low value. Because we were at that time still indoctrinated by the πN people that $f^2 = 0.079(1)$, we prevented our coupling constant from going "too low," and ended up with the value $f^2 = 0.0772$.

It was around 1983 that we became convinced that the $\pi^0 pp$ coupling constant was much lower than 0.079. At the 1983 Few-Body Conference in Karlsruhe we [35] stated that "we believe that f_p^2 is probably more in the neighborhood of 0.075 than of 0.080." This belief was based on our PWA of the low-energy *pp* data. It lasted until 1987 before we could make our suspicions hard, that is, before we could give solid evidence based on statistics. In our energy-dependent PWA [36] of the *pp* scattering data below $T_{lab} = 350$ MeV we found $f_p^2 = 0.0725(6)$. Unfortunately, the magnetic-moment interactions were not yet included in these analyses. When that was finally done [37], we [38,39] obtained $f_p^2 = 0.0749(6)$. At that time we had not made a determination of f_c^2 by ourselves. Therefore we had to assume that the value for f_c^2 determined in πN scattering was correct and that there was thus evidence for a large breaking of charge independence. That raised hell. Many people [40–46] had their own explanation about what was wrong with *our* analyses. Those "explanations" were almost all based on wrong assumptions and ignorance about energy-dependent PWA's. Nobody, on the other hand, questioned seriously the value for f_c^2 from πN analyses.

Of course there are people who are familiar with PWA's. Arndt *et al.* noticed that in the energy-dependent VPI&SU analyses of the *np* data a lower coupling constant was favored. This encouraged them to have a fresh look at the πN data. In 1990 the VPI&SU group [47] came up with the new value $f_c^2 = 0.0735(15)$. Again, many people [44,48–51] had their own "explanation" about what was wrong with *this* analysis. Of course, nothing was really wrong! In order to try to appease the opposition the VPI&SU group [52] included dispersion-relation constraints in their analyses of the πN data. This lead then to the value $f_c^2 = 0.0761(8)$. In our opinion, it is better not to impose such constraints, because it is almost impossible to make the proper corrections for the electromagnetic interactions.

In Nijmegen we were at that time also very busy to get determinations of f_c^2. A study of the backward *np* data was very disappointing. Despite the folklore [18] that this charge-exchange reaction was the best place to determine f_c^2, we were unable to get a sufficiently accurate determination [53] such that we could distinguish between the values 0.075 and 0.079 for f_c^2.

An energy-dependent analysis [54] in 1991 of the charge-exchange reaction $\overline{p}+p \rightarrow \overline{n}+n$ gave us the value $f_c^2 = 0.0751(17)$. To convince ourselves that we were really looking at the OPE potential, we determined the mass of the exchanged particle. This turned out to be $m_{\pi^+} = 145(5)$ MeV, which must be compared with the experimental value $m_{\pi^+} = 139.57$ MeV. This 1991 result was the main motivation for a new accurate measurement of the charge-exchange cross section by the PS206 group at LEAR. The completed $\overline{N}N$ PWA of 1993 gave [55] $f_c^2 = 0.0732(11)$.

Our energy-dependent PWA's of the *np* data required an enormous effort. Finally, in 1991 our first analysis [56] of the combined *pp* and *np* data was completed. We then were able to make independent determinations of various πNN coupling constants. We found $f_p^2 = 0.0751(6)$, $f_0^2 = 0.0752(8)$, and $f_c^2 = 0.0741(5)$. For the charge-independent value we found $f^2 = 0.0749(4)$.

It was now evident that the energy-dependent PWA's of the *pp*, *np*, $\overline{p}p$, and πp data all lead to about the same value $f^2 = 0.075$ for the πNN coupling constant. We were surprised to note the many physicists trying to hold on to the old values. To counter their arguments we published in 1993 a paper [57] in which we studied in detail various aspects of our determinations of coupling constants. In that paper [57] we paid special attention to Chew's proposal [18] to determine f_c^2 from single-energy studies of the backward *np* data. We showed explicitly that extrapolating to the pion pole is *very inaccurate*, and clearly not a good method to make a reliable determination of f_c^2 with a small error.

IV. PRESENT SITUATION

The coupling constant f_p^2 can best be determined from the *pp* scattering data. Our latest published value stems from 1993. In an analysis [58] of 1787 *pp* data below 350 MeV with 21 model parameters, we obtained $\chi^2_{min} = 1787$ and $f_p^2 = 0.0745(6)$.

