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Chiral two-pion exchange and proton-proton partial-wave analysis*

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Abstract

The chiral two-pion exchange component of the long-range *pp* interaction is studied in an energy-dependent partial-wave analysis. We demonstrate its presence and importance, and determine the chiral parameters c_i $(i = 1, 3, 4)$. The values agree well with those obtained from pion-nucleon amplitudes.

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The longest-range part of the strong nucleon-nucleon (NN) interaction is the well-established one-pion exchange (OPE) force [1,2]. Next in range is the two-pion exchange (TPE) force, the formulation of which has been a long-standing problem [3], both in field theory [4,5] and in dispersion theory [6]. In recent years, it has been argued that the key to the solution is the chiral symmetry of QCD [7–9], and that the long-range parts of the TPE potential can be derived model-independently by a systematic expansion of the effective chiral Lagrangian [8]. In this Letter, we will study this long-range chiral TPE force in the proton-proton (pp) interaction and show unambiguously its presence and its importance.

In the energy-dependent Nijmegen partial-wave analyses (PWA's) of the NN and $\overline{N}N$ scattering data [10–13], the long-range forces are taken into account exactly and the short-range forces are parametrized analytically. The partial-wave scattering amplitudes are analytic functions of the energy. The nearby left-hand singularities in the complex-energy plane are due to the long-range forces; these cause the rapid energy dependence of the physical NN scattering amplitudes. The shorter-range forces are responsible for the far-away singularities, which give in the physical region only slow energy variations of the amplitudes. This method of PWA can serve as a sensitive tool to investigate precisely these long-range interactions. It has been used successfully in studies of electromagnetic interactions [14] and of the OPE potential [2,15–17]. Here this tool will again be employed, now to study the long-range chiral TPE component of the pp force.

The methods of the Nijmegen PWA's are described in detail in Refs. [10–13]. The long-range potentials, including the full electromagnetic interaction (relativistic Coulomb, magnetic-moment interaction, and vacuum polarization) and the longest-range strong interactions are used in the relativistic Schrödinger equation which is solved with a boundary condition (BC) at some $r = b$. This BC is parametrized as an analytic function of energy for the various partial waves. The BC parameters, representing short-range physics, and the free parameters in the long-range forces (*e.g.* the pion-nucleon coupling constant) are determined from a fit to the data. In the "standard" Nijmegen PWA's of Refs. [11,12] the boundary is put at $b = 1.4$ fm, and the long-range strong potential outside of 1.4 fm is taken as the OPE potential supplemented by the non-OPE forces of the Nijmegen soft-core potential Nijm78 [18]. These heavy-boson exchanges were included because OPE alone did not allow for an optimal description of the data. In this standard pp PWA, we obtain with 19 BC parameters $\chi^2_{\rm min} = 1968.7$ and $f^2_{pp\pi^0} = 0.0756(4)$, where the error is statistical, on the Nijmegen 1998 pp database below 350 MeV, in which 1951 pp scattering data are included [19]. This result will serve here as a benchmark.

Let us demonstrate our method first with some parts of the electromagnetic interaction. When one omits in the standard 1998 pp PWA the magnetic-moment interaction, both from the potential and in constructing the scattering amplitude, the $\chi^2_{\rm min}$ increases by 390.0 to $\chi^2_{\rm min} = 2358.7$. This is therefore a 19.7 standard deviation (s.d.) effect. Omitting vacuum polarization leads to $\chi^2_{\rm min} = 2181.3$, *i.e.*, a rise in $\chi^2_{\rm min}$ of 212.6, which corresponds to 14.6 s.d. These numbers demonstrate that one can use this method of energy-dependent PWA to show the presence and the importance of these specific well-known parts of the long-range pp interaction.

