THEF-NIJM 97.04

NN potential from chiral SU(3) Lagrangian

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(Proceedings of the 15th International Conference on Few-Body Problems in Physics, July 22–26, 1997, Groningen, The Netherlands;)

Abstract

We present a chiral-invariant SU(3) Lagrangian describing the interactions of the baryon octet with the lowest-mass meson nonets. Empirical estimates for the strengths of these vertices are given. The nonlinear realization of the chiral symmetry generates pair-meson interaction vertices, the coupling constants for which contain no new free parameters. A baryon-baryon potential is evaluated and its quality is examined for the *NN* sector.

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I. INTRODUCTION

The construction of baryon-baryon potentials has a long history. Despite an enormous amount of effort, it has not yet been possible to construct potential models which give a high-quality description of the empirical data, obey the symmetries of QCD, and at the same time contain only a limited number of free parameters. The familiar one-boson-exchange potentials (Nijmegen, Paris, Bonn) contain a relatively small number of free parameters, but their description of the scattering data can only be classified as reasonable. On the other hand, the recent high-quality *NN* potentials (Nijm I, Nijm II, Reid93, CD-Bonn, AV18) give a very good description of the empirical scattering data, but they contain a large number of purely phenomenological parameters. A recent attempt to impose the symmetries of QCD using an effective Lagrangian of pions and nucleons [1] failed in the sense that it only captures the qualitative features of the nuclear interaction and in no way can compete with the much more successful potential models mentioned above.

In our study of two-meson-exchange (TME) contributions [2], we found that the inclusion of TME potentials provides a significant improvement over the more traditional one-meson-exchange models. Furthermore, the pair-meson vertices in the triangle and bubble TME diagrams are in principle dictated by chiral symmetry, and so the TME contributions do not require the introduction of new free parameters. The pair-meson potentials can be viewed as the result of integrating out the heavy-meson and resonance degrees of freedom. For low energies the heavy-meson propagators can very well be approximated by constants, leading directly to pair-meson vertices.

Here we try to combine the idea of a QCD-inspired Lagrangian with the success of traditional meson-exchange potentials. Rather than integrating out all mesons other than the pion, we here keep *all* lowest-mass mesons up to $\sim$1 GeV. The potential is then obtained by evaluating the standard one-meson-exchange contributions, but now also including the box and crossed-box two-meson exchanges and the pair diagrams where at least one of the baryon lines contains a pair-meson vertex. In principle, all coupling constants can be fixed using empirical estimates and symmetry arguments, where the only free parameters are the cutoff masses needed to regularize the potential, and higher-order extensions as allowed by the symmetry. The latter are to be determined in a fit to the *NN* scattering data.

II. THE MODEL

The octet of baryon fields containing the nucleon, Λ, Σ, and Ξ fields is represented by a 3×3 traceless matrix Ψ, where the left- and right-handed components transform as

$$\Psi_L \to L\Psi_L L^\dagger, \quad \Psi_R \to R\Psi_R R^\dagger \tag{1}$$

under the $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R$ symmetry. The octet of pseudoscalar mesons (π, η_8, K) and the nonet of scalar mesons (σ, a_0, f_0, κ) are grouped into

$$\begin{aligned}\Sigma &= \lambda_0\sigma_0 + \lambda_a\sigma_a + i\lambda_a\pi_a \\ &= F + \lambda_0 s_0 + \lambda_a s_a + i\lambda_a\pi_a \quad (a = 1,\ldots,8),\end{aligned} \tag{2}$$

which transforms as $\Sigma \to L\Sigma R^\dagger$. The isoscalar singlet $\sigma_0 = \langle\sigma_0\rangle + s_0$ and octet $\sigma_8 = \langle\sigma_8\rangle + s_8$ fields are assumed to have nonvanishing vacuum expectation values (v.e.v.'s), which are collected in a diagonal matrix $F = \mathrm{diag}(f_1, f_1, f_2)$ with

$$f_1 = \sqrt{\tfrac{2}{3}}\langle\sigma_0\rangle + \sqrt{\tfrac{1}{3}}\langle\sigma_8\rangle, \quad f_2 = \sqrt{\tfrac{2}{3}}\langle\sigma_0\rangle - 2\sqrt{\tfrac{1}{3}}\langle\sigma_8\rangle. \tag{3}$$