This analysis has been updated [59]. The 1997 Nijmegen *pp* database contains 1955 data. These can be described very well with 20 model parameters. We find $\chi^2_{min} = 1962$ and $f_p^2 = 0.0753(5)$. When we leave out the recent data from Haeberli *et al.* [60], then we get $f_p^2 = 0.0747(5)$. We have here a nice example of a *non*-dedicated experiment, which *does* influence the value of the coupling constant.

The *np* scattering data give the combinations f_c^2 and f_0^2. In the 1993 Nijmegen PWA [58] of the *pp* and *np* data below 350 MeV we analyzed 2512 *np* data. In this analysis we found $\chi^2_{min} = 2480$ and $f_c^2 = 0.0748(3)$. We have also been able to do PWA's of the *np* data alone [61]. This analysis uses all *np* data, $N_{data} = 3964$, below 500 MeV. We get $\chi^2_{min} = 4005$ with $f_c^2 = 0.0748(3)$ and $f_0^2 = 0.0745(9)$.

Another useful place to determine these coupling constants is in $\overline{N}N$ scattering. The elastic $\overline{p}p$ data give f_p^2, and the charge-exchange data give f_c^2. In these analyses [54,55,62] all data below 925 MeV/c were analyzed. In 1991 884 observables were analyzed [54]. Using 23 model parameters we reached $\chi^2_{min}/N_{obs} = 1.15$ and we determined $f_c^2 = 0.0751(17)$. The 1993 analysis [55] contained 3646 data (elastic as well as charge exchange) and 30 model parameters were used; $\chi^2_{min}/N_{data} = 1.04$ was reached with $f_c^2 = 0.0732(11)$. This analysis has been updated, where the latest charge-exchange data from the LEAR experiments PS199 [63,64] and PS206 [65] were incorporated. Analyzing now 3847 data with 36 model parameters [62] $\chi^2_{min}/N_{data} = 1.05$ was obtained with $f_c^2 = 0.0736(10)$.

Traditionally, the most important source for the πNN coupling constant has been πN scattering. Neutron exchange in elastic $\pi^+ p$ scattering depends on the combination f_c^2. The analysis of the elastic and charge-exchange $\pi^- p$ scattering is more complicated, and various combinations of the coupling constants come into play. We have already seen that Arndt *et al.* determined in 1990 [47] the value $f_c^2 = 0.0735(15)$. In 1994 they [52] included dispersion-relation constraints. This raised their value for f_c^2 to 0.0761(8). Bugg has redone his 1973 dispersion analysis [66] and his updated value is $f_c^2 = 0.0771(14)$. This determination is *not* based on an energy-dependent PWA.

Another analysis is the Los Alamos-Groningen energy-dependent PWA of the πN data by Timmermans [67]. After a careful selection the $\pi^+ p$ database below 410 MeV contains 1092 data. The PWA uses 14 parameters and reaches $\chi^2_{min}/N_{data} = 0.98$ with $f_c^2 = 0.0743(8)$. Extending this analysis to the coupled $\pi^- p$ and $\pi^0 n$ channels results in a database containing 898 data. In this analysis 17 parameters were used and $\chi^2_{min}/N_{data} = 1.11$ was reached with the charge-independent coupling $f^2 = 0.0756(9)$.

It takes too much space and time for a proper discussion of all determinations of the coupling constants in single-energy PWA's. Such determinations are inherently much more inaccurate than the energy-dependent analyses. One of the reasons is just the number of data that is involved. In single-energy PWA's this is about a few tens, but in energy-dependent PWA's about a few thousands. Roughly a factor of 100 difference. This implies about a factor of 10 difference in the statistical accuracy. Because we feel very strongly that single-energy PWA's are vastly inferior to energy-dependent PWA's, we will not discuss determinations based on such single-energy studies (with the exception of the recent Uppsala work, see below).

It must be clear from the above, that (*i*) the energy-dependent PWA's of the about 12.000 *NN*, $\overline{N}N$, and πN data are all in agreement, that (*ii*) there is at present no evidence for a breaking of charge independence in the coupling constants, and that (*iii*) the charge-independent value is $f^2 \simeq 0.0750$.