A very important part of the energy dependence of the NN phase shifts comes from OPE. In the Nijmegen energy-dependent PWA's the different pion-nucleon coupling constants could be determined accurately and reliably [2,15,16]. In Ref. [16], we recommended for the charge-independent coupling constant the value $f^2_{NN\pi} = 0.0750(9)$, where the error includes statistical as well as systematic effects. As a systematic check, the masses of the exchanged pions were determined, with excellent results: $m_{\pi^0} = 135.6(1.0)$ MeV and $m_{\pi^+} = 139.6(1.3)$ MeV. In this way, the presence of OPE in the NN force was shown with an enormous statistical significance. A more subtle effect is the energy dependence of the OPE potential due to the minimal-relativity factor M/E, where M is the proton mass and E the proton center-of-mass energy. Omitting this factor from the OPE potential results in $\chi^2_{\rm min} = 1977.2$. This is a rise of 8.5 in $\chi^2_{\rm min}$, or an almost 3 s.d. effect. Recently, also the electromagnetic corrections to the OPE potential in np scattering were investigated [17].

The starting point to derive the OPE and TPE potentials is the effective chiral Lagrangian, the leading-order of which is the nonlinear Weinberg model [20],

$$\mathscr{L}^{(0)} = -\overline{N}\left[\gamma_\mu \mathscr{D}^\mu + M + g_A i\gamma_5\gamma_\mu\, \vec{\tau}\cdot\vec{D}^\mu\right] N \ , \tag{1}$$

with the chiral-covariant derivatives [7]

$$\begin{aligned}\vec{D}^\mu &= D^{-1}\partial^\mu\vec{\pi}/F_\pi \ , \\ \mathscr{D}^\mu N &= \left(\partial^\mu + \frac{i}{F_\pi}c_0\,\vec{\tau}\cdot\vec{\pi}\times\vec{D}^\mu\right) N \ .\end{aligned} \tag{2}$$

Here, $D = 1 + \vec{\pi}^2/F^2_\pi$, $g_A = 1.2573$ is the Gamow-Teller coupling, and $F_\pi = 185$ MeV is the pion decay constant; chiral symmetry fixes $c_0 \equiv 1$. Eq. (1) implies that the planar- and crossed-box TPE diagrams should be calculated with the pseudovector (PV) $NN\pi$ Lagrangian. We use the physical $NN\pi$ coupling constant f, *i.e.*, we trade in the Goldberger-Treiman value g_A/F_π for $\sqrt{4\pi}f/m_s$; the scaling mass m_s serves to make f dimensionless and is conventionally chosen to be numerically equal to the charged-pion mass, $m_s \equiv m_{\pi^+}$. In addition to the PV $NN\pi$ interaction, Eq. (1) contains the Weinberg-Tomozawa (WT) $NN2\pi$ seagull interaction [21], resulting in triangle and football TPE diagrams.

In order to derive the TPE potential in subleading order, three more $NN2\pi$ interactions are required [8], *viz.*

$$\begin{aligned}\mathscr{L}^{(1)} = -\overline{N}\Big[\, & 8c_1 D^{-1} m^2_\pi \vec{\pi}^2/F^2_\pi + 4c_3\, \vec{D}_\mu\cdot\vec{D}^\mu \\ & + 2c_4\, \sigma_{\mu\nu}\, \vec{\tau}\cdot\vec{D}^\mu\times\vec{D}^\nu\Big] N \ ,\end{aligned} \tag{3}$$

leading to additional triangle diagrams. The values of the chiral parameters ("low-energy constants") c_i $(i = 1,3,4)$ of order $(1/M)$ are not fixed by chiral symmetry; the c_i's represent "integrated-out" hadrons, such as the heavier mesons like the ε and ϱ, and the N- and Δ-isobars. The definition Eq. (3) of these c_i's [22] agrees with the convention used in heavy-baryon χPT [23,24]; an additional c_2-term does not contribute to the NN force in this order. The c_1-term violates chiral symmetry explicitly. A systematic expansion of Eqs. (1) and (3) to order $(1/M)$ gives the relevant part of the chiral Lagrangian [25].

The OPE and TPE potentials derived from this Lagrangian contain central, spin-spin, tensor, and spin-orbit terms, *viz.*

$$V = V_C + V_S\, \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_T\, S_{12} + V_{SO}\, \boldsymbol{L}\cdot\boldsymbol{S} \ , \tag{4}$$

where $S_{12} = 3\,\boldsymbol{\sigma}_1\cdot\widehat{\boldsymbol{r}}\,\boldsymbol{\sigma}_2\cdot\widehat{\boldsymbol{r}} - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$. With $i = C, S, T, SO$, $\xi = m_\pi/m_s$, and $x = m_\pi r$, we can write

$$V_i(r) = f^{2n}\,\xi^{2n}\,(M/E)\,[v_i(x) + w_i(x)\vec{\tau}_1\!\cdot\!\vec{\tau}_2]\,m_\pi\ , \tag{5}$$

where $n = 1$ for OPE and $n = 2$ for TPE.