The v.e.v. for the singlet field is introduced to break the chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry to $\mathrm{SU}(3)_V$, and the v.e.v. for the isoscalar octet to further break the $\mathrm{SU}(3)_V$ symmetry [3]. The simplest possible interaction Lagrangian then reads

$$\begin{aligned}\mathcal{L}_I &= -g\mathrm{Tr}(\overline{\Psi}_L\Sigma\Psi_R\Sigma^\dagger + \overline{\Psi}_R\Sigma^\dagger\Psi_L\Sigma) \\ &\sim -g\overline{\Psi}_N(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\Psi_N + \ldots\end{aligned} \tag{4}$$

Defining a nonlinear transformation [4] $u(\xi_a)$, we can transform away the nonderivative pseudoscalar coupling while leaving the v.e.v. matrix F unchanged. It is given by

$$\left.\begin{aligned} B_R &= u\Psi_R u^\dagger \\ B_L &= u^\dagger\Psi_L u \end{aligned}\right\}, \quad u(\xi_a) = \exp[i\lambda_a\xi_a] \equiv \exp\left[\frac{i\lambda_a\pi'_a}{2f_1}\right], \tag{5}$$

where the primed fields are complicated expressions in terms of the original scalar and pseudoscalar fields. Since the primed fields behave as pseudoscalar fields, we will simply identify them with the octet of physical pseudoscalar fields, i.e.

$$\frac{1}{\sqrt{2}}\lambda_a\pi'_a = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\dfrac{2\eta_8}{\sqrt{6}} \end{pmatrix}. \tag{6}$$

The new octet of baryons, transforming as $B \to HBH^\dagger$ in this representation, is given by

$$B = \begin{pmatrix} \dfrac{\Sigma^0}{\sqrt{2}} + \dfrac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\dfrac{\Sigma^0}{\sqrt{2}} + \dfrac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & -\Xi^0 & -\dfrac{2\Lambda}{\sqrt{6}} \end{pmatrix}. \tag{7}$$

The kinetic-energy term for the baryons now transforms into

$$\begin{aligned}&\mathrm{Tr}(\overline{\Psi}_L i\gamma^\mu\partial_\mu\Psi_L + \overline{\Psi}_R i\gamma^\mu\partial_\mu\Psi_R) \to \\ &\quad \mathrm{Tr}(\overline{B}i\gamma^\mu D_\mu B) - \mathrm{Tr}(\overline{B}\gamma^5\gamma^\mu u_\mu B),\end{aligned} \tag{8}$$

where the derivative $D_\mu B = \partial_\mu B + i[\Gamma_\mu, B]$ contains the connection

$$\Gamma_\mu = -\tfrac{1}{2}i(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger) \sim \frac{1}{4f_1^2}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi} + \ldots \tag{9}$$

and hence pair interactions. The second term involves

$$u_\mu = -\tfrac{1}{2} i (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) \sim \frac{1}{2f_1} \boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi} + \ldots \tag{10}$$

implying the pseudovector (derivative) coupling for the pseudoscalar fields.

Using the transformation properties of the new fields we can define the combination

$$\chi_+ = \tfrac{1}{2}(u^\dagger \Sigma u^\dagger + u \Sigma^\dagger u), \tag{11}$$

and an interaction Lagrangian

$$\mathcal{L}_I = -g_{s,1} \mathrm{Tr}(\overline{B} \chi_+ B) - g_{s,2} \mathrm{Tr}(\overline{B} B \chi_+). \tag{12}$$

The combination χ_+ behaves like a nonet of scalar fields, and so we can simply *define* these fields to be the physical scalar fields and drop any reference to the original fields. The v.e.v. of the σ_0 field alone would imply a common mass $M = (g_{s,1} + g_{s,2}) f$ for all the baryons in the baryon octet, but the additional assignment of a v.e.v. to σ_8 now allows for the appropriate mass splittings within the baryon octet.