V. BREAKING OF CHARGE INDEPENDENCE

In which way the breaking of charge independence influences the values of the coupling constants is a widely studied topic [68–74]. An earlier paper discussing the radiative corrections was by Morrison [68]. Later, estimates were made using various quark models [69,70] and more recently QCD sum rules [72] were applied. A difficulty with these various methods and models is that the predictions differ (even in sign) from model to model. In some of the simpler models one has the relation $f_c = (f_p + f_n)/2$. Friar, Goldman, and van Kolck studied this issue in chiral perturbation theory [71].

Using the QCD sum rules for the appropriate pion-nucleon three-point functions Meissner and Henley [72] calculated the splitting between f_p, f_n, and f_c due to isospin breaking in the strong interaction. They found a large breaking of isospin in these coupling constants, the lower limit of which will result approximately in $f_n^2 = 0.074$, $f_c^2 = 0.075$, and $f_p^2 = 0.076$.

The present accuracies in the determination of the various coupling constants are such, that with a little improvement in the data and in the analyses these charge-independence breaking effects could be checked.

VI. ENERGY-DEPENDENT PWA VERSUS SINGLE-ENERGY PWA

In the energy region below $\mathrm{T}_{lab} = 350$ MeV *one* energy-dependent PWA of all *NN* data could be performed. Let us take as example the Nijmegen 1993 PWA [58]. We simultaneously analyzed 1787 *pp* data and 2514 *np* data for a grand total of 4301 *NN* data. Or one could divide all these data in about 10 energy intervals, clustered around the energies: 0, 1, 5, 10, 25, 50, 100, 150, 210, and 320 MeV, and perform 10 single-energy PWA's. Both these total analyses will furnish phase shifts at these above energies. The big advantage of the energy-dependent PWA is that the multienergy (m.e.) phases are much smoother as a function of energy. The statistical fluctuations in the single-energy (s.e.) phases are averaged out in the m.e. phases. As a result, the errors in the m.e. phases are much smaller than the errors in the s.e. phases. When new data are added to the database these m.e. phases are much more stable than the s.e. phases. What is needed for a successful energy-dependent PWA is a good description of the energy dependence of the phases.

The difficulty with the single-energy PWA's is that they are *overparametrized*. In the above mentioned m.e. PWA we use 39 parameters. To describe exactly the same data in the 10 s.e. PWA's we need 116 parameters. In both analyses we use 4301 NN data. In the m.e. analysis we reach $\chi^2_{min} = 4276$. In the s.e. analyses we describe all data with $\chi^2_{min} = 4096$. We need 77 more parameters to obtain a drop in χ^2 of *only* 180. This overparametrization of the s.e. phases results in a large noise content in these phases. This makes such s.e. phases rather useless. This is the reason that the Nijmegen group will *not* present any more s.e. phases. We strongly feel that such phases should not be used anymore.

Important in energy-dependent PWA's is a good description of the fast energy dependence of the phases. The slow energy dependence can easily be parametrized. The fast energy dependence is determined by the longest-range forces. It is therefore important to have a good description of the long-range electromagnetic forces. In the Nijmegen analyses we have paid very special attention [75,37] to these longest-range forces. The longest-range nuclear forces are due to one-pion exchange (OPE). An important part of the energy dependence of the phases comes from OPE. This is the main reason that we can determine the coupling constants so accurately. The most accurate data are the pp data. The np data are definitely less accurate. Still, we can determine the charged-pion coupling constant f_c^2 in the np PWA with the very small error 3×10^{-4}. The reason is that in np scattering the electromagnetic force is essentially absent (only magnetic-moment interactions). The long-range OPE force in pp scattering is hidden under the longer-range electromagnetic interaction. This "explains" why we can determine f_p^2 only with the somewhat larger error 6×10^{-4}.

VII. BACKWARD NP SCATTERING

Recently, backward np scattering experiments at $T_{lab} = 162$ MeV were performed in Uppsala [76]. These data were then used in a modified Chew extrapolation method to determine f_c^2. The authors claim a "high" value for f_c^2 with an incredible small error. This value is obtained from only 31 data and is in flagrant disagreement with the values deduced in the energy-dependent PWA's of the 12.000 pp, np, $\overline{N}N$, and πN data. Let us look at what is going on.

A. OPE amplitude

Forgetting about the np mass difference the one-pion-exchange amplitude evaluated in Born approximation reads

$$M_{OPE} = -\frac{4\pi f^2}{m_s^2}\,\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k})}{k^2+m_\pi^2}\ , \tag{7}$$

with $k^2 = 2p^2(1 \mp \cos\theta)$. Here p is the center-of-mass momentum. We see that M_{OPE} is energy (E or p^2) dependent, θ dependent, and mass dependent. The Nijmegen energy-dependent PWA's take *both* the energy dependence and the θ dependence into account. Also the mass dependence has been studied and the masses of the exchanged particles were determined with excellent results [56].