The long-range OPE potential contains an isovector spin-spin part w_S and an isovector tensor part w_T,

$$\begin{aligned} w_S(x) &= e^{-x}/3x\ ,\\ w_T(x) &= \left(1 + x + x^2/3\right) e^{-x}/x^3\ . \end{aligned} \tag{6}$$

For the pp case, the neutral-pion mass m_{π^0} is used in OPE. The coupling $f_p^2 = f_{pp\pi^0}^2$ is a free parameter.

For TPE, the dimensionless isoscalar functions v_i are written as the sum of the leading-order terms $v_{i,1}$ and the subleading-order terms $v_{i,2}$,

$$v_i(x) = (2/\pi)\ v_{i,1}(x) + (m_\pi/M)\ v_{i,2}(x)\ , \tag{7}$$

and similarly for the isovector functions w_i. In the TPE potential, we use the average pion mass $m_\pi = 138.04$ MeV and the fixed charge-independent coupling constant is $f^2 = f_{NN\pi}^2 = 0.0750$. Care must be taken to obtain the appropriate form for the use of Eq. (5) in the relativistic Schrödinger equation. Other forms of the OPE potential or other two-body equations will, in general, give different TPE potentials [5,26].

The leading-order static potential TPE(l.o.) contains isoscalar spin-spin and tensor terms, $v_{S,1}$ and $v_{T,1}$ respectively, and an isovector central component $w_{C,1}$. The long-range parts are

$$\begin{aligned} v_{S,1}(x) &= 12K_0(2x)/x^3 + (12 + 8x^2)K_1(2x)/x^4\ ,\\ v_{T,1}(x) &= -12K_0(2x)/x^3 - (15 + 4x^2)K_1(2x)/x^4\ ,\\ w_{C,1}(x) &= \left(\tilde{c}_0^2 + 10\tilde{c}_0 - 23 - 4x^2\right) K_0(2x)/x^3 +\\ &\quad\ \left(\tilde{c}_0^2 + 10\tilde{c}_0 - 23 + (4\tilde{c}_0 - 12)x^2\right) K_1(2x)/x^4\ , \end{aligned} \tag{8}$$

where the modified Bessel functions have asymptotic behavior $K_n(2x) \sim \sqrt{\pi/4x}\,e^{-2x}$. This TPE(l.o.) is the "TMO" potential [27], supplemented by the diagrams with the WT seagulls [26,28]. In the WT terms we extracted, for ease of presentation, an overall factor f^4, cf. Eq. (5), and defined $\tilde{c}_0 = c_0/\tilde{g}_A^2$, where $\tilde{g}_A = F_\pi\sqrt{4\pi}f/m_s$.

The subleading-order potential TPE(s.o.) contains nonstatic terms from Eq. (1) and the leading-order terms from Eq. (3). The long-range parts read

$$v_{i,2}(x) = \textstyle\sum_{p=1}^{6} a_p\, e^{-2x}/x^p\ , \tag{9}$$

and similarly for $w_{i,2}$, with the coefficients a_p as collected in Table I. Also here a factor f^4 was extracted and the result was rewritten in terms of $\tilde{c}_0$ and $\tilde{c}_i = c_i M/\tilde{g}_A^2$. Our results for TPE(s.o.) agree with Ref. [29].

Remarkably, a large part of the correct TPE potential was already obtained by Sugawara and Okubo [30] in "pre-chiral days," by using PV coupling and two phenomenological $NN2\pi$

interactions: the WT term of Eq. (1) and the c_1-part of Eq. (3). They also pointed out that PV coupling gives a rather strong attractive isoscalar spin-orbit force in subleading order. However, the important additionally required chiral c_3- and c_4-terms were missing; these were for the *NN* case first given in Ref. [8].