Vector and axial-vector mesons are introduced by extending the global chiral symmetry to a local one, which introduces the corresponding left- and right-handed gauge fields. The mass Lagrangian for these spin-1 fields and a fit to the empirical vector and axial-vector meson masses provides values for the f_1 and f_2 v.e.v.'s of the scalar mesons.

Given the transformation properties of the baryon and meson fields and introducing singlets to complete the pseudoscalar and vector nonets, we can define the chiral-invariant combinations

$$\begin{aligned}
\left[\overline{B} B \Phi\right]_D &= \mathrm{Tr}(\overline{B} \Phi B) + \mathrm{Tr}(\overline{B} B \Phi) - \tfrac{2}{3} \mathrm{Tr}(\overline{B} B) \mathrm{Tr}(\Phi), \\
\left[\overline{B} B \Phi\right]_F &= \mathrm{Tr}(\overline{B} \Phi B) - \mathrm{Tr}(\overline{B} B \Phi), \\
\left[\overline{B} B \Phi\right]_S &= \mathrm{Tr}(\overline{B} B) \mathrm{Tr}(\Phi),
\end{aligned} \tag{13}$$

and hence the general interaction Lagrangian reads

$$\begin{aligned}
\mathcal{L}_I = &-g^{\mathrm{oct}} \sqrt{2} \left\{ \alpha \left[\overline{B} B \Phi\right]_F + (1-\alpha) \left[\overline{B} B \Phi\right]_D \right\} \\
&-g^{\mathrm{sin}} \sqrt{\tfrac{1}{3}} \left[\overline{B} B \Phi\right]_S ,
\end{aligned} \tag{14}$$

where α is known as the $F/(F+D)$ ratio. We furthermore have to introduce mixing angles θ to make a connection between the pure singlet and octet isoscalar fields and the physical meson fields. All baryon-baryon-meson coupling constants for each type of meson (scalar, pseudoscalar, vector) can be expressed in terms of the four parameters g^{oct}, g^{sin}, α, and θ. The transformation of the vector-meson fields requires the condition $\alpha_V = 1$, the so-called universality condition proposed by Sakurai [5]. Also, all other parameters can in principle be fixed beforehand [3].

To second order in the meson fields the nonlinear transformation $u(\xi_a)$ gives rise to pair interactions of the form $\gamma^\mu (\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi})$, $\gamma^5 \gamma^\mu (\boldsymbol{\pi} \times \boldsymbol{\rho}_\mu)$, etc., where all coupling constants can be readily expressed in terms of the single-meson coupling constants.

III. RESULTS

The interaction vertices are next used in a potential model, where we restrict ourselves to maximally two mesons in the intermediate state and a total meson mass of about 1 GeV. This means that for the two-meson exchanges we only consider the π-meson, KK, and $\sigma\sigma$ exchanges. (The σ is taken to be a very broad scalar resonance where the potential due to its exchange can be successfully approximated by a sum of two stable effective mesons [6], one of them having a mass less than 500 MeV.) The potential due to each exchange needs to be regularized with an exponential cutoff. We allow for a different cutoff for each type of meson (scalar, pseudoscalar, vector). However, it then still is impossible to arrive at an acceptable fit to the *NN* scattering data. We therefore also introduce a diffractive contribution [7] and a further chiral-invariant interaction piece, quadratic (to lowest order) in the pseudoscalar fields [3]. The latter introduces pair interactions of the form

$$g^{\mu\nu}\partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi}, \quad \sigma^{\mu\nu}\partial_\mu\boldsymbol{\pi}\times\partial_\nu\boldsymbol{\pi}, \tag{15}$$

which *do* have free parameters in front of them.

The 12 free parameters allow for a very satisfactory fit of $\chi^2/\text{data} = 1.35$ for the *NN* data below 300 MeV, which is significantly better than other meson-theoretical models that have appeared in the literature, and which use roughly the same number of free parameters. In contrast to these other models, it should also be realized that in the present model *all* coupling constants satisfy constraints as imposed by chiral symmetry or empirical constraints from other sources. Obviously, further improvements can be made by relaxing some of the constraints on the coupling constants.

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