Recently, there has appeared a new gimmick: *dedicated* experiments. This seems to mean: "*Dedicated to the proposition that all PAC's are equally easily led astray.*" In the analyses of the dedicated experiment [76] *only* the θ dependence of the one-pion-exchange amplitude is taken into account, and that only approximately.

We would like to point out that the π^0-exchange amplitude vanishes in the forward direction ($\theta = 0$), and that the $\pi^\pm$-exchange amplitude (forgetting the np mass difference) vanishes at $\theta = 180°$. This implies that the backward peak ($\theta = 180°$) in np scattering is *not* due to $\pi^\pm$ exchange. The fall-off of the differential cross section for $\theta < 180°$ is due to destructive interference between $\pi^\pm$ exchange and the background.

B. The data

The data, as published [76], have an incredibly large χ^2 with respect to the standard PWA's of the VPI&SU [77] and Nijmegen [78] groups. We find $\chi^2 = 291.6$ for 31 data. We have tried to understand what causes this. The data consist of several independent and unnormalized data sets. When one compares these sets in the regions where they overlap one notices inconsistencies. Next, these sets are *incorrectly* normalized. In this way large systematic errors are introduced. Our conclusion about these data is that, first, one must clean up the systematic deviations between the individual data sets and that, second, the data badly need reanalysis, where better attention should be paid to the normalization.

We want to point out that the problem is not just between the Uppsala data and other backward np cross sections. Even if *all* backward np cross sections are removed from the database, the Uppsala data are still in flagrant disagreement with the remainder of the data. This means that the Uppsala data are also in disagreement with all the spin observables, such as polarization and spin-correlation data. Moreover, as mentioned, the Uppsala data are internally inconsistent.

The Uppsala group has claimed that there are two "families" of np cross sections, and that in the Nijmegen PWA only one "family" is accepted. This unfounded claim we find ludicrous. According to our findings, the second "family" consists *only* of the incorrectly normalized Uppsala data.

C. Extrapolation Method

The authors try to extrapolate the backward differential cross section to the charged-pion pole. They consider several extrapolation procedures and claim that their modified Chew extrapolation method is model independent. Moreover, they think that this method can reduce the statistical errors. We would like to stress that *no method of analysis can reduce statistical errors*: These are inherent to the experimental data.

Arndt *et al.* [79] used exactly the same method as used by the Uppsala group to *all* available backward np data sets. They showed that the *average* of all extrapolations is consistent with $f_c^2 = 0.075$ and that the error in each individual determination is much larger (about a factor of 10) than claimed by the Uppsala group.

The model dependence of the Uppsala method is easily demonstrated. We applied exactly the same method as used by the Uppsala group. For the comparison potential we use the Nijmegen np potential Nijm93, as was done by the Uppsala group. However, we constructed different versions by only changing the πNN coupling constant f_c^2 in this comparison potential.

In Model I we use $f_c^2 = 0.075$. In the fit to the data 2 parameters are needed and we find $f_c^2 = 0.0809(3)$. The error is here the estimate of the statistical extrapolation error only. This is exactly the same model as used by the Uppsala group and our results are therefore exactly the same as the results of the Uppsala group.

In Model II we use $f_c^2 = 0.079$. In the fit to the data we now need 4 parameters and we find $f_c^2 = 0.0808(30)$. This model gives the same value for f_c^2 as Model I, but the error estimate is now a factor 10 larger (and much more realistic).

In Model III we use $f_c^2 = 0.071$. To produce a good fit only 1 parameter is needed. This time we find a much lower value for the coupling constant, $f_c^2 = 0.0747(1)$, but a very unrealistic small error (because in this model we need only 1 parameter: the more parameters, the larger the error).

This simple exercise clearly demonstrates the model dependence introduced by the comparison potential. We have shown here that both the value of the coupling constant and the value of the error are model dependent. We claim that the real error is 0.003 and *not* 0.0003 as given by the Uppsala group. The result then is in total agreement with our previous conclusions [57] and also in agreement with Arndt's results [79].

This large error makes the determination of the coupling constant by the Uppsala group *totally uninteresting* and shows that the label "dedicated" for such experiments is presumptuous and completely unwarranted.