We now come to the results of the TPE studies, in which we again use the 1998 database below 350 MeV, with 1951 data [19]. The main results of the various PWA's are summarized in Table II. We start conservatively with the boundary at $b = 1.8$ fm, since beyond 1.8 fm only OPE and TPE are expected to contribute significantly. When only OPE is included as strong force, $\chi^2_{\rm min} = 1956.6$ is reached at the cost of 29 BC parameters. We want to investigate if the fit can be even further improved when TPE is added. When only the TPE(l.o.) potential of Eq. (8) is used, we obtain $\chi^2_{\rm min} = 1965.9$ with 26 BC parameters. But we can do better. The complete TPE potential, χTPE = TPE(l.o.) + TPE(s.o.), contains three *a priori* unknown constants: the chiral parameters c_i ($i = 1, 3, 4$) from Eq. (3). In the fits we obtain $c_1 = -4.4(3.4)$/GeV. The values of c_1 and c_3, appearing both only in the isoscalar central potential, cf. Table I, are strongly correlated. The correlations between the parameters can be summarized concisely by:

$$\begin{aligned} c_3 &= \left[-5.08 - 0.62(c_1 + 0.76) + 40(f_p^2 - 0.0755)\right]/\mathrm{GeV}, \\ c_4 &= \left[+4.70 + 0.01(c_1 + 0.76) + 250(f_p^2 - 0.0755)\right]/\mathrm{GeV}. \end{aligned}$$

In order to determine reliable values for c_3 and c_4, we use the theoretical estimate [23] for c_1 obtained from the scalar form factor $\sigma(t)$ of the proton [31] at $t = 0$, *viz.*

$$c_1 = -\left[\sigma(0)/4m_\pi^2 + 9f^2\xi^2/16m_\pi\right] \; ; \tag{10}$$

$\sigma(0)$ is the pion-nucleon sigma term, the value of which is uncertain. We take here the plausible "low" value $\sigma(0) = 35(5)$ MeV [32], which is supported by the recent πN PWA of Ref. [33]. This gives

$$c_1 = -[0.46(7) + 0.30]/\mathrm{GeV} = -0.76(7)/\mathrm{GeV} \; ; \tag{11}$$

the error here is theoretical. Our determination of c_1 is consistent with this value. Fixing $c_1 = -0.76$/GeV, we find, with 22 BC parameters, $\chi^2_{\rm min} = 1937.8$ and $f_p^2 = 0.0755(7)$; the resulting values for c_3 and c_4 are

$$c_3 = -5.08(28)/\mathrm{GeV} \; , \quad c_4 = +4.70(70)/\mathrm{GeV} \; , \tag{12}$$

where the errors are statistical. The improvement over only OPE is reflected, even beyond 1.8 fm, in the 18.8 lower $\chi^2_{\rm min}$ and in the 7 fewer BC parameters required.

The result found for f_p^2 is in very good agreement with the value 0.0756(4) determined in the standard 1998 *pp* PWA. Our values for the c_i's can be compared to the determination from the πN scattering amplitudes in Ref. [34]. Here, $c_1 = -0.93(9)$/GeV was obtained using Eq. (10), but with $\sigma(0) = 45(8)$ MeV, along with $c_3 = -5.29(25)$/GeV and $c_4 = +3.63(10)$/GeV. In view of the uncertainties in the πN amplitudes [33], the good agreement is a significant success. It underlines, for the first time quantitatively, that the long-range *NN* and the low-energy πN interactions are governed by the same chiral Lagrangian.

In previous studies of the OPE potential, a good systematic check has been the determination of the masses of the exchanged pions. In order to check explicitly that we are now actually looking at the TPE interaction, we determine the range. This is done by adding the pion mass m_π in the potential χTPE as another free parameter. We first fix the pion coupling in OPE at $f_p^2 = 0.0755$ and the c_i's to their central values given in Eqs. (11) and (12). Then we fit an overall scale factor λ for the potential χTPE, the pion mass m_π, and the BC parameters. The results are: $\lambda = 0.82(16)$ and $m_\pi = 125(10)$ MeV. Alternatively, we fix c_1 and fit m_π together with f_p^2, c_3, c_4, and the BC parameters. This results in $m_\pi = 128(9)$ MeV, again in good agreement with the average pion mass $m_\pi = 138.04$ MeV. The very good $\chi^2_{\rm min}$ obtained, the good values for the c_i's, and this correct pion mass constitute convincing proof for the presence of chiral TPE loops in the long-range *pp* interaction.