Similar conclusions hold for Chew-type extrapolations of the forward $\overline{p} + p \rightarrow \overline{n} + n$ cross sections to determine f_c^2 [80–82]. Such extrapolations cannot compete with determinations in the energy-dependent PWA of all $\overline{p}p$ data [54,55,62].

VIII. FORWARD DISPERSION RELATIONS

Forward dispersion relations are often seen as the ultimate tool in determining coupling constants. This is definitely not correct for the pp forward dispersion relations. One reason for this is the problematic treatment of the electromagnetic interactions [83–85,26]. The scattering amplitude M between charged particles can be written as

$$M = M_{em} + M_N \ , \tag{8}$$

where M_{em} is the electromagnetic amplitude [38] and M_N the nuclear amplitude. Forward dispersion relations cannot be written down for this nuclear amplitude, because it contains too many remnants of the electromagnetic interaction. For example, this amplitude M_N does not have a pion pole, but it has a branchpoint singularity like $1/(k^2+m_\pi^2)^{1+i\eta}$, where $\eta = \alpha/v_{lab}$ is the relativistic Rutherford parameter [86]. By removing all Coulomb effects from M_N one can construct a new amplitude M_N', for which one could possibly write down forward dispersion relations.

A somewhat similar situation exists for the ${}^1S_0$ pp scattering length. Here one also likes to remove all Coulomb effects. This time from the pp scattering length [87] $a_c(pp) = -7.806(3)$ fm, such that the resulting nuclear pp scattering length $a_N(pp) \simeq -17.3$ fm can be compared to the np scattering length [58] $a(np) = -23.738$ fm. The latter is obtained at very low energies from the effective-range function $F(p^2) = p\cot\delta$ by the zero-range expansion $F(p^2) = -1/a$. We will compare $a_N(pp)$ with M_N'. The pp scattering length $a_{em}(pp)$ which is obtained from a much more complicated effective-range function [88–91] (including effects of the full electromagnetic interaction) must then be compared with M_N. Various pp scattering lengths, a_c, a_E, and a_{em}, exist. They all arise from different treatments of the electromagnetic interaction (including vacuum polarization, relativistic effects, etc.) in the construction of effective-range functions. We will explain below that there are also different definitions of M_N.

Very important in the treatments of forward dispersion relations has been the optical theorem. However, big question marks must be placed by the application of the optical theorem in reactions between charged particles. Because of the presence of the long-range Coulomb interaction (with screening effects neglected) the total cross section σ_T is infinite. This is a well-known result, but the consequences are sometimes not properly understood [92,93]. To get finite results one has to remove in one way or another the Coulomb interaction. That this is a far from trivial and model-dependent procedure is often not appreciated. One then defines such theoretical concepts as the "total hadronic cross section" σ_T^h. This is *not* a measurable experimental quantity, but a theorist's invention. Moreover, one has to realize that the hadronic cross section is related to M_N and *not* to M_N'. Therefore the hadronic cross section is *not* related to $\mathrm{Im}\, M_N'$ and cannot be used in the dispersion integrals. Hence, the optical theorem cannot be used in connection with M_N'.

The scattering amplitude M between charged particles was written down in Eq. (8), where the electromagnetic amplitude M_{em} is given by [38]

$$M_{em} = M_{c_1} + M_{c_2} + M_{mm} + M_{vp} \ . \tag{9}$$

Here M_{c_1} is the standard Coulomb amplitude, M_{c_2} a two-photon exchange contribution, M_{mm} the magnetic-moment contribution, and M_{vp} the vacuum-polarization contribution. The standard Coulomb amplitude M_{c_1} can be written either as the divergent partial-wave series [94–96]

$$M_{c_1} = \sum_{\ell=0}^{\infty}(2\ell+1)\ \frac{e^{2i\xi_\ell}-1}{2ip}\ P_\ell(x)\ , \tag{10}$$

where $x=\cos\theta$ and the phases ξ_ℓ will be discussed below, or as the sum

$$M_{c_1} = \frac{\eta}{2p}\ \frac{1}{(1-x)^{1+i\eta}}\ e^{2i\Phi}\ F(k^2)\ . \tag{11}$$

The nuclear amplitude is given by

$$M_N = \sum_{\ell=0}^{\infty}(2\ell+1)\, e^{2i\sigma_{em,\ell}}\, \frac{e^{2i\delta_\ell}-1}{2ip}\, P_\ell(x) \ . \tag{12}$$