In order to investigate further the importance of χTPE, we move the boundary inwards to $b = 1.4$ fm. When only OPE is used as long-range force, it is possible to achieve a reasonable fit: at the cost of 31 BC parameters $\chi^2_{\rm min} = 2026.2$ is reached. We then add to OPE the potential TPE(l.o.). With 28 BC parameters, $\chi^2_{\rm min} = 1984.7$ is obtained. Compared to only OPE, this corresponds to a drop in $\chi^2_{\rm min}$ of 41.5 with 3 fewer parameters, a significant improvement. However, the fit is still not optimal [35]. We next add also the potential TPE(s.o.). With fixed $c_1 = -0.76$/GeV, this gives with 23 BC parameters $\chi^2_{\rm min} = 1934.5$, $c_3 = -4.99(21)$/GeV, and $c_4 = +5.62(59)$/GeV. This shows that OPE together with χTPE gives a very good *NN* force at least as far inwards as 1.4 fm.

In conclusion, we have, for the first time, incorporated and studied chiral TPE in an energy-dependent PWA of the *pp* scattering data. The main result of this Letter is that we have shown the presence of chiral TPE loops in the long-range *pp* interaction. A significant improvement over using just OPE is seen. With OPE and χTPE, an excellent fit to the database becomes possible, even somewhat better than the standard 1998 *pp* PWA. The chiral parameters agree with those found in πN scattering. Especially important in obtaining the very good fit is the isoscalar central attraction from the c_3-term, partly a "chiral van-der-Waals force" due to the axial polarizability of the nucleon [36]. In all, our results provide a big success for chiral symmetry. A novel class of PWA has been established, with such a theoretically well-founded and model-independent chiral TPE potential included in all partial waves.

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TABLES

TABLE I. Coefficients of the subleading-order potential TPE(s.o.) of Eq. (9), for the central, spin-spin, tensor, and spin-orbit terms, both isoscalar and isovector. We defined $\tilde{c}_0 = c_0/\tilde{g}_A^2$, $\tilde{c}_i = c_i M/\tilde{g}_A^2$ for $i = 1, 3, 4$, and $\tilde{c}_{04} = \tilde{c}_0 + 4\tilde{c}_4$.

		a_p					
	i	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$
v	C	$3/4$	$9+48\tilde{c}_1+24\tilde{c}_3$	$27+96\tilde{c}_1+96\tilde{c}_3$	$99/2+48\tilde{c}_1+240\tilde{c}_3$	$54+288\tilde{c}_3$	$27+144\tilde{c}_3$
	S		-3	-9	$-33/2$	-18	-9
	T		$3/2$	$27/4$	15	18	9
	SO			-12	-36	-48	-24
w	C	$3/2$	$4-2\tilde{c}_0$	$14-8\tilde{c}_0$	$31-20\tilde{c}_0$	$36-24\tilde{c}_0$	$18-12\tilde{c}_0$
	S		$-2/3$	$-14/3+8\tilde{c}_{04}/3$	$-31/3+20\tilde{c}_{04}/3$	$-12+8\tilde{c}_{04}$	$-6+4\tilde{c}_{04}$
	T		$1/3$	$17/6-4\tilde{c}_{04}/3$	$26/3-16\tilde{c}_{04}/3$	$12-8\tilde{c}_{04}$	$6-4\tilde{c}_{04}$
	SO				$8-8\tilde{c}_0$	$16-16\tilde{c}_0$	$8-8\tilde{c}_0$

TABLE II. Results for the PWA's with different long-range interactions. #BC is the number of BC parameters.

	$b = 1.4$ fm		$b = 1.8$ fm	
	#BC	$\chi^2_{\rm min}$	#BC	$\chi^2_{\rm min}$
Nijm78	19	1968.7	–	–
OPE	31	2026.2	29	1956.6
OPE + TPE(l.o.)	28	1984.7	26	1965.9
OPE + χTPE	23	1934.5	22	1937.8