Now there are choices that must be made. Do we want to include in Eq. (11) a form factor $F(k^2)$, or do we take $F \equiv 1$? When we include a form factor, then we need to find a way to calculate the corresponding Coulomb phases ξ_ℓ. Only in the case of point charges we know these Coulomb phases analytically: $\xi_\ell = \sigma_\ell \equiv \Gamma(\ell+1+i\eta)$. Because of the uncertainty of how to handle the long-range screening of the Coulomb potential, the overall phase Φ in Eq. (11) is really unknown. There are two standard ways [94–96] to treat this for point charges: either we choose $\Phi = \sigma_0$, then $\xi_\ell = \sigma_\ell$, or we choose $\Phi = 0$, but then $\xi_\ell = \sigma_\ell - \sigma_0$. In Nijmegen we opted for the latter choice. The $\sigma_{em,\ell}$ in Eq. (12) is the sum of the choice for ξ_ℓ of the standard Coulomb amplitude Eq. (10), the two-photon phase [75] ρ_ℓ, the magnetic-moment phase [37] ϕ_ℓ, and the vacuum-polarization phase [97] τ_ℓ.

We hope that we made it clear that the phase of the Coulomb amplitude is "theorist dependent." The same holds then for the nuclear amplitude. This means that therefore also the imaginary part of the nuclear amplitude is theorist dependent. This has repercussions for the optical theorem. Consider the optical theorem in the case of purely elastic scattering. When f_ℓ is the partial-wave nuclear scattering amplitude and $\sigma^{(\ell)}_{elas}$ the hadronic cross section in this partial wave, then

$$\sigma^{(l)}_{elas} \sim |f_\ell|^2 = \sin^2\delta_\ell \quad \text{and} \quad \mathrm{Im}\, f_\ell = \sin\delta_\ell \sin(\delta_\ell + 2\sigma_{em,\ell}) \ . \tag{13}$$

Clearly, $\mathrm{Im}\, f_\ell \neq |f_\ell|^2$: The optical theorem for these partial-wave amplitudes and for the amplitude M_N is obviously *not* valid.

Let us now return to the *pp* forward dispersion relations. One cannot write down a dispersion relation for the forward scattering amplitude $M_N(x=1)$. First, one needs to correct this amplitude for the electromagnetic interaction. This is the same type of correction as for the *pp* scattering length, only it has to be done for *all* partial waves at *every* energy. These corrections are difficult to make and are often not very acurate. Doing this with Gamow factors is certainly too inaccurate. In the physical region below the pion-production threshold the nuclear amplitude is constructed using a PWA. In that PWA a choice was made for the πNN coupling constant. Above the pion-production threshold the treatment makes essential use of the optical theorem. But we did show that the optical theorem is not really valid for M_N nor for M'_N. In the physical region of the $\overline{N}N$ channel one again needs the optical theorem. The largest uncertainty comes from the treatment of the unphysical cut. Many parameters need to be introduced in order that the fixed-t dispersion relations can describe the experimental data with sufficient accuracy. This can only be done in a PWA of the *pp* scattering data. Such work could be performed, but it will require such an enormous effort that it probably will not be done soon. Again such a PWA requires as input a value for the πNN coupling constant. We therefore come to the conclusion that an accurate determination of the $\pi^0 pp$ coupling constant in forward *pp* dispersion relations is not very realistic. It may not be impossible, but it will require many man-years of hard and tedious work.

IX. CONCLUSIONS

We have seen that there is overwhelming evidence that the charge-independent value for the πNN coupling constant is $f^2 = 0.0750$ with a total error of perhaps less than 9 in the last digit. This error includes possible systematic errors. We think that the present accuracy is such that the determination of charge-independence breaking effects is soon within reach.

We would like to comment here on another, much-quoted result of the Karlsruhe-Helsinki analysis [98]: the value $\kappa_\rho = 6.1$ of the vector-to-tensor ratio of the ρNN coupling constants. Vector-meson dominance of the electromagnetic form factors gives $\kappa_\rho = 3.7$. The Nijmegen soft-core potential Nijm78 [34] produces $\kappa_\rho = 4.2$; its update Nijm93 [99] $\kappa_\rho = 4.1$. Because it is clear that the value of πNN coupling constant has changed a lot since 1975, one [43] can place also question marks by the more than 20 years old Karlsruhe value for κ_ρ. We may assume that the newer data will change that value.

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